

Partial Dynamical Symmetry in Deformed Nuclei

Amiram Leviatan

Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel
(Received 9 April 1996)

We discuss the notion of partial dynamical symmetry in relation to nuclear spectroscopy. Explicit forms of Hamiltonians with partial SU(3) symmetry are presented in the framework of the interacting boson model of nuclei. An analysis of the resulting spectrum and electromagnetic transitions demonstrates the relevance of such partial symmetry to the spectroscopy of axially deformed nuclei. [S0031-9007(96)00782-X]

PACS numbers: 21.60.Fw, 21.10.Re, 21.60.Ev, 27.70.+q

Recent years, in particular since the introduction of the interacting boson model of nuclei (IBM) [1], have witnessed substantial progress in developing algebraic symmetry-based models, which are now part of the standard lexicon of nuclear structure [2]. A characteristic and attractive feature in these models is the occurrence of dynamical symmetries. This corresponds to a situation in which the Hamiltonian is written in terms of Casimir operators of a chain of nested groups. A dynamical symmetry provides considerable insight since it allows all properties of the system to be calculated in closed form. The labels of irreducible representations (irreps) of the groups in the chain serve as quantum numbers to classify members of a complete basis in which the Hamiltonian is diagonal. The group-theoretical classification scheme inherent to the dynamical symmetry basis facilitates the numerical treatment and interpretation of the general Hamiltonian.

The merits of having a (dynamical) symmetry are self-evident. However, in detailed applications of group theoretical schemes to the spectroscopy of nuclei, one often finds that the assumed symmetry is not obeyed uniformly, i.e., some levels fulfill the symmetry while other levels do not. Exact symmetries impose severe constraints on the corresponding spectrum (e.g., particular band degeneracies) which are rarely observed in real nuclei. These observations motivate one to consider a particular symmetry breaking that would result in mixing of irreps in some part of the spectrum while retaining a good symmetry to specific eigenstates. We refer to such a situation as partial (dynamical) symmetry. Within such symmetry construction only a subset of eigenstates are pure and preserve the desired features of a dynamical symmetry. IBM Hamiltonians with F -spin partial symmetry were shown in [3]. The mathematical aspects and algorithm for partial dynamical symmetries (pds) were presented in [4]. The purpose of the present work is to show that pds are not just a formal mathematical notion but rather are actually realized in nuclei and thus may serve as a useful tool in realistic applications of algebraic methods to nuclear spectroscopy. In this Letter we consider ^{168}Er as a typical example of an axially deformed prolate nucleus in the rare earth region and show the relevance of SU(3) pds to its description.

The starting point for the IBM description of axially deformed nuclei is the SU(3) dynamical symmetry, corresponding to the chain $U(6) \supset SU(3) \supset O(3)$. The basis states are labeled by $[[N](\lambda, \mu)KLM]$, where N is the total number of monopole (s^\dagger) and quadrupole (d^\dagger) bosons, L the angular momentum, (λ, μ) denote the SU(3) irreducible representations (irreps), and K is an additional label needed for complete classification and corresponds geometrically to the projection of the angular momentum on the symmetry axis. The Hamiltonian in this case involves a linear combination of the Casimir operators of SU(3) and O(3). The corresponding eigenstates are arranged in SU(3) multiplets. The lowest SU(3) irrep is $(2N, 0)$, which describes the ground band $g(K=0)$ of an axially deformed nucleus. The first excited SU(3) irrep $(2N-4, 2)$ contains both the $\beta(K=0)$ and $\gamma(K=2)$ bands. Consequently, states in these bands with the same angular momentum are degenerate. This β - γ degeneracy is a characteristic feature of the SU(3) limit of the IBM which, however, is not commonly observed [5]. In most deformed nuclei the β band lies above the γ band as is evident from the experimental spectrum of ^{168}Er shown in

