Electrostatics of Vortices in Type-II Superconductors

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In a type-II superconductor the gap variation in the core of a vortex line induces a local charge modulation. Accounting for metallic screening, we determine the line charge of individual vortices and calculate the electric field distribution in the half space above a field penetrated superconductor. The resulting field is that of an atomic size dipole $\mathbf{d} \sim ea_B\hat{\mathbf{z}}$, $a_B = \hbar^2/me^2$ is the Bohr radius, acting on a force microscope in the pico- to femto-Newton range. [S0031-9007(96)00683-7]

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The trapping of a magnetic flux $\Phi_0 = hc/2e$ by a vortex line in a type-II superconductor is a well known phenomenon [1]. Less familiar, however, is the fact that a vortex line in general traps an electric charge Q as well. It is the purpose of this Letter to determine this vortex line charge quantitatively and to discuss the feasibility of its experimental observation.

The vortex line charge Q has been discussed before by Khomskii and Freimuth [2] and by Feigel'man *et al.* [3] (see also [4]) within the context of the sign change in the Hall coefficient, as observed in a number of type-II superconductors [5]. Here, we concentrate on the vortex charge and its accompanying electrostatic features, with a specific emphasis on its experimental observability.

The main reason for the charge accumulation around the vortex is found in the particle-hole asymmetry, as quantified by the energy dependence of the density of states (DOS, per spin) $N(E)$ at the Fermi level, $Q \propto dN(E)/dE|_\mu$. In the presence of particle-hole asymmetry, the carrier density $n(\mu, \Delta)$ in the superconductor not only depends on the chemical potential μ, but on the energy gap Δ as well. The singular behavior of the phase at the center of a vortex leads to the formation of a core with a suppressed gap function $\Delta(R \to 0) \to 0$, where R measures the distance from the phase singularity. With the chemical potential fixed, charge carriers [electrons (holes) for a nearly filled (empty) band] are expelled from this core region. Metallic screening drastically reduces the accumulated charge; however, in our analysis below, we show that the residual vortex line charge $|Q| \sim ek_F(\lambda_{\rm TF}/\xi)^2$ is still experimentally observable (here, k_F, $\lambda_{\rm TF}$, and ξ denote the Fermi wave number, the Thomas-Fermi screening length, and the coherence length, respectively; we define $e > 0$). In particular, one may envisage the classic geometry for the observation of vortices via the Bitter-decoration method [6], with the superconductor filling the half space $z < 0$ and penetrated by a magnetic field $\mathbf{B} \parallel \hat{\mathbf{z}}$; see Fig. 1. The vortex line charge produces an electric field in the vacuum above the superconductor ($z > 0$), which corresponds to the one of a surface electric dipole $\mathbf{d} \parallel \hat{\mathbf{z}}$ with unit $\pm$ charges separated by a distance ~ 1 Å. The vortex charge is associated with the core size ξ and therefore a much higher resolution can be expected in an electrostatic experiment as compared to the magnetic experiments probing structures on the scale of the penetration depth λ_L. In the following, we derive the vortex line charge Q and solve the "half-space" electrostatic problem for a single vortex and for the vortex lattice. Next, we discuss the observability of the vortex charge and close with a few more subtle questions regarding our analysis.

