

## Stick-Slip Motion and Force Fluctuations in a Driven Two-Wave Potential

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A model of a particle interacting with two periodic potentials, one of which is externally driven, is analyzed. Three regimes are identified in the motion of the driven plate: (a) stick-slip motion, (b) intermittent stick slip characterized by force fluctuations, and (c) sliding which occurs above a critical driving velocity  $v_c$ . In the vicinity of  $v_c$  the power spectra of the force obey a  $\omega^{-2}$  law and the force fluctuations decrease as  $(v_c - v)^{1/2}$  for  $v < v_c$ . Our calculations suggest that stick-slip dynamics is characterized by chaotic behavior of the top plate and the embedded molecular system. An equation is derived which provides a coarse-grained description of the plate motion near  $v_c$ . [S0031-9007(96)00727-2]

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Stick-slip motion has been a subject of active research related to a broad range of phenomena from friction in nanoscale liquid films [1,2] to geophysics and earthquake faults [3,4]. In recent experiments on friction, in particular on confined molecular systems under shear, stick-slip motion has been carefully analyzed [5,6]. Attention has been paid to the deterministic features of friction and also to force fluctuations in terms of their power spectra. It has been observed [1,2,5,6] that stick-slip behavior is followed by an intermittent stick slip and then by sliding as the shear rate increases. Different models have been proposed to account for this type of motion, including spring-block models [3] and chain or layer motion on a substrate [7,8]. Stick-slip behavior has also been seen in direct molecular dynamical simulations [9–11]. However, the microscopic origin of stick-slip dynamics is still not well understood.

In this Letter we introduce a model of a single particle which interacts with two corrugated plates, one of which is externally driven. We demonstrate that various properties typical of thin liquids under shear are already observed in the framework of our single-particle system. The model can be related to various problems in nonlinear dynamics [12–14].

Consider a one-dimensional model which includes two rigid plates and a single particle of mass  $m$  embedded between them. The interaction between the particle and each of the plates is described by the periodic potential  $U(x) = -U_0 \cos(2\pi x/b)$ . There is no direct interaction between the plates. The top plate of mass  $M$  is pulled by a linear spring with force constant  $K$  connected to a stage

moving with a velocity  $v$  (see Fig. 1 for the sketch of the model geometry).

The coupled equations of motion for the top plate and the particle can be written in a dimensionless form as

$$\ddot{Y} + \epsilon\gamma(\dot{Y} - \dot{y}) + \alpha^2(Y - v\tau) - \frac{\epsilon}{2\pi} \sin[2\pi(y - Y)] = 0, \quad (1a)$$

$$\ddot{y} + \gamma(2\dot{y} - \dot{Y}) + \frac{1}{2\pi} \sin(2\pi y) + \frac{1}{2\pi} \sin[2\pi(y - Y)] = 0, \quad (1b)$$

where  $y$  and  $Y$  are the coordinates of the particle and plate, respectively, in units of the period of the potential  $b$ ,  $\tau = \omega t$  is the dimensionless time,  $\omega = (2\pi/b)\sqrt{U_0/m}$  is the frequency of the small oscillations of the particle in the minima of potential  $U(x)$ ,  $\gamma$  is a dimensionless friction constant which accounts for dissipation due to phonons and/or other excitations,  $\epsilon = m/M$  is the ratio of particle and plate masses, and  $\alpha = \Omega/\omega$  is the ratio of frequencies of the free oscillations of the top plate and the particle,  $\Omega = \sqrt{K/M}$ . The second terms in Eqs. (1a) and (1b) describe the dissipative forces between the particle and the plates and are proportional to their relative velocities. The third term in Eq. (1a) is the driving force due to the stage which moves with the constant velocity  $v$ . Equations (1a) and (1b) relate to the problem of friction in lubricating films [6,7,10], and, in the limit  $\alpha \rightarrow \infty$ , reduce to the problems of a particle in a two-wave potential [12,13] and of a parametric oscillator [14], actively studied in the theory of nonlinear dynamical systems.