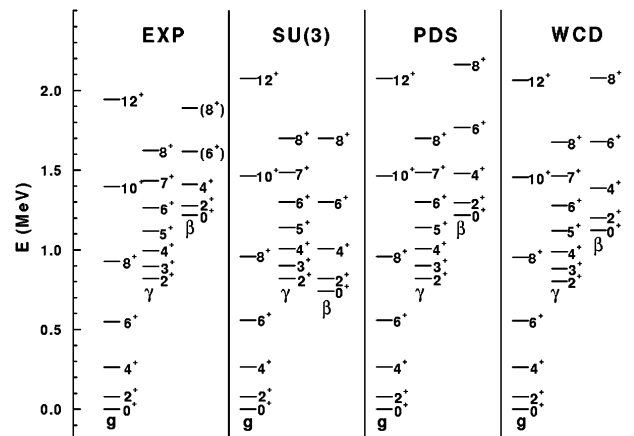


FIG. 1. Spectra of ^{168}Er . Experimental energies (EXP) are compared with an IBM calculation in an exact SU(3) dynamical symmetry [SU(3)], in a broken SU(3) symmetry [6] and in a partial dynamical SU(3) symmetry (PDS). The latter employs the Hamiltonian of Eq. (6) with $h_0 = 0.008$, $h_2 = 0.004$, $\lambda = 0.013$ MeV.

Fig. 1. In the IBM framework, with at most two-body interactions, one is therefore compelled to break SU(3) in order to conform with the experimental data. To do so, the usual approach has been to include in the Hamiltonian terms from other chains so as to lift the undesired β - γ degeneracy. Such an approach was taken in Ref. [6] where an O(6) term was added to the SU(3) Hamiltonian yielding a satisfactory description of the spectroscopic data of ^{168}Er below 2 MeV, as shown in Fig. 1. However, in this procedure, the SU(3) symmetry is completely broken, all eigenstates are mixed, and no analytic solutions are retained. Similar statements apply to the description in the consistent Q formalism [7]. In contrast, partial SU(3) symmetry, to be discussed below, corresponds to breaking SU(3), but in a very particular way so that *part* of the states (but not all) will still be solvable with good symmetry. As such, the virtues of a dynamical symmetry (e.g., solvability) are fulfilled but by only a subset of states.

To consider partial SU(3) symmetry in the IBM framework we examine the following rotational-invariant Hamiltonian:

$$H(h_0, h_2) = h_2[-\hat{C}_{\text{SU}(3)} + 2\hat{N}(2\hat{N} + 3)] \\ + (h_2 - h_0)[-4\hat{N}^2 - 6\hat{N} + \hat{n}_d - \hat{n}_d^2 \\ + 4\hat{N}\hat{n}_d + 2\hat{C}_{\text{O}(6)} - \hat{C}_{\text{O}(5)}], \quad (1)$$

where h_0, h_2 are arbitrary constants and we use the definition of Casimir operators as in Table I of the Appendix in Ref. [8]. Clearly, for $h_0 \neq h_2$ the above Hamiltonian contains a mixture of Casimir operators of all IBM chains, hence it breaks the SU(3) symmetry. However, it respects SU(3) as a partial symmetry. To confirm this nontrivial statement, it is simpler to consider the normal order form [8,9]

$$H(h_0, h_2) = h_0 P_0^\dagger P_0 + h_2 P_2^\dagger \cdot \tilde{P}_2, \quad (2)$$

$$P_{L,\mu} |c; N\rangle = 0, \quad [P_{L,\mu}, P_{2,2}^\dagger] |c; N\rangle = \delta_{L,2} \delta_{\mu,2} 6(2N + 3) |c; N\rangle, \\ [[P_{L,\mu}, P_{2,2}^\dagger], P_{2,2}^\dagger] = \delta_{L,2} \delta_{\mu,2} 24 P_{2,2}^\dagger, \quad L = 0, 2. \quad (4)$$

