Determining the $t\bar{t}$ and ZZ Couplings of a Neutral Higgs Boson of Arbitrary CP Nature at the Next Linear Collider

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The optimal procedure for extracting the coefficients of different components of an observable which takes the form of unknown coefficients times functions of known form is developed. When applied to $e^+e^- \to t\bar{t}+$ Higgs production at $\sqrt{s} = 1$ TeV and integrated luminosity times efficiency of 50 fb^{-1}, we find that the $t\bar{t} \to$ Higgs CP-even and CP-odd couplings and, to a lesser extent, the $ZZ \to$ Higgs (CP-even) coupling can be extracted with reasonable errors. Typically, a standard-model-like CP-even Higgs boson can be distinguished from a purely CP-odd Higgs boson at a high level of statistical significance, and vice versa. [S0031-9007(96)01889-3]

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In this Letter, we present a powerful technique [1] for determining the coefficients c_i appearing in an observable that can be written in the generic form $\mathcal{O}(\phi) = \sum_i c_i f_i(\phi)$, where ϕ denotes (precisely measurable) variables upon which $\mathcal{O}$ depends and the $f_i(\phi)$ are known functions. Variational calculus implies that the technique presented is the optimal one in the limit of Gaussian statistics. We apply this procedure to the extremely important task in elementary particle physics of determining the magnitude and the CP nature of the couplings of a Higgs boson (generically denoted as h). In particular, we focus on the $e^+e^- \to t\bar{t}h$ production process at a next linear e^+e^- collider (NLC), in which case $\mathcal{O}$ is the differential cross section, $d\sigma/d\phi$, ϕ denotes the kinematical variables specifying the final state phase space configuration, and the c_i are functions of the Higgs couplings. By extracting the c_i we can determine all the Higgs couplings and, thence, its CP nature. Since our procedure makes full use of the information contained in the final state distributions, it can significantly improve the statistical precision with which the couplings/CP nature of a Higgs boson can be determined relative to procedures that have been studied in the past (see Ref. [2] for a review). Techniques explored previously include photon polarization asymmetries in $\gamma\gamma \to h$ [3], momentum correlations among the final state τ or t decay products appearing in $e^+e^- \to Zh$ and $\mu^+\mu^- \to h$ with $h \to \tau^+\tau^-$ or $t\bar{t}$, respectively [4,5], and single-variable-weighted cross section integrals in $pp \to t\bar{t}h$ at the LHC [6] and in $e^+e^- \to t\bar{t}h$ at the NLC [7]. (These latter analyses did not take full advantage of the detailed functional form of $d\sigma/d\phi$.) At the very least, application of the optimal analysis procedure to $e^+e^- \to t\bar{t}h$ will result in coupling determinations that can be combined with those from other types of analyses to greatly improve overall errors. To illustrate the power of the technique, we note that if we accumulate $L = 500$ fb^{-1} at $\sqrt{s} = 1$ TeV and if the final state reconstruction efficiency is of order $\epsilon = 0.1$, a standard-model (SM)-like CP-even Higgs boson can be distinguished from a pure CP-odd Higgs boson at roughly the 9.5σ statistical level, a result substantially superior to that achieved using any of the other techniques listed above.

General technique—We assume that

$$\mathcal{O}(\phi) = \sum_i c_i f_i(\phi), \tag{1}$$

where the $f_i(\phi)$ are known functions of the variables ϕ, and the c_i are model-dependent coefficients (taken to be dimensionless in our convention). The coefficients c_i can be extracted by using appropriate weighting functions $w_i(\phi)$ such that $\int w_i(\phi)\mathcal{O}(\phi)d\phi = c_i$. In general, different choices for the $w_i(\phi)$ are possible. However, there is a unique choice such that the statistical error in the determination of the c_i is minimized in the sense that the entire covariance matrix is at a stationary point in terms of varying the functional forms for the $w_i(\phi)$ while maintaining $\int w_i(\phi)f_j(\phi)\,d\phi = \delta_{ij}$. Thus, we require