We focus on the dynamical behavior of the top plate and of the particle as the driving velocity of the stage is varied. Our simulations demonstrate that, within velocity values of interest, there are three different dynamical regimes: (a) At low velocities we observed a regular stick-slip motion of the plate; (b) as the stage velocity increases, the top plate ceases to stop (time intervals

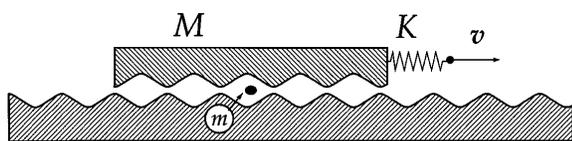


FIG. 1. Schematic sketch of a model geometry.

between rare stop events increase rapidly with  $v$ ), and stick-slip motion becomes more erratic and intermittent; (c) smooth sliding occurs when the stage velocity is above the critical velocity  $v_c$ . Figure 2 illustrates these phenomena showing the time dependence of the spring force which acts on the top plate. In the calculations reported below we use the following values of the parameters:  $\alpha = 0.02$ ,  $\gamma = 0.1$ ,  $\epsilon = 0.125$  for which the system is underdamped. Other ranges of parameter values are discussed elsewhere [15].

The motion of the top plate in the first regime is typical of relaxation oscillations. The top plate is initially at rest, and the spring connecting it to the stage stretches linearly in time. When the force on the plate exceeds the static frictional force  $F_s$ , which in our model equals  $2\pi U_0/b$ , the top plate begins to slide. Since the frictional force in this kinetic state is less than  $F_s$ , the plate accelerates. Owing to the inertia, the velocity of the plate,  $\dot{Y}$ , is initially lower than the driving velocity  $v$ , and the spring will continue to extend until finally  $\dot{Y} > v$ . The maximum spring force will therefore be greater than  $F_s$ . When the plate velocity is  $\dot{Y} > v$ , the spring force decreases until it reaches some value where the motion stops and then the process repeats. We have also noticed that at low stage velocities the amplitude of the spring force does not depend on  $v$ , and the period of oscillations decreases with the increase of  $v$ . In this range of velocities the time averaged velocity and the displacement of the particle are much smaller than the average velocity and the displacement of the top plate.

In the second regime the amplitude of the spring force strongly depends on the stage velocity  $v$ . Here the frictional force is less than the static friction practically for all times. As we show below, the nature of the intermittent behavior in this regime is determined by an effective velocity-dependent friction force. We have also found that in this case the time averaged velocity and displacement of the particle are close to half of those of the top plate. The trajectory of the particle shows that the particle jumps between the two plates. It clings to

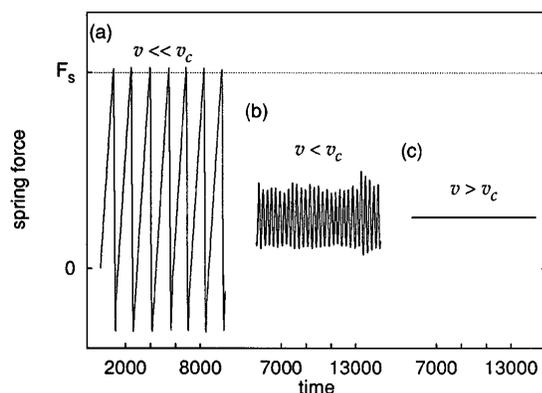


FIG. 2. The three regimes of top plate motion.

each of them for times much longer than the characteristic modulation time induced by the stage motion (natural period),  $1/v$ . We note that windows of sliding motion appear within the intermittent stick-slip region [15].