The state $|c; N\rangle \propto [(s^\dagger + \sqrt{2}d_0^\dagger)]^N |0\rangle$ in Eq. (4) is a condensate of N bosons, which serves as an intrinsic state [10] for the SU(3) ground band. For arbitrary h_0, h_2 coefficients the Hamiltonian $H(h_0, h_2)$ is not an SU(3) scalar. Nevertheless, it has a subset of eigenstates with good SU(3) character. This follows from relations (4) which imply that the sequence of states

$$|k\rangle \propto (P_{2,2}^\dagger)^k |c; N - 2k\rangle \quad (5)$$

are eigenstates of $H(h_0, h_2)$ with eigenvalues $E_k = 6h_2[2N + 1 - 2k]k$. These energies are the SU(3) eigenvalues of $H(h_0 = h_2)$ and identify the states $|k\rangle$ to be in the SU(3) irreps $(2N - 4k, 2k)$ with $2k \leq N$. It can be further shown that they are lowest weight states

TABLE I. $B(E2)$ branching ratios from states in the γ band in 168. The experimental ratios (EXP) and the broken SU(3) calculation of Warner, Casten, and Davidson (WCD) are taken from Ref. [6]. PDS are the partial dynamical SU(3) symmetry calculation reported in the present work.

J_i^π	J_f^π	EXP	PDS	WCD	J_i^π	J_f^π	EXP	PDS	WCD
2_γ^+	0_g^+	54.0	64.27	66.0	6_γ^+	4_g^+	0.44	0.89	0.97
	2_g^+	100.0	100.0	100.0		6_g^+	3.8	4.38	4.3
	4_g^+	6.8	6.26	6.0		8_g^+	1.4	0.79	0.73
3_γ^+	2_g^+	2.6	2.70	2.7	7_γ^+	4_γ^+	100.0	100.0	100.0
	4_g^+	1.7	1.33	1.3		5_γ^+	69.0	58.61	59.0
	2_γ^+	100.0	100.0	100.0		6_g^+	0.74	2.62	2.7
4_γ^+	2_g^+	1.6	2.39	2.5	8_γ^+	5_γ^+	100.0	100.0	100.0
	4_g^+	8.1	8.52	8.3		6_γ^+	59.0	39.22	39.0
	6_g^+	1.1	1.07	1.0		6_g^+	1.8	0.59	0.67
	2_γ^+	100.0	100.0	100.0		8_γ^+	5.1	3.57	3.5
	4_g^+	2.91	4.15	4.3		6_γ^+	100.0	100.0	100.0
5_γ^+	6_g^+	3.6	3.31	3.1	7_γ^+	135.0	28.64	29.0	
	3_γ^+	100.0	100.0	100.0					
	4_γ^+	122.0	98.22	98.5					

where $\tilde{P}_{2,\mu} = (-)^\mu P_{2,-\mu}$. The Hamiltonian is seen to be constructed from boson pair operators with angular momentum $L = 0$ and 2, which are defined as

$$P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2, \\ P_{2,\mu}^\dagger = 2s^\dagger d_\mu^\dagger + \sqrt{7}(d^\dagger d^\dagger)_\mu^{(2)}. \quad (3)$$

These boson pair operators satisfy the following properties:

in these representations. The states $|k\rangle$ are deformed and serve as intrinsic states representing γ^k bands with angular momentum projection ($K = 2k$) along the symmetry axis [11]. In particular, $|k = 0\rangle$ represents the ground-state band ($K = 0$) and $|k = 1\rangle$ is the γ band ($K = 2$). The intrinsic states break the O(3) symmetry but since the Hamiltonian in Eq. (2) is an O(3) scalar, the projected states are also eigenstates of $H(h_0, h_2)$ with energy E_k and with good SU(3) symmetry. For the ground band ($k = 0$) the projected states span the entire SU(3) irrep $(2N, 0)$. For excited bands ($k \neq 0$), the projected states span only part of the corresponding SU(3) irreps. There are other states originally in these irreps (as well as in other irreps) which do not preserve the SU(3) symmetry and therefore get mixed. In particular,

