# Electromagnetically Induced Transparency in an Ideal Plasma

S. E. Harris
*Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305*
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A process analogous to electromagnetically induced transparency in atoms is described for an ideal plasma. Two electromagnetic fields whose frequency difference is near, but not equal, to the plasma frequency will drive a longitudinal plasma oscillation whose contribution to the current density opposes the current density produced by either field, if alone. This creates a passband at frequencies which would otherwise be below cutoff. [S0031-9007(96)02013-3]

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About six years ago, Boller *et al.* demonstrated a technique for making an optically opaque transition transparent to laser radiation [1]. This is done by applying two lasers whose frequencies differ by a nonallowed transition of an atom or molecule. The lasers are applied in a manner which causes the atom to evolve smoothly from its ground state to an antiphased superposition state, often termed as a population trapped or dark state [2]. When in this state, the dipole moment at one field, and sometimes at both fields, is very small, and the atom and the field (or fields) are nearly decoupled. The effect may also be used to produce transparency in the continuum [3], to control the real part of the refractive index, and, at least when near resonance, to eliminate optical self-focusing [4]. This technique, where one electromagnetic field controls the (complex) refractive index of another, is now called electromagnetically induced transparency (EIT). There is literature on this subject, both in its own right [5], and on its relation to lasers without inversion [6] and to nonlinear optics [7].

In this Letter we explore the question of whether a collective excitation of a medium, rather than an internal excitation of an atom or molecule, can be used to control the refractive index and to establish transparency. In particular, we examine the propagation of a laser beam in a plasma where the plasma frequency is higher than that of the laser frequency and where, therefore, without a second laser beam present, the propagation constant is imaginary.

We find that in the plasma, the role of the nonallowed transition of the single atom is replaced by a (collective) longitudinal plasma oscillation. The oscillation is driven by the $\vec{V} \times \vec{B}$ force at the difference of the applied laser frequencies. By allowing for a small two-photon detuning from the plasma frequency, we find that the phase of the plasma oscillation is such that its contribution to the induced current density opposes the primary currrent density, and that the presence of both lasers creates a passband in the otherwise opaque plasma.

Figure 1 is an energy schematic for this work. The quantity $\omega_p$ is the plasma frequency. The frequency which is initially below cutoff and around which a passband is to be created is denoted by $\omega_s$ and will be referred to as the Stokes frequency. The frequencies of the strong driving laser and of the longitudinal plasma oscillation are $\omega_a$ and $\omega_l = (\omega_a - \omega_s)$, respectively.

We first note the assumptions of this work: We will assume that one of the two laser beams is sufficiently weak that the time-varying component of the charge density $\rho$ is small as compared to the static component $n_e$, and that the other beam propagates as if alone in the plasma. In the first portion of what follows the intensity of the Stokes wave is taken to be small; in the second portion, the intensity of the wave with frequency $\omega_a$ is taken as small. We also assume that the pondermotive energy of the strong field is small as compared to $mc^2$ and do the calculation nonrelativistically. Thermal motion and collisions are neglected, thereby assuming that the unperturbed plasma frequency is independent of $\vec{k}$ and has a zero linewidth. Both laser beams are taken as plane waves of infinite extent in the transverse direction. This assumption is important in that it allows the neglect of both relativistic self-focusing [8] and the resonant pondermotive self-focusing mechanism of Joshi *et al.* [9]. The competition of many other plasma nonlinearities and instabilities are also neglected [10].

We take the applied fields at $\omega_a$ and $\omega_s$ to be plane waves with their $\vec{E}$ fields and velocities polarized in the $\vec{x}$ direction, and with their $\vec{B}$ fields polarized in the $\vec{y}$ direction. The plasma oscillation and its associated $\vec{E}$

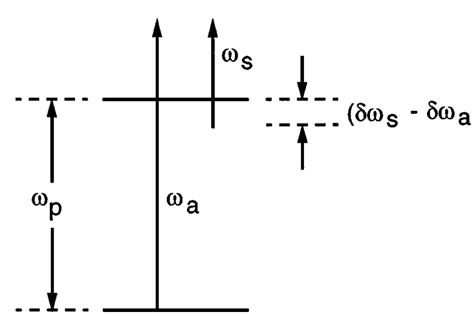


FIG. 1. Energy schematic for EIT in an ideal plasma. The electromagnetic fields have angular frequencies $\omega_a$ and $\omega_s$. The longitudinal plasma oscillation is driven at the frequency $\omega_l = \omega_a - \omega_s$ (not shown). The difference between the plasma frequency $\omega_p$ and $\omega_l$ is $\delta(\omega_p - \omega_l) = (\delta\omega_s - \delta\omega_a)$.