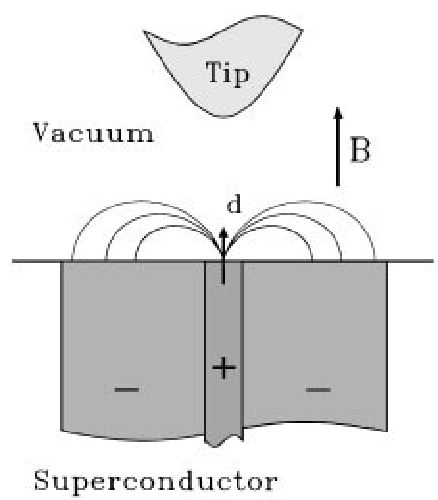

FIG. 1. The superconductor (lower half space) is penetrated by the magnetic field **B**. The resulting vortex line is charged due to the particle-hole asymmetry as quantified by the finite derivative of the density of states at the Fermi level, $dN(E)/dE|_\mu$. Charge carriers are expelled from the vortex core (core radius $\sim\xi$, + region in the figure) and an equal and opposite screening charge on a scale $\min(a_\Delta, \lambda_L)$ accounts for charge neutrality. The electric field generated in the upper half space (see field lines in the figure) is that of a surface dipole **d** of size $\sim ea_B$. A tip approaching the surface is attracted to the dipole with a force depending on the specific setup; see Eqs. (15) for a grounded tip and (17) for a tip biased with a voltage V against the superconductor.

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The origin of the vortex charge can be understood on the basis of a textbook problem [7]: Consider the Sommerfeld free electron model for a metal and determine the particle density n at *fixed chemical potential* μ, $n(T) \approx n(0)[1 + (\pi^2/8)T^2/\mu^2]$. The density increase δn is a consequence of the finite temperature smearing of the Fermi function combined with the finite slope in the density of states $N'_\mu \equiv dN(E)/dE|_\mu$ ($= 3n/8\mu^2$ in a 3D parabolic band). For a general Fermi surface with a smooth DOS we have

$$\delta n \approx (\pi T)^2 N'_\mu/3\,. \tag{1}$$

Note that the sign of δn depends on that of N'_μ, with $\delta n > 0$ for electronlike carriers [8].

Next, consider a BCS superconductor where the pair occupation probability $v_{\mathbf{k}}^2 = (1 - \xi_{\mathbf{k}}/E_{\mathbf{k}})/2$ [$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ and $E_{\mathbf{k}} = (\xi_{\mathbf{k}}^2 + \Delta^2)^{1/2}$ denote the excitation energies in the normal and superconducting state, respectively] determines the density via $n = 2\sum_{\mathbf{k}} v_{\mathbf{k}}^2$. The opening of a gap Δ in the spectrum produces an analogous smearing in the occupation probability and the density changes with Δ according to

$$\delta n \approx \Delta^2 N'_\mu \ln(\hbar\omega_D/T_c)\,, \tag{2}$$

where ω_D denotes the usual frequency cutoff on the attractive interaction. Indeed, the occupation probability in the superconductor resembles a Fermi distribution with a temperature $T \simeq \Delta$ [9], in agreement with the results (1) and (2).

In the presence of a vortex line the gap parameter turns to zero in the core,

$$\Delta^2(R) \approx \Delta_\infty^2 R^2/(R^2 + \xi^2)\,, \tag{3}$$

where $R < \lambda_L$ denotes the radial distance from the phase singularity, Δ_∞ is the magnitude of the gap parameter far away from the core, and the coherence length ξ determines its spatial extent. The slow algebraic decay $\delta\Delta^2 \sim -\Delta_\infty^2(R)^2$ is a consequence of the slow decay of the supercurrent $j(R) \sim j_0\xi/R$ within the London screening length λ_L (j_0 is the depairing current density; $\delta\Delta^2$ drops to zero exponentially for $R > \lambda_L$).

We account for metallic screening within a Thomas-Fermi approximation, substituting μ by the electrochemical potential $\mu + e\varphi$ in the expression for the density n. The density modulation $\delta n(R) = n[\mu + e\varphi(R), \Delta(R)] - n(\mu, \Delta_\infty)$ is driven by the variation in the gap function $\Delta(R)$ and induces a scalar potential $\varphi(R)$, which is obtained from the solution of Poisson's equation

$$\nabla^2\varphi(R) = 4\pi e\,\delta n(R)\,. \tag{4}$$

Linearizing in φ and Δ we arrive at

$$[\nabla^2 - \lambda_{\mathrm{TF}}^{-2}]\varphi(R) = 4\pi e\,\delta n_{\mathrm{ext}}(R)\,, \tag{5}$$

with the Thomas-Fermi length $\lambda_{\mathrm{TF}} = (8\pi e^2 N_\mu)^{-1/2}$ and the "external" density modulation

$$\delta n_{\mathrm{ext}}(R) = -N_\mu \Delta_\infty^2 \frac{\xi^2}{R^2 + \xi^2}\frac{d\ln T_c}{d\mu} \tag{6}$$

