Reducibility and Thermal and Mass Scaling in Angular Correlations from Multifragmentation Reactions

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The azimuthal angular correlations of light charged particles and light intermediate mass fragments emitted from the reaction ^{36}Ar + ^{197}Au at $E/A = 50$ MeV are found to be reducible to the angular distributions of individual fragments. Thermal scaling is also observed in the coefficients of the angular correlations. Furthermore, the observed scaling with fragment mass seems to imply secondary emission from relatively small ($A \approx 15-30$) primary fragments. [S0031-9007(96)00701-6]

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Intermediate-mass-fragment (IMF) emission probabilities and IMF charge distributions were recently shown to be reducible to the corresponding one fragment quantities. Furthermore, a strong thermal scaling was shown to control the energy dependence of the same quantities [1–5].

The multifragmentation scenario painted by these experimental observations is that of a process controlled by a largely independent emission of individual fragments, which in turn is dominated by phase space.

Fragment-fragment angular correlations have been used to study the space-time extension of the emitting source. In particular, small angle repulsion (the Coulomb hole) has been interpreted in terms of the Coulomb repulsion of fragments emitted near each other, in both space and time.

We will show that, except at small angles, the particle-particle angular correlations and their dependence on excitation energy are interpretable in terms of nearly independently emitted fragments whose angular distributions are controlled by phase space. Thus we show that the angular correlations are *reducible* and *thermally scalable.* A mass scaling of the angular correlations will also be demonstrated and its possible implications discussed.

The evidence presented illustrates the role in multifragmentation of angular momentum, a variable not yet explored either experimentally or theoretically.

In pursuit of these ideas, we have explored the azimuthal correlations between emitted particles [6–15] defined by

$$\left.\frac{Y(\Delta\phi)}{Y'(\Delta\phi)}\right|_{\theta,E_t} = C[1 + R(\Delta\phi)]|_{\theta,E_t}\,. \qquad (1)$$

Here, $Y(\Delta\phi)$ is the coincidence yield of two particles emitted with relative azimuthal angle $\Delta\phi$ at a polar laboratory angle θ, and selected by the total transverse energy of an event ($E_t = \sum_i E_i \sin^2\theta_i$, where E_i and θ_i are the kinetic energy and polar angle of particle i in the event [14]); $Y'(\Delta\phi)$ is the background yield constructed by mixing particle yields from different events selected by identical cuts on E_t and θ; C is a normalization constant chosen so that the yields of Y and Y' integrated over $\Delta\phi$ are equal. All azimuthal correlation functions presented in this Letter were constructed from particles detected at $\theta = 31°-50°$. Software energy thresholds of $E_t/A = 3$ MeV were applied to all particles [16]. Pairs of particles extending from protons to carbons were considered.

Figure 1 shows azimuthal correlation functions of particle pairs of He nuclei (solid circles) [17] and mixed pairs consisting of He and Be (open circles) detected for four windows of E_t from the reaction ^{36}Ar + ^{197}Au at $E/A = 50$ MeV (details of the experiment can be found in Ref. [14]). Consistent with previous observations in somewhat different systems [8–13,15], the azimuthal correlation functions exhibit a slightly distorted V-shape pattern with a clear minimum at $\Delta\phi \approx 90°$. At larger excitation energies (assumed proportional to E_t) the correlations become progressively damped.

In an effort to understand the evolution of the correlation functions of Fig. 1, we have considered the exactly solvable problem of thermal particle emission from a rotating source. The classical probability of emitting a particle with reduced mass μ from the surface of a rotating system (of angular momentum I, moment of inertia $\Im$, temperature T, and distance R between the two centers of the "daughter" and emitted nuclei) in a direction given by polar angle θ (in the center of mass frame) and azimuthal angle ϕ (measured with respect to the reaction plane that is perpendicular to $\vec{I}$) is [18]

$$P(\theta,\phi) \propto \exp[-\beta \sin^2\theta \sin^2\phi], \qquad (2)$$

where

$$\beta = \frac{\hbar^2 I^2}{2\Im T}\frac{\mu R^2}{\Im + \mu R^2} = \frac{E_{\rm rot}}{T}\frac{\mu R^2}{\Im + \mu R^2} \qquad (3)$$

and $E_{\rm rot}$ is the rotational energy of the source.