  

field and velocity are $\vec{z}$ polarized. Each of the field quantities and velocities is written as a product of a slowly varying envelope and a propagation factor and are hereafter taken as scalar quantities. For example, the Stokes $E$ field and the charge density are

$$E_s(t) = \frac{E_s}{2}\exp[j(\omega_s t - k_s z)] + \text{c.c.},$$
$$\rho(t) = \frac{\rho}{2}\exp[j(\omega_l t - k_l z)] + \text{c.c.} \tag{1}$$

With the envelopes of the velocities at $\omega_a$, $\omega_s$, and $\omega_l$ denoted by $V_a$, $V_s$, and $V_l$, the equations for the velocities of a single particle, in a fixed frame, are

$$\frac{\partial V_a}{\partial t} + j\omega_a V_a = -\frac{qE_a}{m} + \frac{qB_s V_l}{2m} + \frac{j}{2}k_s V_s V_l\,, \tag{2a}$$

$$\frac{\partial V_s}{\partial t} + j\omega_s V_s = -\frac{qE_s}{m} + \frac{qB_a V_l^*}{2m} + \frac{j}{2}k_a V_a V_l^*, \tag{2b}$$

$$\frac{\partial V_l}{\partial t} + j\omega_l V_l = -\frac{qE_l}{m} - \frac{q}{2m}(B_a V_s^* + V_a B_s^*)\,. \tag{2c}$$

The derivatives of the envelopes at $V_a$ and $V_s$ are taken to vary slowly as compared to $\omega_a$ and $\omega_s$ and are immediately dropped. Because the Stokes field is very small, we have $B_a = k_a E_a/\omega_a$ and $V_a = jqE_a/\omega_a m$. This, in turn, causes the last two terms on the right-hand side (RHS) of Eq. (2b) to cancel so that $V_s = +jqE_s/\omega_s m$. Equation (2c) is then combined with the divergence equation and the equation of continuity, together with $V_s$, to yield the equation of motion for the plasma oscillation

$$\frac{\partial \rho}{\partial t} - j2\delta\omega_s \rho = -\frac{qn_e(k_a - k_s^*)}{2m\omega_p}(V_a B_s^* + B_a V_s^*)$$
$$= j\frac{n_e q^2 (k_a - k_s^*)^2}{2m^2\omega_a\omega_s\omega_p}E_a E_s^*. \tag{3}$$

This oscillation is driven by the $\vec{z}$ directed $\vec{V}\times\vec{B}$ force at a frequency of $(\omega_a - \omega_s)$ and differs from the natural plasma oscillation frequency $\omega_p$ by $\delta\omega_s = \omega_p - (\omega_a - \omega_s)$. In deriving Eq. (3), we have assumed that the envelope of the charge density $\rho$ varies [Eq.(1)] slowly as compared to the plasma frequency.

The current densities which drive the fields at $\omega_a$ and $\omega_s$ are

$$J_a = -qn_e V_a - \frac{q\rho V_s}{2}\,, \tag{4a}$$

$$J_s = -qn_e V_s - \frac{q\rho^* V_a}{2}\,. \tag{4b}$$

The first term on the RHS of these equations, if alone, yields the normal dielectric constant of a plasma. The essence of the transparency effect is to cause $\rho$ to be phased such that the nonlinear term in the current density equations opposes the linear term. From Eq. (4b), Maxwell's equations, and $V_a$ and $V_s$ (retaining the small Stokes field assumption), we obtain

$$\rho^* = -\frac{2\omega_a}{\omega_s\omega_p^2}n_e(c^2k_s^2 + \omega_p^2 - \omega_s^2)\frac{E_s}{E_a}\,. \tag{5}$$

