Motion Induced Radiation from a Vibrating Cavity

Astrid Lambrecht,[1] Marc-Thierry Jaekel,[2] and Serge Reynaud[3]
[1]Max-Planck-Institut für Quantenoptik and Ludwig-Maximilians-Universität München, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany
[2]Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, Université de Paris-Sud, Centre National de la Recherche Scientifique, 24 rue Lhomond, F-75231 Paris Cedex 05, France
[3]Laboratoire Kastler Brossel, Ecole Normale Supérieure, Université Pierre et Marie Curie, Centre National de la Recherche Scientifique, 4 place Jussieu, F-75252 Paris Cedex 05, France
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We study the radiation emitted by a cavity moving in vacuum. We give a quantitative estimate of the photon production inside the cavity as well as of the photon flux radiated from the cavity. A resonance enhancement occurs not only when the cavity length is modulated but also for a global oscillation of the cavity. For a high finesse cavity the emitted radiation surpasses radiation from a single mirror by orders of magnitude. [S0031-9007(96)00676-X]

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Vacuum field fluctuations exert radiation pressure forces on any scatterer placed in empty space. For two mirrors at rest in vacuum, this effect has been known for a long time as the Casimir effect [1]. It has more recently been recognized that dynamical counterparts of this static force appear for moving scatterers. For some types of motion, the field does not remain in the vacuum state, but photons are produced through nonadiabatic processes [2]. Because of energy conservation, the scatterers' motion then has to be damped out, and this damping may be associated with dissipative radiation reaction forces.

Motion induced effects of vacuum radiation pressure do not require the presence of two mirrors but already exist for a single mirror moving in vacuum. In this case, the radiation reaction force is known to arise as soon as the mirror has a nonuniform acceleration [3]. The effects of radiation from a moving mirror and the associated radiation reaction force raise intriguing questions with respect to the standard mechanical description of motion. They imply that dissipative effects are associated with the motion of mirrors in vacuum, although this motion has no further reference than vacuum itself. They thus seem to challenge the principle of relativity of motion. It would therefore be very important to obtain experimental evidence of these dissipative processes associated with motion in vacuum. However, vacuum radiation pressure scales as Planck's constant $\hbar$ and produces therefore only small mechanical perturbations for any macroscopic mirror, so that the feasibility of an experimental demonstration of motion induced dissipation is usually considered to lie out of reach of present technology.

The aim of the present Letter is to show that quantitative figures are greatly improved when the attention is focused onto the emission of radiation from an empty high finesse cavity oscillating in vacuum. Indeed, the number of emitted photons is the ratio of the radiated energy to the photon energy and thus scales as $\hbar^0$. This argument clearly supports a detection of optical rather than mechanical signatures of motion induced dissipation. Furthermore, a cavity configuration should allow one to take advantage of resonance enhancement effects.

Motion induced radiation can be interpreted using analogies with optical parametric processes. It is well known that the cavity field is parametrically excited when the mechanical cavity length is modulated at a frequency equal to an even integer multiple of the fundamental optical resonance frequency. If the cavity field is initially in the vacuum state, this excitation leads to a squeezed vacuum state [4] which differs from the pure vacuum state and, in particular, contains photons. Compared to the situation with a single oscillating mirror, radiation is resonantly enhanced in this cavity configuration. More strikingly, a resonant enhancement also exists when the cavity oscillates as a whole, with its mechanical length kept constant, at frequencies equal to odd integer multiples of the fundamental optical resonance frequency. Motional radiation is in this case reminiscent of photon emission from a single oscillating mirror, however, with the difference that it is enhanced by the cavity finesse.

A number of calculations has been devoted to the energy buildup inside a cavity with perfect mirrors [5]. However, these calculations do not provide satisfactory answers to the previously discussed questions. They do not consider the photons radiated by the cavity since the latter is treated like a closed system. Even for the photons produced inside the cavity, the hypothesis of perfect mirrors amounts to disregarding the important problem of finite lifetime of photons inside the cavity. In this Letter, in contrast, we study the configuration of a cavity built with partly transmitting mirrors. The cavity thus appears as an open system able to radiate into the free field vacuum. At the same time, the influence of the cavity finesse may be quantitatively evaluated.