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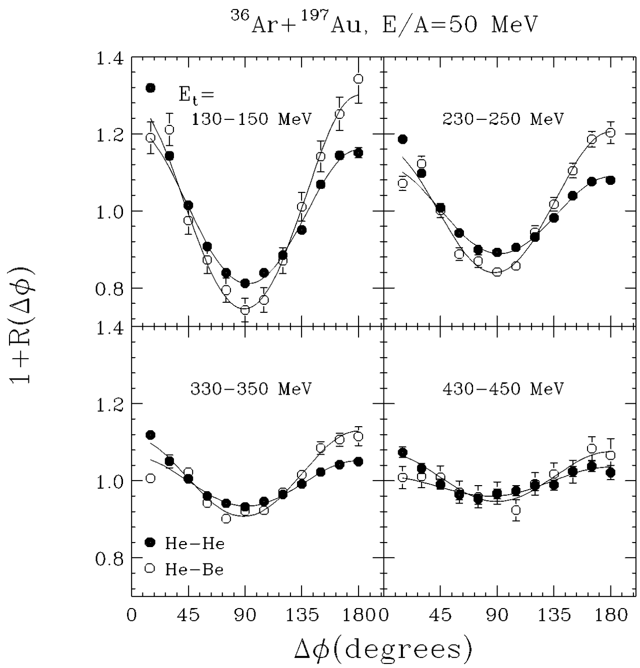

FIG. 1. Evolution of the azimuthal correlation functions of two He particles (solid circles) and He and Be particles (open circles) emitted at $\theta_{\rm lab} = 31°-50°$ for four different cuts on the transverse energy E_t. The solid lines are fits of the form given in Eq. (12).

The singles distribution of Eq. (2) comes from an extension of the angular distributions for fission. As in fission, particle emission in the angular momentum frame follows

$$P(\psi) \propto e^{-\mathcal{E}_{\rm rot}(K)/T}, \tag{4}$$

where $K = I\cos\psi$ is the projection of I on the separation axis of the scission configuration and

$$\mathcal{E}_{\rm rot} = \frac{I^2 - K^2}{2\Im_\perp} + \frac{K^2}{2\Im_\parallel}, \tag{5}$$

with $\Im_\perp = \Im_r + \mu R^2$ and $\Im_\parallel \approx \Im_r$. A straightforward transformation of Eq. (4) into a frame where the z axis coincides with the beam direction (so that a particle's direction is specified by polar angle θ and azimuthal angle ϕ) gives Eq. (2).

To avoid the uncertainty in the reaction plane determination [15,20], we use the azimuthal correlation function [Eq. (1)] which is proportional to the joint probability of observing two particles at a fixed relative angle.

If the fragments are emitted (nearly) independently of one another, the joint probability of observing two particles at a given polar angle θ and different azimuthal angles ϕ and $\phi + \Delta\phi$ is $P(\theta,\phi,\Delta\phi) = P(\theta,\phi)P(\theta,\phi+\Delta\phi)$. The resulting probability distribution must be averaged over the different directions of $\vec{I}$ arising from different orientations of the impact vector. Averaging over the direction of $\vec{I}$ is equivalent to integrating over ϕ,

$$P(\theta,\Delta\phi) \propto \int_0^{2\pi} d\phi\, e^{-\beta\sin^2\theta\sin^2\phi} e^{-\beta\sin^2\theta\sin^2(\phi+\Delta\phi)}. \tag{6}$$

This integral can be performed exactly, and one finds

$$P(\theta,\Delta\phi) \propto I_0\left(\beta\sin^2\theta\sqrt{\frac{1+\cos2\Delta\phi}{2}}\right), \tag{7}$$

where I_0 is the modified Bessel function of zeroth order [21].

These equations, which hold for like particles, can be generalized to unlike particles,

$$P(\theta,\Delta\phi) \propto I_0\left(\frac{\sqrt{\beta_1^2+\beta_2^2}}{2} \times \sin^2\theta\sqrt{1+\frac{2\beta_1\beta_2}{\beta_1^2+\beta_2^2}\cos2\Delta\phi}\right), \tag{8}$$

where β_1 and β_2 are calculated via Eq. (3) for particles of reduced mass μ_1 and μ_2, respectively.

It is useful to consider the Taylor expansion of $I_0(z)$

$$I_0(z) = 1 + \frac{\frac{1}{4}z^2}{(1!)^2} + \frac{(\frac{1}{4}z^2)^2}{(2!)^2} + \frac{(\frac{1}{4}z^2)^3}{(3!)^2} + \cdots. \tag{9}$$

For small z we can keep only the first three terms of the expansion and find that the joint probability (for $\beta_1 = \beta_2 = \beta$) is

$$P(\theta,\Delta\phi) \propto 1 + \frac{D}{1+D/2}\cos2\Delta\phi + \frac{D^2}{(D+2)^2}\cos^2 2\Delta\phi \tag{10}$$

$$= 1 + \lambda_2\cos2\Delta\phi + \lambda_4\cos^2 2\Delta\phi, \tag{11}$$

where $D = (\beta^2\sin^4\theta)/8$.