A sharp boundary at  $v = v_c$  is observed between the intermittent stick slip and sliding regimes. When the velocity approaches the critical velocity  $v_c$  from below, the amplitude of oscillations of the spring force decreases as  $\sqrt{v_c - v}$  and sliding sets in. (The spring force amplitude exhibits hysteretic behavior around  $v_c$  [15].) In the sliding regime the spring force performs “microscopic” oscillations with a period of the order  $1/v$ , and with amplitudes much smaller than in regimes (a) and (b). The critical velocity  $v_c$  depends on the mass of the particle, particle-plate interaction, and the friction coefficient  $\gamma$ . Within the accuracy of our calculations we have found no dependence of  $v_c$  on the mass of the top plate and on the spring constant, in contrast to previous findings [9,11]. In the sliding regime the particle does not jump between the two plates but rather clings to one of them and oscillates within one cell of the corrugated potential  $U(x)$ . The transition from regular to intermittent stick slip occurs through a sequence of period doubling bifurcations and windows of chaotic behavior and depends on the mass of the top plate and the spring constant.

The trajectories of the top plate and the particle in regimes (a) and (b) demonstrate high sensitivity to initial conditions, which is a manifestation of the dynamical chaos in the system. Positive values of the largest Liapunov exponent have been obtained, which provide a quantitative measure of the degree of stochasticity of the trajectories. As the stage velocity increases and approaches  $v_c$ , the largest Liapunov exponent decreases. This concurs with the reductions of the amplitude of the spring force oscillations. It should be mentioned that Liapunov exponents can be extracted from experimental data on time dependencies of the spring force or the velocity of the top plate [16].

We have also calculated the power spectra of the spring force, and of the velocities of the top plate and the particle. The power spectra  $S(\omega)$  depend on the stage velocity, and for  $v < v_c$  show a power law behavior  $S(\omega) \sim \omega^{-2}$  for frequencies above some cutoff (see Fig. 3).

It should be emphasized that, although our model is a single-particle model, the observed phenomena of stick slip, intermittent stick slip, critical  $v_c$ , and  $\omega^{-2}$  power spectra are in qualitative agreement with recent experiments on sheared nanoscale liquids [1,2,6].

It is possible to give an analytical description of the motion of the top plate connected to the spring which predicts the transition at  $v_c$ . We introduce two assumptions for the top plate dynamics in the vicinity of  $v_c$ : (a) The characteristic frequency of the large scale plate motion is much smaller than both the characteristic

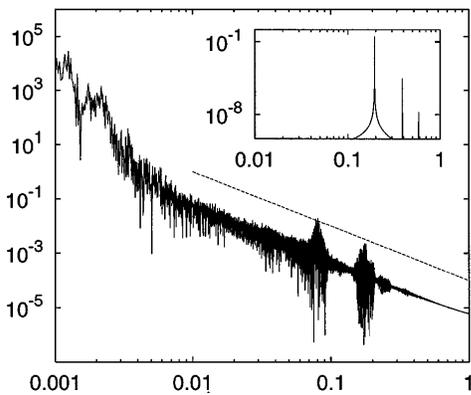


FIG. 3. The power spectrum of the spring force fluctuations,  $v = 0.36$ ,  $v < v_c$ . The dotted line of slope  $-2$  is provided for reference. The inset shows the power spectrum for  $v = 0.4$ ,  $v > v_c$ .

frequency of the particle oscillations and the natural frequency  $\nu$ ; (b) the mass of the particle is smaller than the mass of the top plate, i.e.,  $\epsilon < 1$ . Hence the top plate and the particle display “slow” and “fast” motions, and there is a separation of time scales, namely, the adiabatic approximation prevails. Under these assumptions, we solve Eqs. (1a) and (1b). For Eq. (1b) we assume that the plate moves with a constant velocity  $\dot{Y} = V$ . For the particle motion we get

$$\ddot{y} + \gamma(2\dot{y} - V) + \frac{1}{2\pi} \sin(2\pi y) + \frac{1}{2\pi} \sin[2\pi(y - V\tau)] = 0. \quad (2)$$

Equation (2) has been used to describe a dissipative parametrically driven pendulum and a dissipative motion of a particle in two waves. In spite of its apparent simplicity, Eq. (2) is not integrable and predicts a rich set of phenomena ([12–14], and references therein).

