Experimental Observation of a Large Excess Quantum Noise Factor in the Linewidth of a Laser Oscillator Having Nonorthogonal Modes

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Several authors have predicted a substantial excess noise factor K_p multiplying the quantum-limited Schawlow-Townes linewidth for laser oscillators having nonorthogonal spatial modes, such as unstable-resonator lasers. Experimental observations of this factor have to date, however, been limited in both number and detail. We report here a detailed experimental measurement of this factor in a diode-pumped hard-edged unstable-resonator Nd:YVO$_4$ laser. The measured excess noise factor $K_p \simeq 330$ is in reasonable agreement with the theoretically predicted value. [S0031-9007(96)00714-4]

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Since the earliest theoretical analyses of laser oscillators [1,2], it has been widely accepted that the rate of spontaneous emission from an atom into any one cavity mode is on average equal to the rate of stimulated emission that would be caused by one noise photon in the same mode. This result can be derived either from an analysis of the interaction between an atom and the quantized electromagnetic field or from considerations of the thermal equilibrium between a collection of atoms and the blackbody radiative environment. From this fundamental result one can derive the well known Schawlow-Townes laser linewidth expression [1]. In 1979, however, Petermann first proposed the existence of an excess spontaneous emission or a so called excess noise factor K_p with a value greater than one which should multiply the Schawlow-Townes expression for the fundamental linewidth in gain-guided semiconductor lasers [3]. This prediction was initially controversial because it seemed to imply the existence of K_p noise photons per mode in contrast to the widely accepted one noise photon per mode. This seeming paradox was later resolved by Haus and Kawakami [4] who pointed out that the noise terms for different propagating modes in a loss-guided or gain-guided system are correlated, as contrasted to the familiar situation with power-orthogonal modes where the noise terms driving each eigenmode are uncorrelated. They showed that for a loss-guided system at thermal equilibrium the correlations between noise signals in different modes act to cancel any excess in the total spontaneous emission, so that thermal equilibrium with blackbody radiation is not violated. Each individual mode nonetheless experiences an excess spontaneous emission such that the Schawlow-Townes linewidth of a single-mode oscillator is multiplied by the Petermann excess noise factor K_p.

One of the present authors later noted that this excess noise factor, and the associated correlations between noise signals for different eigenmodes, should be general properties of all open-sided laser resonators or lens-guide systems with significantly nonorthogonal eigenmodes [5,6], rather than being limited to the gain-guided cases considered by Petermann and by Haus and Kawakami. The excess noise factor for an nth order eigenmode, as given in Eq. (21) of Ref. [6], then becomes

$$K_{pn} = \left[\frac{1}{|\gamma_n|^2} \left(\frac{1 - |\gamma_n|^2}{2\ln(|\gamma_n|)} \right)^2 \right] \iint \phi_n^*(\mathbf{s})\phi_n(\mathbf{s})\, d\mathbf{s}\,, \quad (1)$$

where γ_n is the mode eigenvalue and $\phi_n(\mathbf{s})$ is the adjoint transverse eigenmode with $d\mathbf{s} = dx\,dy$ representing the transverse coordinates across the resonator or waveguide. The expression within the square brackets results from longitudinal mode nonorthogonality, as restated by Hamel and Woerdman [7,8]. Even with large output coupling this longitudinal value is usually only slightly larger than unity. The overlap integral, which accounts for transverse mode nonorthogonality, has a value of one for stable resonator lasers or index guided systems, and a value of $\sqrt{2}$ for purely gain-guided systems. This nonorthogonality factor can, however, become as large as hundreds or even thousands for unstable-resonator lasers or unstable lens-guide systems [5,6].

Even though these excess noise effects have been the topic of many theoretical papers [3–7], experimental observations of this excess noise factor, particularly the transverse mode factor, have to date been limited in both number and detail [8–11]. Previous measurements on gain-guided or partially gain-guided unstable-resonator semiconductor diode lasers [9,10] were limited by large uncertainties concerning other parameters of the lasers in question. Earlier measurements from our group [11] made use of an unusual type of primarily gain-guided unstable resonator laser together with a sensitive but not truly self-calibrating measurement procedure for the quantum linewidth. In this Letter, therefore, we report a detailed experimental confirmation of a large Petermann excess noise factor as seen in the fundamental laser linewidth of an axially diode-pumped hard-edged unstable-resonator Nd:YVO$_4$ laser with no significant gain-guiding effects present. The hard-edged unstable resonator was chosen because its transverse-mode nonorthogonality gives it a large excess noise factor which provides a good signal to noise ratio and can also be calculated with reasonable accuracy. We are also now using a substantially more direct measurement method to observe the quantum limited linewidth.

A diode-pumped solid state laser scheme was chosen because lasers of this type are well understood and their properties are by now well characterized. In particular, solid state lasers have much less nonlinearity than semiconductor lasers for which Henry's α factor [12] and possible filamentation can increase measurement uncertainties. Diode pumping of such lasers can also be quieter and more stable than lamp pumping of similar lasers. Neodymium vanadate was chosen as the laser crystal because, compared with Nd:YAG, it has a much larger pump absorption coefficient and a higher gain cross section, which together allow the cavity to be short enough to ensure single longitudinal mode oscillation. Its anisotropic gain further ensures oscillation in a single polarization without need of extra optical components.