[we have substituted the expression $N'_\mu \ln(\hbar\omega_D/T_c)$, see (2), by the phenomenological parameter $N_\mu d\ln T_c/d\mu$ using the BCS expression $T_c \approx \hbar\omega_D \exp(-1/N_\mu V)$]. The integration of (6) over the planar coordinate $\mathbf{R}$ provides the total external line charge

$$Q_{\mathrm{ext}} \approx 2\pi e \Delta_\infty^2 \xi^2 N_\mu \frac{d\ln T_c}{d\mu}\ln\frac{\lambda_L}{\xi}\,. \tag{7}$$

For a BCS model in the clean limit $\Delta_\infty^2\xi^2 N_\mu = k_F\tilde{\mu}/\pi^4$ with $\tilde{\mu} = m_{\mathrm{eff}} v_F^2/2$ (for a nontrivial Fermi surface we have $\mu \neq \tilde{\mu}$ in general), and using $d\ln T_c/d\ln\tilde{\mu} \approx \ln(\hbar\omega_D/T_c) \sim 1 - 10$, we obtain an external line charge of order ek_F, with only a weak dependence on the superconducting parameters T_c and λ_L.

We determine the real charge distribution $\rho(R) = -e\,\delta n(R) = -\nabla^2\varphi(R)/4\pi$ (positive for electronlike carriers) by solving the screened Poisson equation (5). In the limit $\lambda_{\mathrm{TF}} \ll \xi$ we obtain

$$\rho(R) = \frac{ea_B}{\pi^3}\frac{d\ln T_c}{d\ln\tilde{\mu}}\frac{\xi^2 - R^2}{(R^2 + \xi^2)^3}\,, \tag{8}$$

where a_B denotes the Bohr radius. Overall charge neutrality requires the total charge $\int d^2R\,\rho(R)$ to vanish: The line charge accumulated within the vortex core is $Q_\xi = ea_B(d\ln T_c/d\ln\tilde{\mu})/(4\pi\xi)^2 \approx Q_{\mathrm{ext}}\lambda_{\mathrm{TF}}^2/\xi^2$ and an equal and opposite charge is provided by the screening outside the core region; see Fig. 1.

Next, we solve the electrostatic problem for a single vortex line penetrating a superconductor filling the lower half space $z < 0$; see Fig. 1. The potential $\varphi(R,z)$ generated by the charge density $\rho_{\mathrm{ext}}(R,z) = -e\,\delta n_{\mathrm{ext}}(R)\Theta(-z)$ is obtained by solving the (screened) Poisson equation $[\nabla^2 - \lambda_{\mathrm{TF}}^{-2}\Theta(-z)]\varphi(R,z) = -4\pi \times \rho_{\mathrm{ext}}(R,z)$. We decompose the potential into a bulk and an interface term, $\varphi(R,z) = \varphi_\infty(R)\Theta(-z) + \varphi_0(R,z)$, where $\varphi_\infty(R)$ denotes the bulk solution of (5). The interface term $\varphi_0(R,z)$ can be obtained from the Fourier ansatz

$$\varphi_0(R,z) = \int \frac{d^2K}{(2\pi)^2}\varphi_0^\pm(\mathbf{K})\exp[i\mathbf{K}\cdot\mathbf{R} \mp k_z^\pm z]\,, \tag{9}$$

with $k_z^+ = K$ and $k_z^- = \sqrt{K^2 + \lambda_{\mathrm{TF}}^{-2}}$ referring to values $z > 0$ above and $z < 0$ below the vacuum-superconductor interface. Requiring continuity of φ and $\nabla\varphi$ across the interface, we can express $\varphi_0^\pm(\mathbf{K})$ through the source term $\varphi_\infty(K) = 4\pi\rho_{\mathrm{ext}}(K)/(K^2 + \lambda_{\mathrm{TF}}^{-2})$. After transformation back to real space we arrive at the final expression

$$\varphi(R, z > 0) = \int \frac{d^2K}{(2\pi)^2}\varphi_\infty(K)\frac{\sqrt{K^2 + \lambda_{\mathrm{TF}}^{-2}}}{K + \sqrt{K^2 + \lambda_{\mathrm{TF}}^{-2}}} \times \exp(i\mathbf{K}\cdot\mathbf{R} - Kz)\,. \tag{10}$$