We will require the quantities

$$\mathcal{E}_a = \frac{q^2|E_a|^2}{4m\omega_a^2}\,; \qquad \mathcal{E}_s = \frac{q^2|E_s}{4m\omega_s^2}\,,$$
$$k_{a0}^2 \equiv \frac{\omega_a^2 n_{a0}^2}{c^2} = \frac{1}{c^2}(\omega_a^2 - \omega_p^2)\,, \tag{6}$$
$$k_{s0}^2 \equiv \frac{\omega_s^2 n_{s0}^2}{c^2} = \frac{1}{c^2}(\omega_s^2 - \omega_p^2)\,,$$

where $\omega_p^2 = n_e q^2/\epsilon_0 m$. $\mathcal{E}_a$ and $\mathcal{E}_s$ are the pondermotive (oscillatory) energies of an electron in the presense of either field if alone; $k_{a0}^2$ and $k_{s0}^2$ are the square of the propagation constants of either of the beams, again, if alone; and $n_{a0}$ and $n_{s0}$ are the corresponding refractive indices.

We will first give the dispersion relation for monochromatic fields and then return to a discussion of the conditions which are necessary to approximately establish this solution with time-varying fields. The dispersion relation for a weak Stokes beam is obtained by setting the derivative in Eq. (3) to zero and setting the steady-state value of $\rho$ equal to $\rho$ from Eq. (5). With the definitions of Eq. (6) we obtain [10]

$$\frac{(k_{a0}^* - k_s)^2}{2}\left(\frac{\mathcal{E}_a}{mc^2}\right)\omega_p + (k_{s0}^2 - k_s^2)\delta\omega_s = 0\,. \tag{7}$$

The solution of Eq. (7) may be expressed in terms of characteristic frequencies $w_{\text{pole}}$ and $w_{\text{crit}}$:

$$k_s(\delta\omega_s) = \frac{-\omega_{\text{pole}}k_{a0} \pm k_{s0}[\delta\omega_s(\delta\omega_s - \omega_{\text{crit}})]^{1/2}}{(\delta\omega_s - \omega_{\text{pole}})}\,, \tag{8a}$$

where

$$\omega_{\text{pole}} = \frac{1}{2}\left(\frac{\mathcal{E}_a}{mc^2}\right)\omega_p\,,$$
$$\omega_{\text{crit}} = \frac{(\omega_a^2 - \omega_s^2)}{(\omega_p^2 - \omega_s^2)}\omega_{\text{pole}}\,. \tag{8b}$$

The quantity $w_{\text{pole}}$ is in effect the "Stark" shift of the unperturbed plasma frequency. The critical frequency, $w_{\text{crit}}$, is that frequency where the refractive index $(n_s = ck_s/\omega_s)$ changes from imaginary to real. The plus and minus sign in Eq. (8a) apply for $\delta\omega_s$ below and above the critical frequency, respectively.

Figure 2(a) shows the imaginary part of the refractive index as a function of the detuning $\delta\omega_s$ of the Stokes frequency from two-photon resonance. The parameters for Fig. 2(a) are $\mathcal{E}_a/mc^2 = 0.02$, $\omega_p = 1$, $\omega_s = 0.75$, and $\omega_a = 1.75$. This yields $\omega_{\text{pole}} = 0.01$, and $\omega_{\text{crit}} = 0.057$. With $\mathcal{E}_a = 0$, the Stokes frequency is below cutoff and has a propagation constant of $-0.88j$. The presence of the driving laser creates a passband with