the ground (g) and γ bands retain their SU(3) character $(2N, 0)$ and $(2N - 4, 2)$, respectively, but the β band is mixed. This situation corresponds precisely to that of partial SU(3) symmetry. A Hamiltonian $H(h_0, h_2)$ which is not an SU(3) scalar has a subset of *solvable* eigenstates which continue to have good SU(3) symmetry. All of the above discussion is applicable also to the case when we add to the Hamiltonian (2) the Casimir operator of O(3) ($\hat{C}_{O(3)}$), and by doing so convert the partial SU(3) symmetry into partial dynamical SU(3) symmetry. The additional rotational term contributes just an $L(L + 1)$ splitting but does not affect the wave functions.

The experimental spectra [6] of the ground (g), β , and γ bands in ^{168}Er is shown in Fig. 1. We now attempt a description in terms of an IBM Hamiltonian with partial dynamical SU(3) symmetry

$$H = H(h_0, h_2) + \lambda \hat{C}_{O(3)}. \quad (6)$$

According to the previous discussion, the spectrum of the ground and γ bands is given by

$$\begin{aligned} E_g(L) &= \lambda L(L + 1), \\ E_\gamma(L) &= 6h_2(2N - 1) + \lambda L(L + 1). \end{aligned} \quad (7)$$

The Hamiltonian in Eq. (6) is specified by three parameters ($N = 16$ for ^{168}Er according to the usual boson counting). We extract the values of λ and h_2 from the experimental energy differences $E(2_g^+) - E(0_g^+)$ and $E(2_\gamma^+) - E(2_g^+)$, respectively. For an exact SU(3) dynamical symmetry, $h_0 = h_2$, implying $E_\beta(L) = E_\gamma(L)$ for even values of $L \geq 2$. The corresponding spectrum (shown in Fig. 1) deviates considerably from the experimental data since empirically the β and γ bands are not degenerate. On the other hand, when the dynamical SU(3) symmetry is only partial, one can vary h_0 so as to reproduce the β bandhead energy $E_\beta(L = 0)$. Having determined the parameters λ, h_0, h_2 from three experimental energies, the prediction for other rotational members of the ground β and γ bands is shown in Fig. 1. No further attempt to improve the agreement between theory and experiment was made since the philosophy of this calcu-

lation was to investigate the validity of the SU(3) pds. Clearly, the SU(3) pds spectrum is an improvement over the schematic, exact SU(3) dynamical symmetry description since the β - γ degeneracy is lifted. The good SU(3) character, $(32, 0)$ for the ground band and $(28, 2)$ for γ band, is retained in the pds calculation, while the β band contains 10% $(26, 0)$ and 3% $(24, 4)$ admixtures into the dominant $(28, 2)$ irrep. The quality of the calculated pds spectrum is similar to that obtained in the broken SU(3) calculation [6] also shown in Fig. 1.

Electromagnetic transitions are a more sensitive probe to the structure of states, hence they are an important indicator for verifying the relevance of partial SU(3) symmetry. To calculate such observables we need to specify the wave functions of the initial and final states as well as the operator that induces the transition. For the Hamiltonian in Eq. (6), with partial dynamical SU(3) symmetry, the solvable states are those projected from the intrinsic states $|k\rangle = |(\gamma)^k (2N - 4k, 2k) K = 2k\rangle$ of Eq. (5), and are simply selected members of the Elliott basis $\phi_E((\lambda, \mu)KLM)$ [12]. In particular, the states belonging to the ground and γ bands are the Elliott states $\phi_E((2N, 0)K = 0, LM)$ and $\phi_E((2N - 4, 2)K = 2, LM)$, respectively. Their wave functions can be expressed in terms of the Vergados basis $(\Psi_V(\lambda, \mu)\chi LM)$ [13], which is the usual (but not unique) choice for orthonormal SU(3) basis. The most general IBM one-body $E2$ operator may be written as

$$T(E2) = \alpha Q^{(2)} + \theta (d^\dagger s + s^\dagger \tilde{d}), \quad (8)$$