$$\delta V_{ij} \propto \int \delta[w_i(\phi)w_j(\phi)]\mathcal{O}(\phi)\,d\phi = 0, \tag{2a}$$

$$\int \delta w_i(\phi)f_j(\phi) = 0, \tag{2b}$$

where a given entry V_{ij} in the covariance matrix is $\propto \int w_i(\phi)w_j(\phi)\mathcal{O}(\phi)\,d\phi$. The weighting functions which satisfy these conditions are

$$w_i(\phi) = \frac{\sum_j X_{ij}f_j(\phi)}{\mathcal{O}(\phi)}, \quad \text{with } X_{ij} = M_{ij}^{-1},$$

$$\text{where } M_{ik} \equiv \int \frac{f_i(\phi)f_k(\phi)}{\mathcal{O}(\phi)}\,d\phi, \tag{3}$$

since, for the $w_i(\phi)$ so defined, the constraint (2b) implies the minimization condition (2a).

We may then compute c_i as

$$c_i = \sum_k X_{ik}I_k = \sum_k M_{ik}^{-1}I_k,$$

$$\text{where } I_k \equiv \int f_k(\phi)\,d\phi. \tag{4}$$

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It can then be demonstrated that the covariance matrix is

$$V_{ij} \equiv \langle \Delta c_i \Delta c_j \rangle = \frac{M_{ij}^{-1} \sigma_T}{N}, \quad (5)$$

where $\sigma_T = \int \mathcal{O}(\phi)\, d\phi$ and N is the total number of events integrating over all ϕ. (For $\mathcal{O} = d\sigma/d\phi$, σ_T would be the integrated cross section and $N = L_{\rm eff}\sigma_T$, with $L_{\rm eff}$ being the luminosity times efficiency.) The result of Eq. (5) applies only for the optimal weighting functions.

We note that the above procedure is the optimal one regardless of the relative magnitudes of the c_i. Various limits of the optimal weighting functions for selected elementary particle cross sections have previously appeared in the literature, see, e.g., Refs. [4,5,8].

Our procedure is not altered if cuts are imposed on the portion of ϕ space over which one integrates. Although such cuts may be required in the actual experimental analysis, we have not included cuts in our model computations to follow, other than through the inclusion of an efficiency factor.

Extracting Higgs couplings in $e^+e^- \to t\bar{t}h$.—We now apply the above procedure to the extraction of Higgs couplings using the process $e^+e^- \to t\bar{t}h$. In order to fully define a point in phase space we must distinguish between the t and $\bar{t}$ and require that only one have invisible energy in its decay (together implying that one t must decay leptonically and the other hadronically); further, the h must decay to a fully reconstructible final state such as $b\bar{b}$ or $ZZ \to 4j$, 4ℓ or $W^+W^- \to 4j$ [9]. The overall efficiency for the mixed leptonic-hadronic final state decays and reconstruction of both t's and the h will be denoted by ϵ, the maximum value for which is $2\sum_{\ell=e,\mu,\tau} B(t \to jjb)B(t \to \ell^+\nu_\ell b) \sim 0.44$ times the appropriate Higgs branching ratio. The effective luminosity is given by $L_{\rm eff} = \epsilon L$, where L is the total integrated luminosity. We shall take $L_{\rm eff} = 50$ fb^{-1}, as could be achieved for $L = 500$ fb^{-1} ($2\frac{1}{2}$ years of running at $L = 200$ fb^{-1} per yr) and $\epsilon = 0.1$ [10].

We could also apply our technique to $e^+e^- \to t\bar{t}h$ in the double-hadronic $t\bar{t}$ decay mode; we only lose sensitivity to the *CP*-odd $d\sigma/d\phi$ component. Formally, if $\overline{\phi}$ is the subset of the kinematical variables ϕ that cannot be determined, we would use the variables, $\hat{\phi}$, that *can* be observed and the functions $\hat{f}_i(\hat{\phi}) \equiv \int f_i(\phi)\, d\overline{\phi}$ (implying $\hat{f}_5 = 0$ below). Including these modes would improve the statistical significance with which the Higgs couplings could be measured beyond the results obtained below using only the mixed hadronic/leptonic $t\bar{t}$ decay channel.

