Precise Determination of the Pion-Nuclear Coupling Parameter from Weak Processes in ^{3}He

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We utilize precise weak interaction experiments on atomic muon capture and beta decay in the $A = 3$ nuclei and take into account the effects of nuclear "anomalous thresholds" to extract the pseudoscalar π-^{3}He-^{3}H coupling parameter, $G^{\text{eff}}(m_\pi^2) = 45.8 \pm 2.4$. This is an order of magnitude improvement in precision over that from the use of pion-nuclear scattering data and dispersion relations. [S0031-9007(96)02021-2]

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Weak interaction processes, in which atomic muons are captured by the nucleus [1], or nuclear muon capture (NMC), are clean ways to study the semileptonic hadron form factors at low q^2 in a nuclear environment of interest to QCD [2]. These also give important insights into meson exchange currents (MEC) [3]. NMC can be, in special cases, useful to give precise information on the strong pion-nuclear coupling strength, as we demonstrate below in the $A = 3$ nuclear system.

The process [3–5]

$$^3\text{He} + \mu^-(1S) \rightarrow {}^3\text{H} + \nu_\mu \tag{1}$$

is attractive theoretically for a number of reasons: (a) The weak hadronic current in (1) has the same Lorentz structure as the fundamental nucleon process, $p + \mu^-(1S) \rightarrow n + \nu_\mu$ [1]. (b) The nuclear physics of the $A = 3$ system has been carefully studied. Thus explicit wave functions can be computed with great reliability [3,6]. (c) The MEC contributions [3] can be determined in a parallel fashion to that of the nuclear β decay [7]:

$$^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e\,. \tag{2}$$

Recently an experimental breakthrough has been achieved [5] for the study of (1) at the "muon factory" of the Paul Scherrer Institut (PSI). The atomic boundary conditions in the $2S$ and $1S$ states [1] have been carefully controlled, confirming a statistical hyperfine atomic population in the $1S$ state before muon capture by the ^{3}He nucleus. This yields a precise capture rate:

$$\Lambda_c = 1496 \pm 4 \text{ s}^{-1}. \tag{3}$$

This can be nicely understood theoretically in terms of the weak and electromagnetic form factors that are known in the $A = 3$ nuclei [4]. The importance of the MEC is also demonstrated by the fact that the impulse approximation [4] yields a rate about 15% smaller than (3), and the margin is provided by the MEC [3].

This brings us to the subject of this Letter: use of the newly obtained precise NMC rate (3) to determine the π-^{3}He-^{3}H coupling strength at a q^2 characteristic of the process (1). We shall compare this to its value from the β-decay process (2), and that at the pion pole as extracted from the strong $\pi^\pm$-^{3}He scattering [8–13]. Contrary to our naive expectations, the present precision of the extracted coupling strength from the strong processes (Table I) is actually much worse than that obtained from the NMC and the nuclear beta decay. The main point of this Letter is the following: Even though this pion-nuclear coupling strength parameter makes a relatively small contribution to the muon capture rate, through the induced pseudoscalar form factor, the recent PSI experiment on the ^{3}He is so precise that it can be used to yield a value of this parameter that is not only consistent with its values extracted from the pion-nucleus scattering experiments, *but far more accurate.* All we need is

TABLE I. A comparison of the effective π-^{3}He-^{3}H coupling parameter obtained from different processes: the strong $\pi^\pm$-^{3}He scattering and dispersion relation (first column), the Goldberger-Treiman relation and weak β decay (second column) and via the pseudoscalar coupling from nuclear muon capture (third column). References, from which the numbers in the first column have been extracted, are explicitly given. The second and third columns contain results of this work, yielding a coupling parameter of 45.8 $\pm$ 2.4 at the pion pole, to be compared with entries in the first column.

G from $\pi^{\pm 3}$He scatt.	G from β decay	G from muon capture
38 $\pm$ 16 (Spencer [10])	36.8 $\pm$ 0.2	31.9 $\pm$ 1.3
45 $\pm$ 19 (Mach and Nichitiu [12])		
49 $\pm$ 14 (Nichitiu and Sapozhnikov [12])		
57 $\pm$ 13 (Kopeliovich [11])		