The first two terms of Eq. (10) have the familiar form of $1 + \lambda_2\cos2\Delta\phi$ often used to describe rotational features of azimuthal correlations [14,15,22]. Positive values of λ_2 produce the V-shaped signature of the data in Fig. 1. The third term can be considered a small perturbation to the general shape of the correlation function determined by λ_2 (for $D \le 0.5, \lambda_4 \ll \lambda_2$). Generally a term $\lambda_1\cos\Delta\phi$ is also included in the fit to describe either the kinematic focusing from a recoiling source ($\lambda_1 < 0$) or directed flow effects ($\lambda_1 > 0$) in the azimuthal correlations [15,22].

Fits of the form

$$P(\theta, \Delta\phi) \propto N\left(1 + \lambda_1 \cos\Delta\phi + \frac{D}{1 + D/2} \cos 2\Delta\phi + \frac{D^2}{(D + 2)^2} \cos^2 2\Delta\phi\right) \tag{12}$$

are shown in Fig. 1. Equivalent fits are produced if one uses the Bessel function I_0 instead of its approximation in Eq. (12). The fits have been limited to $\Delta\phi \geq 45°$ in an effort to remove the sensitivity of the fit parameters to strong resonances ($^8\mathrm{Be} \rightarrow 2\alpha$) and to the Coulomb repulsion between the particle pair. Both may strongly affect the correlation in the region of small $\Delta\phi$. Extracted values of λ_1 are small, typically a factor of 10 smaller than the values of D, and show no strong dependence on E_t [14]. The quality of the fits using Eq. (12) is sufficiently good that the parameters λ_1 and D may be used to characterize the main features of the evolution of the azimuthal correlations with increasing excitation energy.

According to Eq. (3), the parameter D is predicted to have a specific temperature dependence

$$D \propto \beta^2 \propto \left(\frac{E_{\mathrm{rot}}}{T}\right)^2 \mu^2 \tag{13}$$

which can be explored in this data set. In previous work [2–4] we have used the transverse energy of an event as a measure of its total excitation energy. Assuming that the transverse energy E_t is proportional to the excitation energy one expects $D \propto 1/T^2 \propto 1/E_t$.

A plot of D as a function of $1/E_t$ is given in the left panel of Fig. 2 for identical emitted particles. The correlations are remarkably linear. We are not limited in this analysis to particle pairs of equal mass [see Eq. (8)]. The right panel of Fig. 2 shows D as a function of $1/E_t$ for particle pairs of different masses (one member of the pair is a He nucleus), where, again, the thermal scaling is evident.

In order to verify the robustness of the procedure we have also extracted D by setting $\lambda_1 = 0$ and limiting the angular range to $\Delta\phi \geq 75°$. The two procedures produce comparable results.

The simplest explanation for the observed linear behavior is that the fragmenting system attains an average rotational energy which is largely independent of E_t. While this assumption is not intuitive, it is supported by the constant slope Arrhenius plots of Refs. [2,3]. The slopes of the Arrhenius plots in those works are proportional to the effective barrier for fragment emission, and are found to be independent of E_t. This may indicate that collective rotation does not change significantly with E_t.

While this is the simplest explanation, the observed linear trends of Fig. 2 could instead come from a more complicated dependence of the rotational energy E_{rot} and of the temperature T on E_t. The observation of a finite intercept (the data do not extrapolate to zero at large E_t) indicates the presence of open questions with regard to this effect.

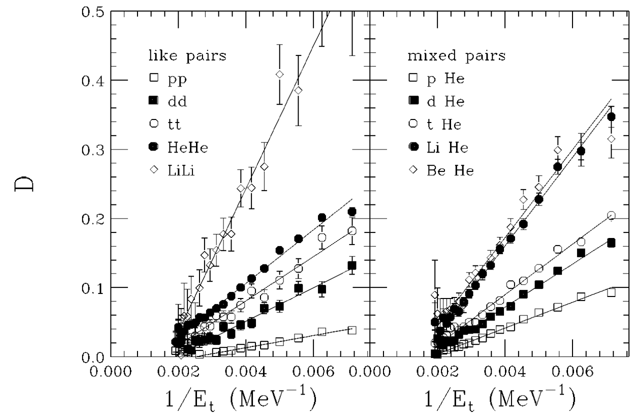

FIG. 2. Mass and "temperature" dependence of D. Left panel: The fit parameter D as a function of $1/E_t$ ($\propto 1/T^2$) for the indicated identical particle pairs. Solid lines are linear fits to the data. Right panel: Same as left panel but for particle pairs of different masses.

The strength of the correlation in Fig. 2 increases with increasing mass of the particle pair. This is consistent with previous observations [8–12,14,19,23–26] where the azimuthal anisotropies show a strong dependence on the mass of the emitted particles.