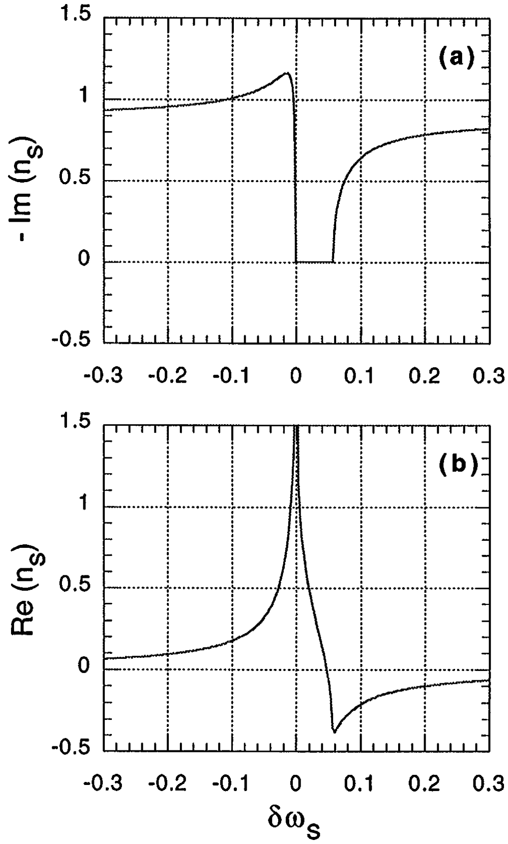


FIG. 2. Refractive index of the Stokes beam $n_s$ vs the detuning from two-photon resonance $\delta\omega_s$. (a) $-\mathrm{Im}(n_s)$, (b) $\mathrm{Re}(n_s)$. The parameters are $\omega_p = 1$, $\omega_s = 0.75$, $\omega_a = 1.75$, and $\omega_{\mathrm{pole}} = 0.01$. A passband is created between $\omega_{\mathrm{crit}} = 0.057$ and the origin.

a width proportional to its power density, in this case, 5.7% of the plasma frequency. Figure 2(b) shows the real part of the refractive index. As described below, the negative group velocity and steep slope of $n_s$ vs $\omega_s$ imposes an energy requirement on the driving laser pulse. One may also note that, even without the inclusion of damping, for forward propagating waves, the response at $\omega_{\mathrm{pole}}$ is not infinite. As the pole is approaced, the numerator of Eq. (8a) approaches zero and $k_s(\mathrm{pole}) = (k_{a0}^2 + k_{s0}^2)/2k_{a0}$.

We next examine the case where the electromagnetic field at $E_s$ is strong, and that at $E_a$ is weak. Proceeding as previously, we obtain the dispersion relation

$$\frac{(k_{s0}^* \mp k_a)^2}{2}\left(\frac{\mathcal{E}_s}{mc^2}\right)\omega_p - (k_{a0}^2 - k_a^2)\delta\omega_a = 0. \quad (9a)$$

The minus sign in Eq. (9a) applies when both waves propagate in the same direction; the plus sign applies when the waves are oppositely directed. Here,

$$\omega_{\mathrm{pole}} = -\frac{1}{2}\left(\frac{\mathcal{E}_s}{mc^2}\right)\omega_p, \qquad \omega_{\mathrm{vac}} = \frac{(\omega_a \mp \omega_s n_{s0})^2}{\omega_p^2}\omega_{\mathrm{pole}}. \quad (9b)$$

The quantity $\omega_{\mathrm{vac}}$ is the detuning of $\omega_a$ such that the beam propagates as if in vacuum.

In Fig. 3 we take both fields to be above the plasma frequency, with parameters $\mathcal{E}_s/mc^2 = 0.02$, $\omega_p = 1$, $\omega_s = 1.25$, and $\omega_a = 2.25$. This yields $\omega_{\mathrm{pole}} = -0.01$, $\omega_{\mathrm{crit}} = -0.0086$, and $\omega_{\mathrm{vac}} = -0.0225$. Here [Fig. 3(a)] a stop band appears between $\omega_{\mathrm{crit}}$ and the origin. For detunings less than $\omega_{\mathrm{crit}}$ the group velocity is positive and increasing. Noting Eq. (9b), we see that if $\omega_s$ and $\omega_a$ are chosen to be nearly equal, then backward propagation may greatly increase the strength of the interaction and the magnitude of $\omega_{\mathrm{vac}}$.

We turn next to the requirements on the power and energy of the laser pulses which are necessary to approximately establish the solutions of Eqs. (7), (9a), and (9b). Using Eq. (5), Eq. (3) may be rewritten as

$$\frac{1}{2}\frac{\partial\rho}{\partial t} - j(\delta\omega_s - \omega_{\mathrm{pole}})\rho = jr(t), \qquad r(t) = \frac{(\mathcal{E}_a\mathcal{E}_s)^{1/2}}{m\omega_p}[(k_{a0} - k_s^*)^2 - (k_s^2 - k_{s0}^2)](\angle E_a E_s^*). \quad (10)$$

First, the power/area of the strong field must be sufficient that $|\omega_{\mathrm{pole}}|$ substantially exceeds the linewidth of the