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the hypothesis of the partial conservation of the nuclear axial current (nuclear PCAC) [4,13] to obtain the NMC obervables in the so-called "elementary particle approach" (EPA) [13]. One consequence of the nuclear PCAC, the Goldberger-Treiman relation (GTR) between the π-^{3}He-^{3}H coupling parameters, the nuclear β-decay axial form factor and the pion decay constant, will be exploited in the presence of *anomalous thresholds* [8–12] in the $A = 3$ nuclei. The latter are obviously absent for the nucleon [1], and its GTR is known to be largely immune from effects of the three-pion cut [14] or chiral symmetry breaking corrections [15], making it an excellent test for PCAC. Given the nuclear PCAC and GTR, we shall determine the pion-nuclear coupling parameter accurately at the pion pole from weak processes, by a linear extrapolation. Conversely, by comparing this extracted coupling parameter with that from pion-nuclear scattering and a suitable dispersion relation, we can, in effect, test the *nuclear* PCAC and GTR. Despite large structural differences between the nucleon and the $A = 3$ nuclei (^{3}He, ^{3}H), PCAC and GTR may work well in both.

We begin with the nuclear weak hadron current for the process (1). It is characterized by a Lorentz structure *identical* to that of the nucleon, since both the $A = 3$ nucleus and the nucleon are $J^\pi = \frac{1}{2}^+$, $I = \frac{1}{2}$ objects. This is exploited in the EPA. The hadron current is given by [3]

$$j^\mu = \bar{u}(k')\left[F_V\gamma^\alpha + iF_M\frac{\sigma^{\mu\nu}q_\nu}{2M} + F_A\gamma^\mu\gamma^5 + F_P\gamma^5\frac{q^\mu}{m}\right]u(k)\,, \tag{4}$$

with $q^\mu = (k' - k)^\mu$, m, the muon mass; M is the mean nucleon (nuclear) mass, $\bar{u}(k')$, $u(k)$ are the spin-$\frac{1}{2}$ nucleon (nuclear) spinors, and F_i's are the usual [1] weak form factors. We assume conserved vector current (CVC) and ignore "second-class" terms [1]. The muon capture rate (3) is given by

$$\Lambda = \frac{G_F^2}{2\pi}|V_{ud}|^2N'^2C|\varphi_\mu(0)|^2\nu^2\left(1 - \frac{\nu}{\sqrt{s}}\right)G_0^2\,, \tag{5}$$

where we follow the notation of Congleton and Fearing (CF) [4]. The weak interaction physics from nuclei is contained in the effective coupling constant squared G_0^2:

$$G_0^2 = G_V^2 + 2G_A^2 + (G_A - G_P)^2. \tag{6}$$

The effective vector, axial vector, and pseudoscalar coupling combinations in terms of the F_i's in (4) are standard [1,4]. Our interest first lies in the determination of G_P from (3), fixing G_V and G_A from experiment, and translating it into F_P. From this we shall extract the π-^{3}He-^{3}H coupling parameter for the NMC. The GTR will give us this parameter from the nuclear β decay. From these two kinematic points, we shall extrapolate it to the pion pole.

To proceed further, we write the dispersion relation [9] by exploiting the PCAC *Ansatz* that the divergence $D(t)$ of the hadronic axial-vector current is proportional to the pion field:

$$D(t) = \left[2MF_A + \frac{t}{m}F_P\right] = -\frac{\sqrt{2}f_\pi m_\pi^2 G(t)}{t - m_\pi^2} + \frac{1}{\pi}\int_a^{+\infty}dt'\frac{\mathrm{Im}[D(t')]}{t' - t}\,. \tag{7}$$

For the muon capture reaction (1), $t \approx -0.96m^2$, f_π is the pion decay constant (130.8 ± 0.3 MeV [15]); $G(t)$ is the π-^{3}He-^{3}H pseudoscalar coupling parameter, related to the pseudovector one by the usual [11] way. The integration threshold a above is set by the so-called "anomalous" cuts [8,9,11] beginning at $t = (1.8m_\pi)^2$ and $(2.1m_\pi)^2$, coming from the deuteron-nucleon and three-nucleon breakup thresholds, respectively. *These are new features of the nuclear process (1),* compared to the NMC by the proton. Letting $t = 0$, and ignoring the contribution from all cuts, we get the standard GTR:

$$f_\pi = \sqrt{2}M\frac{F_A}{G(0)}\,. \tag{8}$$

At this stage, we recall the nucleon case of the GTR. There are no anomalous cuts here. Estimating the three-pion cut following Wolfenstein [14], its effect is found to be small. Thus the relation (8) can be used to estimate the value of $G(0)$. Taking $G(0) \approx G({m_\pi}^2)$ by PCAC, $t = m_\pi^2$ corresponding to the physical pion pole, using the neutron β-decay value of $F_A(0) = 1.2601 \pm 0.0025$ [16], and $[G(m_\pi^2)]^2/4\pi = 14.28 \pm 0.36$ [17], the GTR is found to be fulfilled within 5%. Using (7), one can then estimate $F_P = 7.25 \pm 0.09$ for muon capture by protons, for which $t \approx -0.88m_\pi^2$. At present, this prediction for the nucleon is poorly tested through NMC [1]. In radiative muon capture (RMC), there is a disagreement with it in a pioneering RMC experiment at TRIUMF [18].