The Higgs couplings are defined via the Feynman rules,

$$t\bar{t}h : -\bar{t}(a + ib\gamma_5)t\frac{gm_t}{2m_W}, \qquad ZZh : c\,\frac{gm_Z}{\cos(\theta_W)}\, g_{\mu\nu}, \quad (6)$$

where g is the usual electroweak coupling constant. Thus, a, b, and c are defined relative to couplings of SM magnitude. The SM Higgs boson has $a = c = 1$ and $b = 0$. A purely *CP*-odd Higgs boson has $a = c = 0$ and $b \neq 0$; the magnitude of b depends upon the model—we will display results for $b = 1$, which would correspond to $\tan\beta = 1$ in a two-Higgs-doublet model of type II (see Refs. [2,12] for details). In our illustrative calculations, we shall assume that there is only one light h in the model. In this case, the only contributing Feynman diagrams involve radiation of the h from the t, $\bar{t}$, or Z lines.

The $t\bar{t}h$ cross section then contains five distinct terms, $d\sigma(\phi)/d\phi = \sum_{i=1}^{5} c_i f_i(\phi)$, where

$$c_1 = a^2; \quad c_2 = b^2; \quad c_3 = c^2; \quad c_4 = ac; \quad c_5 = bc. \quad (7)$$

Of these, the only term in $d\sigma(\phi)/d\phi$ that is actually *CP* violating is that proportional to bc; this is the term upon which Ref. [7] focused. Our approach makes use of the fact that the full cross section contains additional information regarding both b and c.

We have considered three distinct Higgs coupling cases:

(I) The standard model Higgs boson, with $a = c = 1$, $b = 0$.

(II) A pure *CP*-odd Higgs boson, with $a = c = 0$, $b = 1$.

(III) A *CP*-mixed Higgs boson, with $a = b = c = 1/\sqrt{2}$.

For unpolarized beams, $\sqrt{s} = 1$ TeV, $m_h = 100$ GeV, and $m_t = 176$ GeV, the integrated cross sections in cases I, II, and III are $\sigma_T = 2.71$, 0.53, and 1.62 fb, respectively. Adopting $L_{\rm eff} = 50$ fb^{-1}, we then computed

$$\chi^2 = \sum_{i,j=1}^{5} (c_1 - c_i^0)(c_j - c_j^0)V_{ij}^{-1}, \quad \text{with } V_{ij}^{-1} = \frac{M_{ij}N}{\sigma_T} \quad (8)$$

[see Eq. (5)] as a function of location in a, b, c parameter space, where the c_i^0 for a given case are computed from the model input values of a, b, c (given above) using Eq. (7). Surfaces of constant $\chi^2 = 1$ and 36 are displayed in Fig. 1 for each of the three cases. We have indicated the parameter space location of models I, II, and III by a solid circle, square, and star, respectively. The $\chi^2 = 36$ (or 6σ) surfaces will be useful as a reference in assessing the level at which we can distinguish the above three model cases from one another.

Because of the fact that the five c_i are functions of only the three parameters, a, b, c, the $\chi^2 = 1$ surfaces in Fig. 1 are not perfect ellipsoids. Nonetheless, we follow the usual procedure of defining the $\pm 1\sigma$ errors in any one of the a, b, c parameters by the largest and smallest values that the given parameter takes as one moves about the $\chi^2 = 1$ surface. (These extrema define the locations of the two planes of constant parameter value that are tangent to the $\chi^2 = 1$ surface.) The resulting 1σ errors are tabulated in Table I. (The upper and lower limits for a, b, c employed for the $\chi^2 = 1$ surface plots of Fig. 1 are only *just* beyond the extrema