According to Eq. (13) one would expect the quantity D to have a μ^2 dependence (for identical particles) on the mass of the emitted particles. For a sufficiently massive source $\mu^2 \sim A^2$ (see the dashed curve of Fig. 3). Instead, a nearly linear scaling with A is observed in Fig. 3 where we have plotted the extracted slopes from Fig. 2 as a

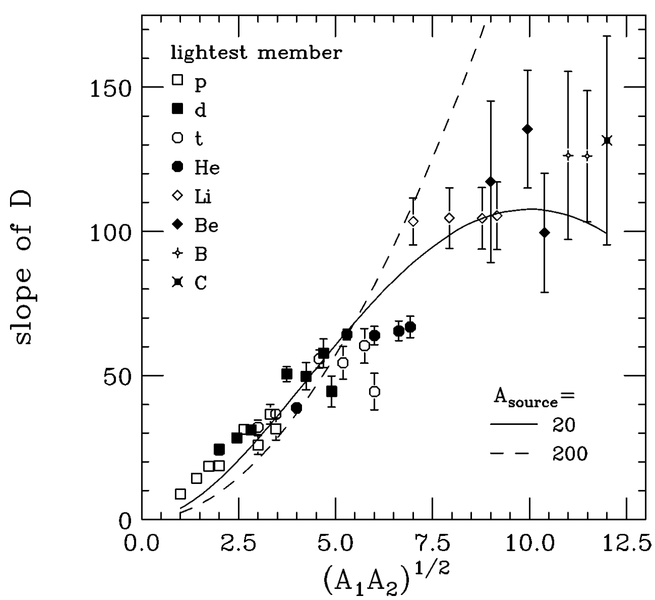

FIG. 3. Slope of D (see Fig. 2) as a function of $\sqrt{A_1A_2}$ for particles with mass numbers between 1 and 12. The most abundant isotope in the periodic table is assumed for the mass numbers of the indicated elements. The lightest member of the particle pair is indicated by the different symbols. The solid (dashed) line is a prediction of the mass scaling assuming emission from a source of size $A_{\mathrm{source}} = 20$ (200).

function of the geometric mean of the mass numbers of the emitted particles [27]. We have also included data for all mass combinations ranging from protons to carbon nuclei. These data show a nearly linear dependence on $\sqrt{A_1 A_2}$ (as opposed to $\propto A_1 A_2$).

One possible way of resolving this contradiction is to assume that the mass of the emitting source(s) is very small, of the order of $A_{\text{source}} \approx 15-30$. In this case μ^2 is approximately linear with the mass of the emitted fragment in the mass range considered here. The solid curve shown in Fig. 3 is calculated assuming $A_{\text{source}} = 20$.

It is important to point out that such a tantalizing explanation requires *multiple* sources of size A_{source}, all corotating (rigidly) with the same angular velocity. A *single* small source would give rise to strong recoil effects (the correlations would be suppressed at $\Delta\phi = 0°$ and enhanced at $\Delta\phi = 180°$), washing out the V-like signature in the azimuthal correlations [9]. With a simple simulation for a single small source, we have studied this effect of preferred emission to opposite sides of the beam axis. By adding three or more sources to the simulation, each rotating with its share of the total angular momentum [28], the contamination to the correlation function from kinematic focusing of the recoiling source is strongly diminished and the V-like signature of rotation becomes quite pronounced.

In this regard it is interesting to notice that a variety of instabilities, like the Rayleigh instability relevant to the rupture of necklike structures, or the sheet instability associated with disklike objects, or even the spinodal instability predict the early formation of several small fragments. A recent calculation [29] demonstrates that a spinodal breakup would produce several excited primary fragments of nearly equal size ($Z = 10-20$) which then undergo statistical sequential decay. The observed mass scaling (Fig. 3) is consistent with such a prediction and may be a surviving signature of such dynamical processes.

In summary, the following conclusions may be made: (1) Particle-particle azimuthal angular correlations are *reducible* to independent particle distributions. (2) The coefficients of these angular distributions show a dependence on E_t consistent with the expected temperature dependence (thermal scaling). (3) A mass scaling is observed in the azimuthal angular correlation coefficients. (4) The above mass scaling is consistent with theoretical expectations under the assumption that the (several) rotating emitting sources are objects of size $A \sim 15-30$.

These observations of thermal angular distributions, combined with the previously observed thermal IMF emission probabilities [1–3] and thermal charge distributions [4], add to the body of evidence illustrating the strong role of phase space in describing multifragmentation. However, the observed mass scaling (along with the possible interpretation of multiple small sources) may indicate the important role of dynamics as well.

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