For simplicity, we limit ourselves here to two-dimensional space-time calculations. As is well known from the analysis of squeezing experiments, the transverse structure of the cavity modes does not change appreciably the results obtained from this simplified model. Each transverse mode is correctly described by a two-dimensional model as soon as the size of the mirrors is larger than the spot size associated with the mode. The two-dimensional model thus corresponds to a conservative estimate where one transverse mode is efficiently coupled to the moving mirrors. A more precise evaluation for a realistic configuration should take diffraction into account and would probably lead to a result obtained by multiplying the two-dimensional result by the Fresnel number, i.e., the number of efficiently coupled transverse modes [6].

Before studying the cavity configuration, we consider briefly the case of a single moving mirror and calculate the photon flux as well as the spectrum of the emitted radiation. To derive the radiation, we use general arguments associated with scattering theory, without specific assumptions on the form of the interaction between mirror and field. This approach does not rely on a detailed microscopic analysis and is therefore applicable to any type of mirror as long as its internal dissipation is negligible. We disregard the recoil of the mirror which is supposed to have a macroscopic mass. To specify the scattering properties of the mirror, we introduce column matrices $\Phi[\omega]$ which contain the components at a given frequency ω of the free fields propagating in opposite directions,

$$\Phi(\omega) = \sqrt{\frac{\hbar}{2(\omega)}} \begin{bmatrix} \theta(\omega) a_{+,\omega} + \theta(-\omega) a^{\dagger}_{+,-\omega} \\ \theta(\omega) a_{-,\omega} + \theta(-\omega) a^{\dagger}_{-,-\omega} \end{bmatrix}. \quad (1)$$

Field components with positive or negative frequencies correspond, respectively, to annihilation ($a_{\pm,\omega}$) and creation ($a^{\dagger}_{\pm,\omega}$) operators (θ is the Heaviside step function). The transformation from the input field $\Phi_{\rm in}$ to the output one $\Phi_{\rm out}$ is described by a unitary S matrix which contains the transmission and reflection amplitudes $s(\omega)$ and $r(\omega)$ at a given frequency

$$\Phi_{\rm out}[\omega] = \begin{bmatrix} s[\omega] & r[\omega] \\ r[\omega] & s[\omega] \end{bmatrix} \Phi_{\rm in}[\omega]. \quad (2)$$

The scattering of the field on a motionless mirror does not change the field frequency, and the vacuum state is then preserved, as a consequence of unitarity.

When the mirror is moving, the frequency of the field is changed by the scattering process, and the S matrix now describes this frequency change,

$$\Phi_{\rm out}[\omega] = \int \frac{d\omega'}{2\pi} S[\omega, \omega'] \Phi_{\rm in}[\omega']. \quad (3)$$

Assuming that the incoming field is in the vacuum state, one obtains the following expression for the number N of photons radiated into vacuum by the moving mirror

$$N = \int_0^{\infty} \frac{d\omega}{2\pi} \int_0^{\infty} \frac{d\omega'}{2\pi} n[\omega, \omega'],$$

$$n[\omega, \omega'] = \frac{\omega}{\omega'} \mathrm{Tr}\, (S[\omega, -\omega'] S[\omega, -\omega']^{\dagger}). \quad (4)$$

$n[\omega, \omega']$ is the spectral density which describes the number of particles present in the output field. Photon creation from vacuum is associated with a scattering process from a negative frequency $-\omega'$ to a positive frequency ω. The radiation has to be summed over the two output ports as indicated by the trace Tr.