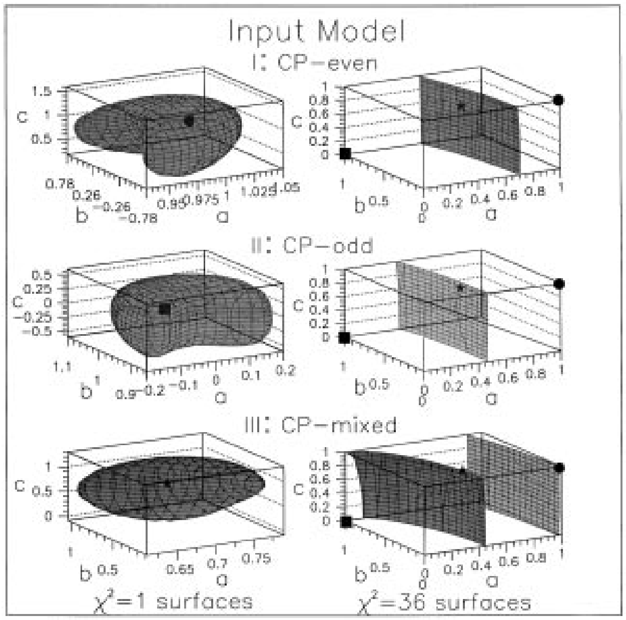

FIG. 1. Surfaces of constant $\chi^2 = 1$ and 36 are displayed for (I) $a = c = 1$, $b = 0$; (II) $a = c = 0$, $b = 1$; and (III) $a = b = c = 1/\sqrt{2}$. The parameter space locations for I, II, and III are indicated by a solid bullet, square, and star, respectively. Results are for unpolarized beams, $\sqrt{s} = 1$ TeV, $m_h = 100$ GeV, $m_t = 176$ GeV, and $L_{\rm eff} = 50$ fb^{-1}.

values.) We observe that a is well determined in all cases, but especially for the $a \neq 0$ cases I and III. Similarly, b is well determined in the $b \neq 0$ cases II and III. The magnitude of the error in c is similar for all three cases, and is never especially small. Of course, a much better measurement (e.g., $\pm 5\%$ for a SM-like h) of or bound on c will be available from inclusive Zh production; however, this does not lead to reduced errors for a and b. Some improvement in the errors is possible if the electron beam can be negatively polarized without loss of luminosity.

Most important is the ability to distinguish different Higgs CP mixtures from one another. Referring to Fig. 1, we observe the following [13]:

(i) If the Higgs is the CP-even SM Higgs boson, then the pure CP-odd case is well beyond even the $\chi^2 = 36$ surface, and, in fact, lies on roughly the $\chi^2 \sim 90$ surface, corresponding to discrimination at the 9.5σ statistical level. Even the equal CP mixture case III (the parameter location of which appears behind the $\chi^2 = 36$ surface in the figure) is ruled out at the 4.8σ level.

(ii) If the Higgs is pure CP odd, with SM $t\bar{t}$ coupling *magnitude*, then the CP-mixed and CP-even cases lie 17σ and 34σ away, respectively.

(iii) If the Higgs is an equal mixture of CP even and CP odd, with coupling strengths specified by $a = b = c = 1/\sqrt{2}$, then the SM CP-even and pure CP-odd cases I and II are both about 6.3σ away, i.e., just a bit further away than the $\chi^2 = 36$ surfaces plotted.

These results improve if the e^- beam has negative polarization. The discrimination abilities are summarized in Table II.

The errors and discrimination abilities slowly worsen as the Higgs mass increases and the $t\bar{t}h$ cross section, i.e. event rate, declines. As m_h increases, it could also happen that the fraction of the decays of the h that are fully reconstructible decreases, causing a decline in ϵ. Results for $m_h = 200$ and 300 GeV, using $(\cdots)$ and $[\cdots]$ notation (respectively), appear in Tables I and II along with the $m_h = 100$ GeV results discussed above. For $L_{\rm eff} = 50$ fb^{-1} and unpolarized beams, discrimination between our three models declines to $\sim 8\sigma (\sim 2\sigma)$ in the best (worst) case at $m_h = 300$ GeV, compared to $\sim 34\sigma$ $(\sim 5\sigma)$ at $m_h = 100$ GeV.