Our axially diode-pumped unstable cavity design is shown in Fig. 1. It consists of a 1 mm thick slab of 1% Nd-doped vanadate and a 10.2 mm divergent radius of curvature AR-coated mirror blank with a 450 μm diameter circular gold mirror spaced 0.8 mm away from the vanadate slab. The pump face of the vanadate slab is HR coated at the operating wavelength and AR coated at the pump wavelength, while the front facet is AR coated for 1.06 μm. The Nd:YVO_4 crystal is held by a heat sink with active temperature stabilization. The round-trip insertion loss of the Nd:YVO_4 slab due to imperfect HR coating and any possible scattering loss was found to be $\approx 1.5\%$ by measuring the reflective to incident power ratio of a beam from a Nd:YAG laser incident on the AR 1.06 μm coating side of the vanadate slab. The gold mirror was produced photolithographically and consists of 150 Å of Cr and 4000 Å of Au evaporated onto an AR-coated lens blank. Profilometer traces show the evaporated gold surface reproduces with excellent fidelity the curvature of the underlying mirror surface. This gold coating was measured to have a reflectivity of 97% at 1.06 μm. The laser diode pump light comes directly from the end of a 400 μm diameter fiber with a numerical aperture of 0.37 brought to within a fraction of a millimeter of the vanadate slab. The effective pump spot is therefore at least as large as the oscillating mode so that the oscillating mode profile is predominantly determined by the cold cavity parameters. This laser oscillates in a single polarization, longitudinal, and transverse mode, as checked by both a monochrometer and a scanning confocal Fabry-Pérot spectrum analyzer, up to a maximum power output of 240 mW as limited by the available pump power of 4.5 W.

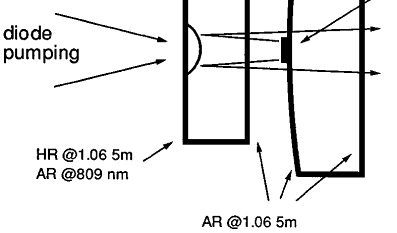

FIG. 1. Design of the unstable-resonator Nd:YVO_4 laser.

To calculate the theoretical excess noise factor for this unstable resonator, we consider the following details. First, at around 3.5 W pump power level, where we performed our laser linewidth measurement, the pump light induces a thermal lens in the vanadate slab with a focal length of ≈ 150 mm, as estimated theoretically and confirmed interferometrically. This thermal focusing slightly modifies the cold cavity parameters of the laser. In addition, the uniaxial character of the vanadate crystal breaks the cavity azimuthal symmetry, introducing a small amount of astigmatism in exact eigenmodes of the cavity. In practice, the equivalent cavity lengths for waves propagating in the x-z and y-z planes (where the crystal c axis is set along the x axis, and waves propagate along the z axis) are 1.26 and 1.36 mm, differing by $\approx 8\%$, and so the corresponding magnifications and equivalent Fresnel numbers [13] are 2.02 and 1.97, and 13.4 and 13.8, respectively. The excess noise factor K_p for our slightly astigmatic unstable cavity laser was then obtained from a 2D paraxial eigenmode calculation with both astigmatism and thermal lensing taken into account. Given about 10% uncertainty in our measured cavity parameters, including cavity length and thermal lensing in particular, and also including the longitudinal excess noise factor which is only 1.1 for calculated eigenvalue $\gamma_0 = 0.57$, the total excess noise factor for our unstable cavity Nd:YVO_4 laser is calculated to lie between 350 and 500.

With the excess noise factor included, the fundamental laser linewidth Δf_l in Hz is given by

$$\Delta f_l = K_p \frac{\pi \hbar \omega_l \Delta f_c^2}{P_l} \frac{\delta_e}{\delta_e + \delta_0}, \tag{2}$$

where ω_l is the laser frequency in the unit of radians per second, P_l is the laser output power, and Δf_c is the cold cavity linewidth in Hz. The factor $\delta_e/(\delta_e + \delta_0)$ accounts for the reduction in laser output power due to the existence of internal loss δ_0 in addition to the useful external loss δ_e for output coupling, where $\delta_e \simeq 63\%$ and $\delta_0 \simeq 4.5\%$ from calculation and measurement. The value of Δf_c was estimated to be 9.2 GHz from the numerical eigenvalue calculations. To verify this value, we measured the relaxation oscillation frequency ω_r versus normalized pump rate r, where $\omega_r^2 = (r - 1)\gamma_a\gamma_c - r^2\gamma_a^2/4$ [13] with γ_c and γ_a the cavity decay rate and

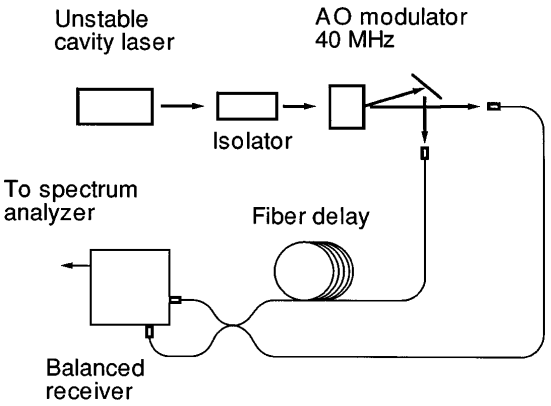

FIG. 2. Schematic of the self-heterodyne measurement.