When the S matrix is evaluated in a first order expansion in the displacement, which is valid for small displacements in which we are interested here, the spectral density $n[\omega, \omega']$ is proportional to the square modulus of the frequency component $\delta q[\omega + \omega']$ of the displacement

$$n[\omega, \omega'] = \frac{\omega \omega'}{c^2} \gamma[\omega, \omega'] |\delta q[\omega + \omega']|^2,$$

$$\gamma[\omega, \omega'] = 2(1 - s[\omega]s[\omega'] + r[\omega]r[\omega'] + 1 - s[\omega]^* s[\omega']^* + r[\omega]^* r[\omega']^*). \quad (5)$$

This expression results from a linear approximation of the motional perturbation of the field, but it is valid without any restriction on the motion's frequency. It is directly connected to the general relation which exists between the motional perturbation of the scattering matrix and the radiation pressure force exerted upon the mirror [7].

In the following, we consider the case of a mirror following a harmonic motion at a frequency Ω. Since we expect the radiation of photons to be proportional to time, we focus our attention on a harmonic motion of amplitude a during a time T,

$$\delta q(t) = 2a \cos(\Omega t), \qquad 0 < t < T. \quad (6)$$

For a long oscillation time T, we find the number N of radiated photons to be defined per unit time,

$$\frac{N}{T} = \frac{a^2}{c^2} \int_0^{\Omega} \frac{d\omega}{2\pi} \omega(\Omega - \omega) \gamma[\omega, \Omega - \omega]. \quad (7)$$

This result is similar to the expression one would obtain for the number of photons spontaneously emitted by an atom coupled to vacuum fluctuations, calculated with Fermi's golden rule. Here the emission is generated by the parametric coupling of the mirror's mechanical motion to vacuum radiation pressure rather than by the coupling of the atomic dipole to the vacuum field. Hence, photons are emitted through a two-photon parametric process rather than through a one-photon process. As is well known, spontaneous emission is not accompanied by absorption processes because vacuum is the field ground state. Here the same property entails that photons are only emitted at frequencies ω and ω' smaller than the frequency Ω of the mechanical motion. Each parametric process corresponds to the emission of two photons carrying away an energy $\hbar\Omega = \hbar(\omega + \omega')$, so that the radiated energy may be obtained as $\frac{1}{2} N \hbar \Omega$. This energy corresponds exactly to the work supplied by the mirror against the radiation reaction force, in absence of other dissipative mechanisms. This consistency between mechanical dissipation and optical radiation is ensured by expression (7) where N appears to be proportional to the noise spectrum at frequency Ω of the fluctuations of vacuum radiation pressure experienced by the mirror [7].

In the limiting case of a nearly perfect mirror ($s \to 0; r \to -1$), we obtain a simplified expression for the number of radiated photons

$$\frac{N}{T} = \frac{8a^2}{c^2} \int_0^{\Omega} \frac{d\omega}{2\pi}\, \omega(\Omega - \omega) = \frac{2a^2\Omega^3}{3\pi c^2},$$
$$N = \frac{\Omega T}{6\pi}\left(\frac{v}{c}\right)^2, \qquad v = 2\Omega a. \tag{8}$$

Expression (8) for N is a product of two dimensionless factors, namely, the number of mechanical oscillation periods during the time T and the square of the maximal velocity v of the mirror divided by the velocity of light c. A characteristic feature of motion induced radiation, which could be used in an experiment to distinguish it from spurious effects, is the parabolic shape of its spectral density with a maximum at $\omega = \Omega/2$.

The derivation of motion induced radiation is similar in the case of two moving mirrors. Assuming the two mirrors follow a harmonic motion at the same frequency Ω with respective amplitudes a_i ($i = 1, 2$), we deduce the number of photons radiated per unit time to be

$$\frac{N}{T} = \sum_{ij} \frac{a_i a_j}{c^2} \int_0^{\Omega} \frac{d\omega}{2\pi}\, \omega(\Omega - \omega)\gamma_{ij}[\omega, \Omega - \omega]. \tag{9}$$