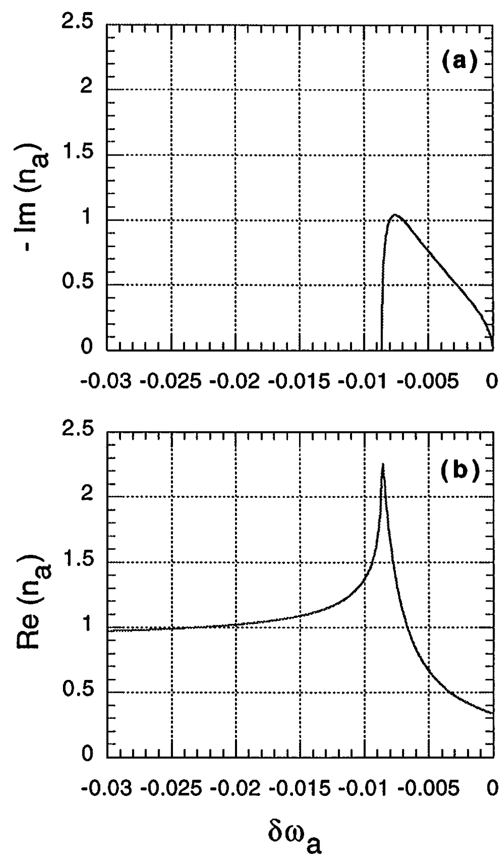


FIG. 3. Refractive index of the laser beam $n_a$ vs the detuning from two-photon resonance $\delta\omega_a$. (a) $-\mathrm{Im}(n_a)$, (b) $\mathrm{Re}(n_a)$. The parameters are $\omega_p = 1$, $\omega_s = 1.25$, $\omega_a = 2.25$, and $\omega_{\mathrm{pole}} = -0.01$. For a detuning $\delta\omega_a = \omega_{\mathrm{vac}} = -0.0225$, the beam propagates as if in vacuum.

plasma resonance. This linewidth may be caused by collisions, plasma inhomogeneities, and, perhaps, other instabilities. For the steady-state solution to evolve smoothly from the zero-field solution, the pulse must vary slowly as compared to the detuning of $\delta\omega_s$ from the pole. Because the magnitude of the pole varies linearly with the power density, for a smooth pulse, this leads to an energy invariant

$$\text{pulse energy/area} > \frac{4\omega_s^2}{\omega_p^3}(n_e c)\,(mc^2)\,. \qquad (11)$$

In most cases, the requirement of Eq. (11) is superceded by the requirement that the pulse not be substantially distorted by the dispersion of the two-photon resonance. If the pulse is not to be substantially distorted after propagating through a length $L$, then the pulse must be sufficiently long that its energy density exceeds

$$\text{pulse energy/area} > \frac{4\omega_a\omega_s^2}{\omega_p^3}|\omega_{\text{pole}}|\left(\frac{\partial n_a}{\partial\omega_a}\right)n_e L(mc^2) \cong n_e L(mc^2)\,. \qquad (12)$$

For example, for propagation through a sample with the parameters of Fig. 2 with $\delta\omega_s$ set near to band center at $\omega_{\text{crit}}/2$, Eq. (12) yields an energy density requirement of $1.61mc^2n_eL$. The comparable requirement for the parameters of Fig. 3 with $\delta\omega_a$ set to $\omega_{\text{vac}}$ is $0.88mc^2n_eL$.

It is of interest to compare EIT in atoms with EIT in an ideal plasma. Both result from an interference between a two-photon resonance and a primary process. Both act to simultaneously reduce the dipole moment at both fields. In atoms, for doubly resonant transitions both fields may simultaneously propagate as in vacuum. In plasmas, with only two fields included, only one field may do so. The pulse energy which is necessary to overcome dispersion in atoms is equal to $\hbar\omega$ multiplied by the oscillator strength weighted number of atoms in the laser path [11]. In the plasma the necessary energy is much larger and is proportional to $mc^2$ times the number of atoms in the laser path.

Applications of EIT in plasmas include focusing of one laser beam by another, propagation in dense plasmas, and application to ladder systems where the sum of two incident frequencies is near to the plasma frequency. This effect in a plasma is also a first example of EIT based on a collective, rather than single particle, excitation of a medium.

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