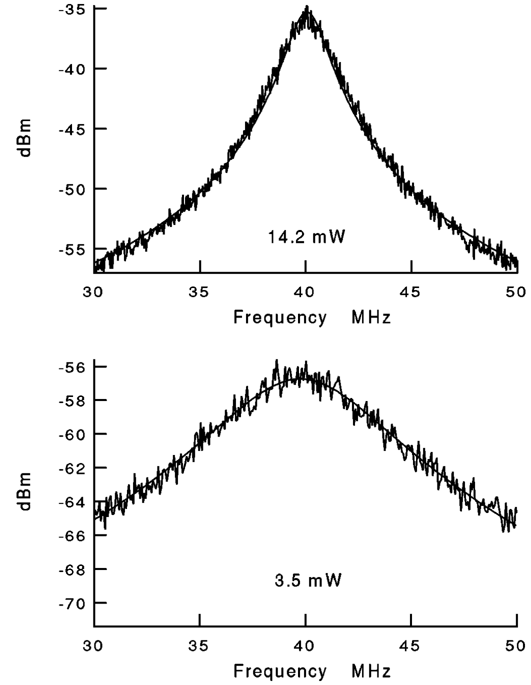

FIG. 3. Typical measured self-heterodyne spectra and superimposed Lorentzian fits.

laser upper level decay rate, respectively. From this relaxation oscillation frequency measurement together with a measured laser upper level lifetime of 95 μs, we obtained a cavity lifetime of 17 ps, which corresponds to a cold cavity linewidth of 9.6 GHz, in good agreement with the estimated value. Since this unstable-resonator laser has negligible gain-guiding effect, the relaxation oscillation complication observed in our previous experiment does not apply here [11,14]. We also note that this cold cavity linewidth is an order of magnitude smaller than the 240 GHz gain bandwidth of the neodimium doped vanadate crystal and that we are therefore justified in using the quantum limited linewidth expression Eq. (2) derived using the good-cavity approximation.

The fundamental laser linewidth was measured using a self-heterodyne technique [15,16] as illustrated in Fig. 2. The zero- and first-order diffractive laser beams generated by an acousto-optic modulator were coupled into the input ports of a single-mode-fiber Mach-Zehnder interferometer, with the zero order signal having an extra 155 m length of fiber inserted as a time delay. The two output signals of the fiber coupler were then detected by a balanced receiver. When the delay time between the two optical paths is significantly larger than the coherence time of the measured signal and smaller than the characteristic time scale of technical noise fluctuations, as is the case in our measurement, the self-heterodyne spectrum for a quantum-noise-limited laser should be Lorentzian with a FWHM equal to twice the laser linewidth. Several self-heterodyne spectra were recorded at different laser power levels, with two typical examples shown in Fig. 3. The instrument noise was dominated by intensity shot noise and was 20 dB below the signal level in the worst case. We then performed least square log Lorentzian fits to the recorded spectra to obtain laser linewidths. The curve fits, as indicated by the solid lines, show good agreement with the experimental data. The measured laser linewidths determined in this fashion are plotted versus laser power in Fig. 4. As predicted, the linewidth is inversely proportional to laser power. Finally, a least square fit to these data points by Eq. (2) was done. This curve fit, combined with our knowledge of the cold cavity linewidth from the relaxation oscillation frequency measurement mentioned above, allows us to calculate the excess noise factor. Three sets of laser linewidth versus laser power measurements were performed, with the unstable cavity slightly realigned between each experiment to obtain maximum power. The resulting excess noise factor in each case was $K_p \simeq 330$ with a variation of less than 10% from measurement to measurement.

In conclusion, detailed measurements of the fundamental laser linewidth for a diode-pumped hard-edged unstable-cavity Nd:YVO_4 laser using a self-heterodyne technique confirm that this laser has a substantial excess quantum linewidth with a value of approximately 330 times the usual Schawlow-Townes value, in reasonable agreement with the theoretically predicted value. We believe this measurement experimentally confirms

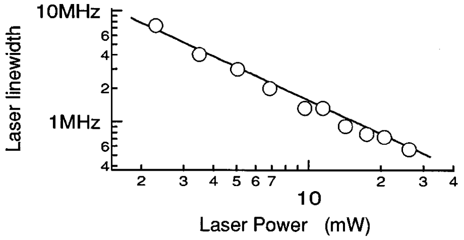

FIG. 4. Laser linewidth versus laser power.

the predicted excess noise factor due to transverse mode nonorthogonality in laser oscillators, and puts this interesting and somewhat unusual quantum noise effect on a solid experimental footing.

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