As in the case of a single mirror, the functions γ_{ij} already appear in the evaluation of motional forces, and they have been studied previously [8]. We introduce here simplifying assumptions allowing one to obtain analytical expressions for the motional radiation. In the frequency range $[0, \Omega]$ one can in a good approximation assume the reflection coefficients r_1 and r_2 of the two mirrors to be real and frequency independent. In the following we are concentrating on the most interesting case where the cavity has a high finesse which implies that both r_1 and r_2 are close to unity. Since the functions $\gamma_{ij}[\omega, \omega']$ exhibit resonances when one of the emission frequencies ω or ω' corresponds to a cavity mode, we will keep the reflection coefficients, which appear in their denominators and thus determine their resonant behavior. In contrast, we will set to unity the reflection coefficients acting only as weighting factors in the numerators. With these assumptions the functions γ_{ij} only depend on the product $r_1 r_2$ of the reflection coefficients which we denote

$$r_1 r_2 = e^{-2\rho}, \qquad \rho \ll 1, \tag{10}$$

where $1/\rho$ measures the cavity finesse. They then read

$$\gamma_{11}[\omega, \omega'] = \gamma_{22}[\omega, \omega'] = 4 + 4D_+[\omega]D_+[\omega'], \qquad \gamma_{12}[\omega, \omega'] = \gamma_{21}[\omega, \omega'] = -4D_-[\omega]D_-[\omega'],$$
$$D_+[\omega] = \frac{\sinh(2\rho)}{\cosh(2\rho) - \cos(2\omega\tau)} = \sum_{k=-\infty}^{\infty} \frac{\rho}{\rho^2 + (\omega\tau - k\pi)^2}, \tag{11}$$
$$D_-[\omega] = \frac{2\sinh(\rho)\cos(\omega\tau)}{\cosh(2\rho) - \cos(2\omega\tau)} = \sum_{k=-\infty}^{\infty} \frac{(-1)^k \rho}{\rho^2 + (\omega\tau - k\pi)^2}.$$

τ is the time of flight of a photon from one mirror to the other. With the exception of the first term in γ_{11}, all terms contain denominators clearly associated with the presence of the cavity.

We can now calculate the emitted photon number by performing the integration (9) for the various Lorentzian components of the spectrum. Using the assumption of a high finesse cavity we find

$$\frac{N}{T} = \frac{\Omega^3(a_1^2 + a_2^2)}{3\pi c^2} + \sum_{k,k'=1}^{\infty} \frac{N_{k,k'}}{T},$$
$$\frac{N_{k,k'}}{T} = \frac{1}{\tau}\frac{k\pi}{c\tau}\frac{k'\pi}{c\tau} \frac{4\rho[a_1 - (-1)^{k+k'}a_2]^2}{4\rho^2 + (\Omega\tau - k\pi - k'\pi)^2}. \tag{12}$$

The photon flux outside the cavity can also be written by resumming the contributions of all modes,

$$\frac{N}{T} = \frac{\Omega^3}{3\pi c^2}(a_1^2 + a_2^2) + \frac{\Omega}{6\pi c^2}\left(\Omega^2 - \frac{\pi^2}{\tau^2}\right)\frac{\sinh(2\rho)\,(a_1 + a_2)^2}{\cosh(2\rho) + \cos(\Omega\tau)} + \frac{\Omega}{6\pi c^2}\left(\Omega^2 - \frac{\pi^2}{\tau^2}\right)\frac{\sinh(2\rho)\,(a_1 - a_2)^2}{\cosh(2\rho) - \cos(\Omega\tau)}. \tag{13}$$

The first term in these expressions is a nonresonant contribution coming from direct reflection of vacuum fluctuations on both sides of the cavity. All other terms describe resonances of the motional radiation occurring when the mechanical excitation frequency Ω is close to an integer multiple of the fundamental optical resonance frequency π/τ. $N_{k,k'}$ describes parametric emission of radiation into the optical modes of frequencies $k\pi/\tau$ and $k'\pi/\tau$. Compared to the result obtained for a single mirror, the radiated photon flux is enhanced by a resonance factor which is essentially the cavity finesse. For the lowest mechanical resonance at $\Omega = 2\pi/\tau$, only one intracavity mode is excited ($k = k' = 1$). This corresponds to the situation studied in most works on intracavity field buildup [5]. In the more general frame developed in the present Letter, higher resonance frequencies exist, giving rise to several emission peaks. The emission peaks all have the same spectral width given by the cavity finesse, and their relative intensities reproduce a parabolic spectrum, as the one obtained for a single moving mirror, however, with a large resonant enhancement. The information contained in the set of peaks can again be used to distinguish motion induced radiation from spurious effects.