The solutions of Eq. (2)  $y(\tau, \dot{Y})$  depend parametrically on  $\dot{Y}$ . Substituting  $y(\tau, \dot{Y})$  into Eq. (1a), we get

$$\ddot{Y} - \epsilon F(\tau, Y, \dot{Y}) + \alpha^2(Y - \nu\tau) = 0, \quad (3)$$

where the particle-plate interaction force

$$F(\tau, Y, \dot{Y}) = \frac{1}{2\pi} \sin[2\pi(y - Y)] - \gamma(\dot{Y} - \dot{y}) \quad (4)$$

contains fast-oscillating components. Averaging Eqs. (3) and (4) over the fast oscillations, we obtain an equation for the slow-oscillating component of the spring length  $L(\tau) = Y(\tau) - \nu\tau$ ,

$$\ddot{L} - \epsilon \phi(\dot{L} + \nu) + \alpha^2 L = 0, \quad (5)$$

where the time-averaged force  $\phi(\dot{Y}) = \langle F(\tau, Y, \dot{Y}) \rangle$  depends only on the velocity of the plate, and presents the effective friction for the plate motion.

Before we solve Eq. (5), we discuss the velocity dependence of the time-averaged force. The effective

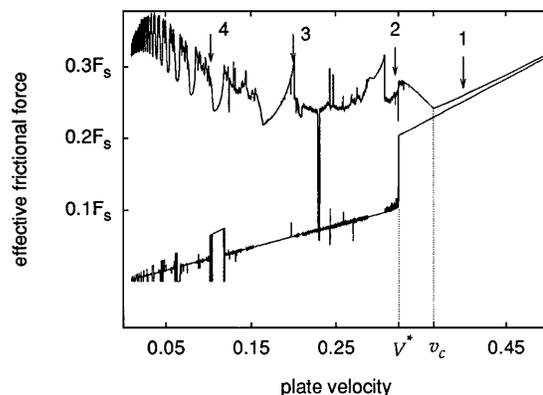


FIG. 4. Friction forces acting on the top plate as a function of plate velocity. The lower curve is the dissipative contribution; the upper curve is the net force. The arrows indicate velocities of the plate corresponding to particle trajectories shown in Fig. 5.

friction  $\phi(\dot{Y})$ , given by the averaged Eq. (4), contains two terms. The first one is the potential component of the frictional force and the second one describes the dissipative contribution (see Fig. 4). The structure in the velocity dependence of the frictional force  $\phi(\dot{Y})$  corresponds to different types of particle trajectories, as shown in Fig. 5. We see that the motion of the particle has two characteristic behaviors: At low velocities  $V < V^*$ , the average velocity of the particle predominantly equals  $\frac{1}{2}V$ , except for short windows where the particle is trapped by one of the plates; for  $V > V^*$ , the particle always clings to one of the plates. This is illustrated clearly by the dissipative component of the frictional force presented in Fig. 4.

There are three types of particle trajectories for  $V < V^*$ : (1) The particle jumps between two plates being trapped by each of them for time much longer than  $V^{-1}$  (curve 2 in Fig. 5); (2) the particle undergoes fast oscillations with the period  $V^{-1}$ , around the trajectory

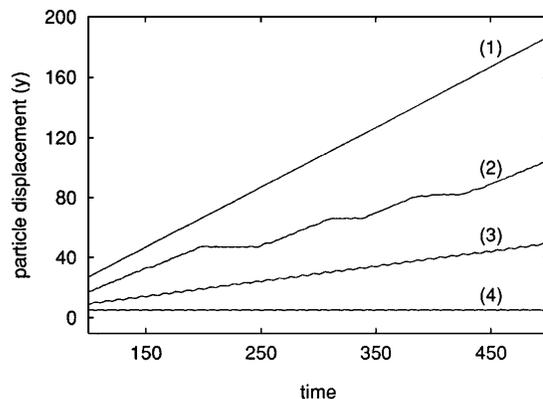


FIG. 5. Particle trajectories for selected values of the plate velocities.