The integral is dominated by small wave vectors and we may neglect K^2 as compared to $\lambda_{\mathrm{TF}}^{-2}$. Using $\rho_{\mathrm{ext}}(K) \approx Q_{\mathrm{ext}} K_0(K\xi)\ln(\lambda_L/\xi)$, with K_0 the modified Bessel function, the integration over $\mathbf{K}$ yields

$$e\varphi(R,z)\,[\mathrm{eV}] \approx 0.8\frac{m}{m_{\mathrm{eff}}}\frac{d\ln T_c}{d\ln\tilde{\mu}}\ln\frac{\min(z,\lambda_L)}{\xi} \times \frac{z[\text{Å}]}{(R^2+z^2)^{3/2}}, \qquad (11)$$

where we have chosen $z > \xi$ and all lengths are taken in angstroms [note that (11) is independent of the DOS, the latter appearing both in δn_{ext} and in the screening length λ_{TF}^2, and only weakly depends on the superconducting properties]. The result (11) is the potential generated by a surface dipole $\mathbf{d} \parallel \hat{\mathbf{z}}$ smeared on the scale ξ, $\varphi(\mathbf{r}) = \mathbf{d}\cdot\mathbf{r}/r^3$,

$$\mathbf{d} = \frac{ea_B\hat{\mathbf{z}}}{\pi^2}\frac{m}{m_{\mathrm{eff}}}\frac{d\ln T_c}{d\ln\tilde{\mu}}\ln\frac{\min(z,\lambda_L)}{\xi}. \qquad (12)$$

With the logarithms roughly compensating for the numerical π^{-2}, we find a dipole with unit $\pm$ charges separated by ~ 1 Å. The charge and field geometry are illustrated in Fig. 1.

The corresponding electrostatic problem for a vortex lattice is solved in the same manner. The integral $\int d^2K \exp[i\mathbf{K}\cdot\mathbf{R} - Kz]$ producing the dipole field has to be replaced by the sum over reciprocal lattice vectors $\mathbf{K_n}$ of the vortex lattice, $(2\pi/a_\Delta)^2\sum_{\mathbf{n}}\exp[i\mathbf{K_n}\cdot\mathbf{R} - K_{\mathbf{n}}z]$, where $a_\Delta = (2/\sqrt{3})^{1/2}(\Phi_0/B)^{1/2}$ denotes the lattice constant. Usually the sum can be restricted to the six nearest neighbor lattice vectors and the result reads

$$e\varphi_{\mathrm{VL}}(R,z)\,[\mathrm{eV}] \approx 15.0\frac{m}{m_{\mathrm{eff}}}\frac{a_B^2}{a_\Delta^2}\frac{d\ln T_c}{d\ln\tilde{\mu}} \times \ln\frac{\min(z,a_\Delta,\lambda_L)}{\xi} \times \left[1 + \exp(-2\pi z/a_\Delta)\sum_{\mathrm{n.n.}}\cos\mathbf{K_n}\cdot\mathbf{R}\right]. \qquad (13)$$

This completes our derivation of the charge and the electrostaticfield distribution for the individual vortex and the vortex lattice.