In Eq. (12), even modes $\Omega = 2\pi/\tau, 4\pi/\tau, \ldots$ appear as elongation modes which correspond to a periodic modulation of the mechanical cavity length. In contrast, odd modes $\Omega = 3\pi/\tau, 5\pi/\tau, \ldots$ are excited by a global translation of the cavity with its length kept constant. The latter effect is thus reminiscent of radiation of a single oscillating mirror, since the cavity moves in vacuum without any further reference than vacuum itself. However, radiation is now enhanced by the cavity finesse. These two kinds of vibration modes, which appear to be contrasted in a mechanical point of view, have been obtained in a unified manner in our scattering approach which deals with the field bouncing back and forth in the cavity. The basic reason for this similar description within the scattering formalism is that the optical length as seen by the field varies in the same way for both kinds of modes, although the mechanical cavity length is modulated in one case and constant in the other one.

To estimate the stationary number of photons inside the cavity, we may use a simple balance argument. Each photon has a probability 4ρ of escaping from the cavity during each round-trip time 2τ. As we know the photon flux emitted by the cavity per unit time, we can deduce the number of photons $\mathcal{N}_{k,k'}$ produced by the oscillation in a pair of cavity modes,

$$\mathcal{N}_{k,k'} = \frac{k\pi}{c\tau} \frac{k'\pi}{c\tau} \frac{2[a_1 - (-1)^{k+k'} a_2]^2}{4\rho^2 + (\Omega\tau - k\pi - k'\pi)^2} . \quad (14)$$

So far we have given a quantitative estimate for the number N of radiated photons as well as for the number $\mathcal{N}$ of photons produced inside the cavity. The model of a cavity with partly transmitting mirrors allows us to evaluate now the resonance enhancement factor in terms of the cavity finesse. A mechanical excitation at exact resonance leads to the following orders of magnitude for N and $\mathcal{N}$:

$$N \simeq \frac{\Omega T}{2\pi} \frac{v^2}{c^2} \frac{1}{\rho} , \qquad \mathcal{N} \simeq \frac{v^2}{c^2} \frac{1}{\rho^2} , \quad (15)$$

where v measures either the sum or the difference of the peak velocity of the vibrating mirrors, depending on the mode parity. We may emphasize that not only the number of photons inside the cavity, but also the number of radiated photons, diverge at the limit of perfectly reflecting mirrors where the finesse of the optical and mechanical resonances goes to infinity. This shows that the simple model which treats the cavity as a closed system misses important physical phenomena.

To be more specific about the orders of magnitude, let us recall that we have assumed the input fields to be in the vacuum state. This assumption requires the number of thermal photons per mode to be smaller than 1 in the frequency range of interest ($\hbar\omega < k_B\Theta$ with k_B the Boltzmann constant and Θ the temperature). Low temperature technology thus points to experiments using small mechanical structures with optical resonance frequencies as well as mechanical oscillation frequencies in the GHz range. In this frequency range, the finesse of a superconducting cavity can reach 10^9 [9]. A peak velocity $v \simeq 1$ m/s, corresponding to an amplitude in the nm range, would thus be sufficient to obtain a radiated flux of 10 photons per second outside and a stationary number of 10 photons inside the cavity. It is important to emphasize that the peak velocity considered in the present analysis is only a small fraction of the typical sound velocity in materials so that fundamental breaking limits do not oppose these numbers. The photons may be detected outside the cavity by performing sensitive photon-counting detection of the radiated flux. Inside the cavity the state of the field could be probed with the help of Rydberg atoms [9]. Therefore, if a technique is found to excite a vibrating motion with the above characteristics, the challenge of an experimental observation of motional radiation in vacuum can be taken up.

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