where $Q^{(2)}$ is the quadrupole SU(3) generator. The matrix elements of such $E2$ operator in the Vergados basis are known [14,15]. It is therefore possible to obtain *analytic* expressions for the $E2$ rates between the subset of solvable states. For the ground band and for members of the γ band with L odd, the Vergados and Elliott bases are identical. Accordingly, the corresponding $B(E2)$ values in the two bases are the same. The Elliott states in the $\gamma(K = 2)$ band with even values of L are mixtures of Vergados states in the $\beta(\chi = 0)$ and $\gamma(\chi = 2)$ bands. The corresponding $B(E2)$ value is

$$\begin{aligned} B_E(E2; \gamma K = 2, L \rightarrow g K = 0, L') \\ = \left[\frac{\sqrt{B_V(E2; \gamma \chi = 2, L \rightarrow g \chi = 0, L')} \pm x_{20}^{(L)} \sqrt{B_V(E2; \beta \chi = 0, L \rightarrow g \chi = 0, L')}}{x_{22}^{(L)}} \right]^2, \end{aligned} \quad (9)$$

where the $+$ ($-$) sign applies to a transition with $L' = L$ ($L' = L \pm 2$). In Eq. (9) the notation $B_V(E2)$ and $B_E(E2)$ stands for $B(E2)$ values calculated in the Vergados and Elliott bases, respectively. The $x_{20}^{(L)}, x_{22}^{(L)}$ are coefficients which appear in the transformation between the two bases [13]. Analytic expressions of $B_V(E2)$ values for $g \rightarrow g$ and $\gamma \rightarrow g$ transitions have been derived [14,15].

To compare with experimental data on $B(E2)$ ratios, we adopt the procedure of Ref. [6] and extract the parameters α and θ of the $E2$ operator in Eq. (8) from the experimental values of $B(E2; 0_g^+ \rightarrow 2_g^+)$ and $B(E2; 0_g^+ \rightarrow 2_\gamma^+)$. The corresponding ratio for ^{168}Er is $\theta/\alpha = 4.261$. As shown in Table I, the resulting SU(3) pds $E2$ rates for transitions originating within the γ band are found to be in excellent agreement with experiment and are

similar to the calculation by Casten, Warner, and Davidson [6] [where the SU(3) symmetry is broken for all states]. In particular, the SU(3) pds calculation reproduces correctly the ratio of $(\gamma \rightarrow \gamma)/(\gamma \rightarrow g)$ strengths. The only significant discrepancy is that for the $8_\gamma^+ \rightarrow 7_\gamma^+$ transition which is very weak experimentally, with an intensity error of 50% and an unknown M1 component [6]. For transitions from the β band the overall agreement is good (better for $\beta \rightarrow \gamma$ transitions) although not as precise as for the γ band. The calculation exhibits the observed dominance of $\beta \rightarrow \gamma$ over $\beta \rightarrow g$ transitions. As an example, for $2_\beta^+ \rightarrow J_f$ transitions with $J_f = (0_g^+, 4_g^+, 2_\gamma^+, 3_\gamma^+, 0_\beta^+)$ the calculated and experimental $B(E2)$ ratios are (0.42 : 1.44 : 2.59 : 4.77 : 100.0) and (0.23 : 1.4 : 4.0 : 4.9 : 100.0), respectively. A comparison with the prediction of an exact SU(3) symmetry for these ratios: (0.47, 1.62, 0.93, 1.66, 100.0) highlights the importance of SU(3) mixing in the β band. If we recall that only the ground band has SU(3) components $(\lambda, \mu) = (2N, 0)$ and that $Q^{(2)}$ in Eq. (8) is a generator of SU(3) [hence cannot connect different (λ, μ) irreps], it follows that $\beta, \gamma \rightarrow g$ $B(E2)$ ratios are independent of both α and θ . Furthermore, since the ground and γ bands have pure SU(3) character, $(2N, 0)$ and $(2N - 4, 2)$, respectively, the corresponding wave functions do not depend on parameters of the Hamiltonian and hence are determined solely by symmetry. Consequently, the $B(E2)$ ratios for $\gamma \rightarrow g$ transitions quoted in Table I are parameter-free predictions of SU(3) pds. The agreement between these predictions and the data confirms the relevance of partial dynamical SU(3) symmetry to the spectroscopy of ^{168}Er .