$x = \frac{1}{2}Vt$  (curve 3 in Fig. 5); (3) the particle clings to one of the plates (curve 4 in Fig. 5). In the first two cases the time-averaged velocity of the particle equals  $\frac{1}{2}V$ , and in the third case it equals  $V$  or  $0$ . The dissipative component of the friction (lower curve in Fig. 4) reflects clearly these features of the motion. The local minima and maxima in the velocity dependence of the net frictional force shown in Fig. 4 correspond to the trajectories of types (3) and (4). It should be stressed that for all trajectories in the region  $V < V^*$  (except for  $V = 0.23$ , which corresponds to a stable motion of the particle with the velocity  $\frac{1}{2}V$ ) the fluctuations of the particle velocity are of the order of, or even larger than, the velocity of the top plate. Curve 1 in Fig. 5 describes the case  $V > V^*$ .

The velocity-dependent features described above are similar to those discussed within our original model. Note that the transition velocity  $V^*$  found in the reduced model is somewhat smaller than the previously determined critical velocity  $v_c$ . However, motion with small fluctuations in the particle velocity occurs only for  $V > v_c$ . In spite of the particle being trapped by one of the plates in the region  $V^* < V < v_c$ , the fluctuations of the velocity are large, being of the order of  $V$ . The fluctuations decrease when we approach  $v_c$  from below. The decay of the potential component of the frictional force in the region  $V^* < V < v_c$ , which is proportional to the square of the amplitude of the velocity fluctuations, manifests the transition from erratic to smooth sliding. The sharp decrease of the potential component of the effective friction corresponds to the disappearance of global chaos in the dynamics of the particle.

In the range  $V > V^*$  we approximate the effective friction force  $\phi(V)$  by a cubic polynomial

$$\phi(\dot{Y}) = a + b\dot{Y} + c\dot{Y}^2 + d\dot{Y}^3. \quad (6)$$

We now return to Eq. (5) which, upon substituting the approximated force of Eq. (6), is of the Rayleigh-type differential equation that describes the stage motion. For stage velocities  $v < v_c$ , Eq. (5) has solutions which correspond to an oscillating spring force (limit cycle). For  $v > v_c$  it has a static solution (fixed point) which describes the sliding regime. An analytical solution of Eq. (5) can be obtained using the Bogoliubov-Krylov technique. One finds that the critical velocity coincides with the position of the minimum of the effective friction force in Eq. (6). The value of the critical velocity found from the adiabatic approximation, Eqs. (5) and (6), agrees well with the results of the numerical analysis of Eqs. (1a) and (1b). For velocities slightly less than  $v_c$  the amplitude of the force oscillations really scale as  $L \sim \sqrt{v_c - v}$ , as observed numerically.

The above considerations demonstrate that the adiabatic approach reasonably describes the dynamics of the top plate when the driving velocity is close to  $v_c$ . Within this picture, the presence of velocity intervals where the friction force decreases with increasing velocity is a

crucial condition for the existence of force fluctuations. It should be mentioned that Eq. (5) does not account for chaotic character of motion, but correctly describes the amplitudes of force oscillations.

To summarize, a single-particle model has been proposed which demonstrates the dynamical features observed experimentally and through simulations in nanoscale liquid films under shear. Our calculations suggest that the information obtained following the macroscopic motion of a plate does not allow one to draw an unambiguous conclusion on the dynamical structure of a molecular system embedded between the plates. Preliminary results indicate that the general characteristics obtained for a single particle hold when an embedded chain is considered [15]. For a wide range of system parameters we find that the motion is chaotic. Therefore the use of recently proposed chaos-controlling approaches is possible in order to convert chaos into periodic motion.

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