Is this vortex charge observable in an experiment? The most straightforward attempt to identify the vortex charge is based on (scanning) force microscopy. Indeed, the observation of single charge carriers by force microscopy has been reported by Schönenberger and Alvarado [10]. Below we consider two experimental setups: (i) A grounded metallic tip is approached to an individual vortex. The vortex (surface) dipole induces a second dipole in the tip, resulting in a dipole-dipole attraction between the vortex and the tip. The expected force is estimated to be of the order of $F_{\mathrm{ind}} \sim 10^{-17}$ N. (ii) The metallic tip is biased against the superconductor. In this capacitor geometry, the bias voltage V drives a charge transfer from the superconductor to the tip, leading to a tip-surface attraction F_{ts} which has to be compensated in the experiment. As the tip is approached to the vortex, the vortex-dipole–tip-charge interaction produces an additional force on the tip, which is the desired signal. The estimated force is proportional to the bias voltage V and is of the order of $F_{\mathrm{bias}} \sim 10^{-14}V[\mathrm{V}]$ N. Note that the mobile vortex dipole can be distinguished from static charged surface defects, as produced by adsorbed atoms and molecules, by driving the vortex with an external ac force and using a lock-in technique.

In order to estimate the dipole-dipole attraction in the first setup (i) we model the tip as a metallic sphere of radius ρ. Its center is chosen a distance $\zeta > \rho$ right above the vortex, thus producing the maximal tip-vortex attraction. A straightforward calculation using the image charge technique (see, e.g., Ref. [11]; we ignore higher order images) provides the result

$$F_{\mathrm{ind}} = 2\rho\zeta d^2\frac{2\rho^2+\zeta^2}{(\zeta^2-\rho^2)^4}. \qquad (14)$$

Inserting the expression (12) for the vortex dipole, using $e^2a_B^2 = 8.23\times10^{-8}$ N, and choosing a typical geometry with $\rho \sim 2$, we find

$$F_{\mathrm{ind}} \sim 2.5\times10^{-8}\left(\frac{a_B}{\rho}\right)^4\,\mathrm{N}, \qquad (15)$$

where we have assumed that $(m/m_{\mathrm{eff}})(d\ln T_c/d\ln\tilde{\mu}) \times \ln[\min(\zeta,\lambda_L)/\xi] \sim 10$. With $\rho \sim 100$ Å the resulting force is $F_{\mathrm{ind}} \sim 2\times10^{-17}$ N.

Next, we consider the capacitor setup (ii) where the tip is biased against the superconductor. It is convenient to model this geometry with a tip of spheroidal shape. Using elliptic coordinates (see, e.g., Ref. [12]), we define the superconductor and tip surfaces through the coordinates $\eta_{\mathrm{sc}} = 0$ and $\eta_{\mathrm{tip}} = \eta_0$, respectively. Of the three parameters, the surface-tip distance ζ, the tip radius of curvature ρ, and the tip aperture 2ϑ, only two can be freely chosen, $\rho/\zeta = \tan^2\vartheta = (1-\eta_0^2)/\eta_0^2$. Solving Poisson's equation $\Delta V = 0$, we find $V(\eta) = Vg(\eta)/g(\eta_0)$, resulting in an electric field $\mathbf{E}_{\mathrm{tip}}(a,R) = -2V\hat{\mathbf{z}}/g(\eta_0)(a^2+R^2)^{1/2}$ at the superconductor-vacuum interface. Here, $g(\eta) = \ln(1+\eta)/(1-\eta)$ and $a = \zeta/\eta_0$ is the scale factor in the transformation to elliptic coordinates. The energy of the vortex dipole in the electric field of the biased tip is given by $U(a,R) = -\mathbf{d}\cdot\mathbf{E}_{\mathrm{tip}}(a,R)/2$ (only half space) and for the forces perpendicular and parallel to the surface we obtain

$$(F_{\mathrm{bias}}^{\perp}, F_{\mathrm{bias}}^{\parallel}) = \frac{dV}{g(\eta_0)}\frac{1}{(R^2+a^2)^{3/2}}(a,R) \qquad (16)$$

(see Ref. [10] for the description of an ac technique used to separate the small modulation $F_{\mathrm{bias}}^{\perp}$ due to the vortex dipole from the large base force F_{ts} due to the image charge attracting the tip). The above results apply for distances $>\xi$; upon moving the tip closer to the vortex the details of the charge distribution become relevant and the forces change; e.g., in $F_{\rm bias}^{\perp}$ we have to replace the scale a by the spread of the vortex charge ξ. Using $\zeta \sim \rho$ and $\vartheta = \pi/4$ we obtain the numerical expression

$$F_{\rm bias} \sim 10^{-9} V[{\rm V}] \left(\frac{a_B}{\rho}\right)^2 {\rm N}\,. \qquad (17)$$

With $\rho \sim 100$ Å the force amounts to $F_{\rm bias} \sim 3 \times 10^{-14} V[{\rm V}]$ N. At present, forces in the pN range are observed in state of the art atomic force microscopy experiments and the fN regime will be taken on in the near future.