Returning to the nuclear system $A = 3$, let us rewrite the dispersion relation (7) as

$$D(t) = -\frac{\sqrt{2}f_\pi m_\pi^2 G({m_\pi}^2)}{t - m_\pi^2}[1 + \delta(t)], \tag{9}$$

where $\delta(t)$ is the nuclear correction from the anomalous cuts. Thus we can introduce an *effective* pion-nuclear coupling parameter $G^{\mathrm{eff}}(t)$ by the relation [9,11]

$$G^{\mathrm{eff}}(t) = G({m_\pi}^2)[1 + \delta(t)]. \tag{10}$$

This yields for the nuclear β decay an effective GTR that takes implicitly into account the effects of the nuclear anomalous cuts:

$$G^{\mathrm{eff}}(0) = \frac{\sqrt{2}MF_A(0)}{f_\pi}\,, \tag{11}$$

by substituting (10) in (9) and taking the $t \to 0$ limits, where M is the mean ^{3}He-^{3}H mass, $M \approx 2808.7$ MeV, and $F_A(0)$ is obtained from ^{3}H β decay [7]:

$$F_A(0) = 1.212 \pm 0.004\,. \tag{12}$$

This gives, using (12) in (11),

$$G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(0) = 36.81 \pm 0.15\,. \tag{13}$$

Note that this extraction *does not* require our explicit knowledge of the anomalous cut contribution $\delta(0)$.

We now discuss the NMC reaction (1) and see how the latest precision measurement of Λ_c yields a value of $G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(t_{\mathrm{cap}})$, where t_{cap} is the characteristic value of t in the capture process (1). Using Eqs. (7), (9), and (10), we get a nuclear PCAC equation, implicitly including the effects of anomalous cuts:

$$G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(t_{\mathrm{cap}}) = -\frac{(t_{\mathrm{cap}} - m_\pi^2)}{\sqrt{2} f_\pi m_\pi^2} \times \left[2MF_A(t_{\mathrm{cap}}) + \frac{t_{\mathrm{cap}}}{m} F_P(t_{\mathrm{cap}})\right]. \tag{14}$$

Using the newly measured rate (3), we can determine a range of the *least known* weak nuclear form factor $F_P(t_{\mathrm{cap}})$, holding the others to their known values [4]. To do this, we exploit the experimentally known vector form factors and take the t dependence of F_A from the vector one [4], as is conventionally done. (Future neutrino experiments at facilities like KARMEN [19] would eliminate this approximation.) This yields, using (3), (5), and (6),

$$F_P = 20.80 \pm 1.6\,, \tag{15}$$

with the parameter C, the correction factor in (5) due to the nuclear finite size effect taken to be 0.98. The nuclear PCAC equation (14) gives us

$$G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(-0.954m^2) = 31.9 \pm 1.3\,. \tag{16}$$

Equations (13) and (16) are two crucial results of this Letter. The effects of anomalous cuts in the $A = 3$ nuclei are *implicitly included* in these numerical values.

Before discussing the significance of these results, we come to the determination of the π-^{3}He-^{3}H coupling parameter from the strong interaction process in the $A = 3$ system directly. The principle has been reviewed by Ericson and Locher long ago [8]. One writes down a dispersion relation for the amplitude, antisymmetric under crossing [8]:

$$Ref^-(\omega) = \sum_i \frac{2\omega r_i}{\omega^2 - \omega_i^2} + \frac{2\omega}{\pi} P \int d\omega' \frac{\mathrm{Im} f^-(\omega')}{\omega'^2 - \omega^2}\,, \tag{17}$$

where ω is the pion lab energy, the poles come from the neighboring nuclei, and r_i is the residue of the ith pole, the coupling constant of interest. Several authors [10–12] have made use of the $\pi^{\pm 3}$He total cross section data in the physical region and analytic extrapolation in the unphysical region, using relation (17), wherein the sum over i gets replaced by a single term, the effective residue at the pion pole, earlier denoted by us as $(G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}})^2$. The results of these authors yield a broad range of values and are summarized in Table I, along with the values obtained from the weak interaction processes (1) and (2). An important point to note here is *the large errors associated with the strong interaction determinations* of the $G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}$ parameter, compared with the precision at the weak interaction values of t, $t \approx 0$, and $t = t_{\mathrm{cap}}$ for the β decay and muon capture, respectively.