To summarize, we have analyzed IBM Hamiltonians with SU(3) pds. Such Hamiltonians are not invariant under SU(3) but have a subset of eigenstates with good SU(3) symmetry. The special states are solvable and span part of particular SU(3) irreps. Their wave functions, eigenvalues, and $E2$ rates are known analytically. An application of the scheme to ^{168}Er has demonstrated that the empirical spectrum and $E2$ rates conform with the predictions of partial SU(3) symmetry. These observations point at the relevance of partial SU(3) symmetry to the spectroscopy of axially deformed nuclei, at least as a starting point for further refinements.

The notion of partial dynamical symmetry is not confined to SU(3). A general algorithm is available for constructing Hamiltonians with pds for any semi-simple group [4]. The occurrence of partial (but exact)

symmetries implies that part of the eigenvalues and wave functions can be found analytically but not the entire spectrum. As such, pds can overcome the schematic features of exact dynamical symmetries (e.g., undesired degeneracies) and simultaneously retain their virtues (i.e., solvability) for some states. We also wish to point out that Hamiltonians with partial symmetries are not completely integrable and may exhibit chaotic behavior. This makes them a useful tool to study mixed systems with coexisting regularity and chaos [16]. It will be of great interest to explore the ramifications of partial symmetries both for discrete spectroscopy and statistical aspects of nuclei.

This research was supported by the Israel Science Foundation administered by the Israel Academy of Sciences and Humanities.

-
- [1] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, England, 1987).
 - [2] R. F. Casten, in *Algebraic Approaches to Nuclear Structure*, edited by R. F. Casten (Harwood Academic Publishers, Chur, Switzerland, 1993).
 - [3] A. Leviatan, J. N. Ginocchio, and M. W. Kirson, *Phys. Rev. Lett.* **65**, 2853 (1990).
 - [4] Y. Alhassid and A. Leviatan, *J. Phys. A* **25**, L1265 (1992); A. Leviatan, in *Symmetries in Science VII*, edited by B. Gruber and T. Otsuka (Plenum Press, New York, 1992), p. 383.
 - [5] R. F. Casten and D. D. Warner, *Rev. Mod. Phys.* **60**, 389 (1988).
 - [6] D. D. Warner, R. F. Casten, and W. F. Davidson, *Phys. Rev. C* **24**, 1713 (1981).
 - [7] D. D. Warner and R. F. Casten, *Phys. Rev. C* **28**, 1798 (1983).
 - [8] A. Leviatan, *Ann. Phys. (N.Y.)* **179**, 201 (1987).
 - [9] A. Leviatan, *Z. Phys. A* **321**, 467 (1985); M. W. Kirson and A. Leviatan, *Phys. Rev. Lett.* **55**, 2846 (1985).
 - [10] J. N. Ginocchio and M. W. Kirson, *Nucl. Phys.* **A350**, 31 (1980).
 - [11] H. T. Chen and A. Arima, *Phys. Rev. Lett.* **51**, 447 (1983).
 - [12] J. P. Elliott, *Proc. Roy. Soc. London A* **245**, 128 (1958); **245**, 562 (1958).
 - [13] J. D. Vergados, *Nucl. Phys.* **A111**, 681 (1968).
 - [14] A. Arima and F. Iachello, *Ann. Phys. (N.Y.)* **111**, 201 (1978).
 - [15] P. Van Isacker, *Phys. Rev. C* **27**, 2447 (1983).
 - [16] N. Whelan, Y. Alhassid, and A. Leviatan, *Phys. Rev. Lett.* **71**, 2208 (1993).