A number of alternative experiments detecting the vortex line charge Q look promising as well. Here we mention only the basic ideas. (i) One of the most sensitive electrometers is the single-electron transistor (SET); e.g., see Ref. [13], where the small central island is connected via tunnel junctions to the two leads. A capacitively coupled gate takes the device to its optimal working point. The device has to be fabricated onto the superconductor surface with only a thin insulating layer ($\sim$100 Å) decoupling the two systems electronically. Vortices driven across the central island act to modulate the gate voltage via their line charge and the signal can be picked up via a lock-in technique. Using setup (i) above we can estimate the induced charge on the island to be $\sim d\rho/\zeta^2 \sim 10^{-2} e$, which should be well detectable by a SET with a charge resolution of $\sim 10^{-4} e/\sqrt{\rm Hz}$ [13].

Another straightforward idea is to imitate the original decoration technique of Träuble and Essmann [6], using electric rather than magnetic particles. It seems difficult, however, to imagine an electric analog of the small ferromagnetic (Fe, Co, or Ni) particles being assembled and spread onto the superconductor surface in a similar fashion. Alternatively, one may resort to the use of electrons, separated from the superconductor surface by a thin ^{4}He film, and setting up a Wigner crystal. The interaction between the surface dipole array due to the vortex lattice with the 2D Wigner crystal will lead to new features affecting the physics of the vortex lattice and the Wigner crystal as well.

We turn to some more subtle issues. On a phenomenological level we can express the external density modulation (6) through the Ginzburg-Landau energy density $\mathcal{F} = N_\mu[\alpha|\Delta|^2 + \frac{\beta}{2}|\Delta|^4 + \gamma|\nabla\Delta|^2]$, with the parameters α, β, and γ depending on the chemical potential μ, $\delta n_{\rm ext}(R) = -\partial_\mu \mathcal{F}|_R^\infty$. The term $N_\mu(\partial_\mu\alpha)\,|\Delta|^2 \propto N_\mu' \ln(\hbar\omega_D T_c)\,(1 - T/T_c)$ produces the main contribution due to particle-hole asymmetry. A second term $\propto N_\mu'(1 - T/T_c)^2$ originates from taking the derivative of the prefactor N_μ in $\mathcal{F}$. Finally, a third contribution from the gradient term, $(N_\mu/\mu)\,(1 - T/T_c)\,|\xi\nabla\Delta|^2 \propto (N_\mu/\mu)\,(1 - T/T_c)^2$, is present even without particle-hole asymmetry and produces the well known London electric field $\mathbf{E}_L \propto \nabla v_s^2(R)$, compensating the centripetal force of the rotating superfluid. In our analysis above we have concentrated on the leading contribution in $1 - T/T_c$ and in $\ln(\hbar\omega_D T_c)$. Away from T_c the other terms will slightly modify our result.

In summary, we have determined the line charge associated with the formation of a vortex in a type-II superconductor. The charge is mainly driven by the particle-hole asymmetry, its bare value is $Q_{\rm ext} \sim ek_F$, and screening reduces this value down to $Q \sim ek_F\lambda_{\rm TF}^2/\xi^2$. We have solved the electrostatic problem related to the observation of the vortex charge on a superconductor surface and found the associated electric field to be that of a microscopic dipole $\mathbf{d} \sim ea_B\hat{\mathbf{z}}$. With our results we hope to motivate new experiments looking for the vortex charge itself (with an interesting relation to the sign change of the Hall effect), we propose a new imaging technique able to address the structure of vortex systems on the scale ξ, and we suggest use of the charge array set up by the vortices in other experimental areas such as the problem of electrons on ^{4}He films.

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