Let us now return to the significance of the determination of the G^{eff} from the weak interaction in (13) and (16) and their implications at the pion pole. Direct investigations of the effects of anomalous cuts have been made by Jarlskog and Yndurain [9] and Kopeliovich [11]. They both find significant variations between $t = 0$ and $t = t_{\mathrm{cap}}$ in the value of G^{eff}, due to the presence of these cuts. Thus

$$G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(0) \approx 1.09 G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(t_{\mathrm{cap}})\,, \tag{18a}$$

according to Jarlskog and Yndurain, and

$$G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(0) \approx 1.19 G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(t_{\mathrm{cap}})\,, \tag{18b}$$

according to Kopeliovich. We find, from (13) and (16),

$$G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(0) = (1.15 \pm 0.05) G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(t_{\mathrm{cap}})\,, \tag{19}$$

in qualitative agreement with both theoretical estimates (18a) and (18b), but are unable to distinguish between them. However, *the deviation from unity in the value of the numerical coefficient on the right-hand side of Eq. (19) is a confirmation, from the weak interaction experiments, of the role of the anomalous cuts in the π-^{3}He-^{3}H coupling.*

We can now use our pion-nuclear coupling values, obtained from the weak interaction studies, to extrapolate to the pion pole. With a linear extrapolation [9],

$$G(t_{\mathrm{cap}}) = G(0) + \frac{t_{\mathrm{cap}}}{m_\pi^2}[G(m_\pi^2) - G(0)]\,, \tag{20}$$

we get the π-^{3}He-^{3}H coupling constant at the pion pole:

$$G^{\mathrm{eff}}_{\pi^3\mathrm{He}^3\mathrm{H}}(m_\pi^2) = 45.8 \pm 2.4\,, \tag{21}$$

consistent with the numbers obtained from the $\pi^{\pm 3}$He scattering (Table I), but far more accurate. Here we have achieved an improvement in precision of the determination of the strong pion-nuclear coupling by an order of magnitude, compared with the current accuracy of its inference from the pion-nuclear scattering. This extraction of a precise pion-nuclear coupling parameter from the weak interaction processes is the *central result* of this Letter.

Further theoretical studies are needed to understand the dynamical significance of the value of the coupling constant in Eq. (21). This much is already clear: *The square of the coupling constant obtained above is about 30% bigger than the impulse approximation estimate of Ericson and Locher [8].*

In summary, we have studied here the weak interaction observables, the nuclear β-decay rate of ^{3}H to ^{3}He, and the inverse muon capture rate, recently measured at PSI with a great precision, and used them to determine the pion-nuclear coupling parameter. We have utilized the nuclear PCAC and Goldberger-Treiman relation, taking the effects of nuclear breakup channels in the intermediate state, through the anomalous cuts, into account. We have extrapolated the π-^{3}He-^{3}H coupling parameter from the weak interaction kinematics to the pion pole and extracted the π-nuclear coupling constant. The resultant parameter, 45.8 $\pm$ 2.4, is much more precise than the values obtained from the $\pi^{\pm 3}$He dispersion relations (Table I). Its dynamical implications need further theoretical exploration beyond the impulse approximation. Conversely, its consistency with the values from the existing pion-nuclear scattering analyses is *new proof* of the validity of *nuclear* PCAC and Goldberger-Treiman relation. This test could be considerably strengthened with new high-quality experiments on the pion-nuclear scattering such that the resultant error on the coupling parameter, inferred from such experiments, would be at least of the same order of magnitude as that in (21).

Further improvement in precision of the pion-nuclear parameter is expected, when the new experimental studies [20] on polarization observables in the NMC are finished. The polarization observables are far more sensitive [4,21] to the pseudoscalar coupling F_P, hence to the pion-nuclear coupling parameter, than the rates of the nuclear muon capture. Thus we are going to obtain information on strong interaction physics from the on-going weak interaction studies at a level of precision *even higher* than what we have reported here. Therefore it would be useful to have a corresponding improvement in accuracy in the application of the dispersion relations to the pion-nuclear scattering. This would need new precise experiments at the pion factories on the total π^3He scattering cross sections. For this, the current database is not good enough.

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