We can also analyze our ability to determine that the CP-violating component of $d\sigma(\phi)/d\phi$, proportional to $c_5 \equiv bc$ is nonzero. We consider model III (the only one of our three models for which $bc \neq 0$). We plot the $\chi^2 = 1$ (1σ) surface in a, b, and bc space and look for the extrema of bc. We find that these extrema occur for $a \sim b \sim 1/\sqrt{2}$ and that bc can range from -0.05 to $+0.91$, assuming $m_h = 100$ GeV, $L_{\rm eff} = 50$ fb^{-1} and unpolarized beams. Clearly, we are not far from establishing a nonzero signal at the 1σ level For twice as much effective luminosity, $L_{\rm eff} \sim 100$ fb^{-1}, the extrema of bc on the 1σ surface are $+0.15$ and $+0.79$, and a nonzero value of bc would have been established at better than the 1σ level. At the 1σ level, $m_h = 100$ GeV,

TABLE I. We tabulate the 1σ errors, as defined in the text, in a, b, and c for the three Higgs coupling cases I ($a = c = 1$, $b = 0$), II ($a = c = 0$, $b = 1$), and III ($a = b = c = 1/\sqrt{2}$), assuming $\sqrt{s} = 1$ TeV, $m_h = 100$ (200) [300] GeV (respectively), $m_t = 176$ GeV and $L_{\rm eff} = 50$ fb^{-1}. Results are for unpolarized beams; errors for 100% negative e^- polarization are typically 10%–15% smaller.

Case	$\pm\Delta a$	$\pm\Delta b$	$\pm\Delta c$
I	$^{+0.043}_{-0.066}\left(^{+0.07}_{-0.14}\right)\left[^{+0.12}_{-0.32}\right]$	$^{+0.76}_{-0.76}\left(^{+1.0}_{-1.0}\right)\left[^{+1.44}_{-1.44}\right]$	$^{+0.51}_{-0.82}\left(^{+0.56}_{-1.04}\right)\left[^{+0.72}_{-1.76}\right]$
II	$^{+0.19}_{-0.19}\left(^{+0.30}_{-0.30}\right)\left[^{+0.45}_{-0.45}\right]$	$^{+0.093}_{-0.14}\left(^{+0.12}_{-0.28}\right)\left[^{+0.20}_{-2.16}\right]$	$^{+0.58}_{-0.58}\left(^{+0.68}_{-0.68}\right)\left[^{+0.96}_{-0.96}\right]$
III	$^{+0.075}_{-0.087}\left(^{+0.12}_{-0.21}\right)\left[^{+0.17}_{-0.53}\right]$	$^{+0.31}_{-0.62}\left(^{+0.49}_{-1.89}\right)\left[^{+0.81}_{-2.15}\right]$	$^{+0.57}_{-0.80}\left(^{+0.63}_{-1.44}\right)\left[^{+0.77}_{-2.15}\right]$

TABLE II. We tabulate the number of standard deviations, $\sqrt{\chi^2}$, at which a given input model (I, II, or III) can be distinguished from the other two models, assuming $\sqrt{s} = 1$ TeV, $m_h = 100$ (200) [300] GeV, $m_t = 176$ GeV, and $L_{\rm eff} = 50$ fb^{-1}. Results for unpolarized beams and for 100% negative e^- polarization are given.

Input Model	Unpolarized e^- Trial Model I	II	III	$P(e^-) = -1$ Trial Model I	II	III
I	···	9.5 (5.6) [3.5]	4.8 (2.8) [1.8]	···	11 (6.1) [4.0]	5.5 (3.2) [2.0]
II	34 (15) [8.3]	···	17 (7.3) [4.2]	40 (17) [9.5]	···	20 (8.4) [4.8]
III	6.3 (3.6) [2.3]	6.3 (3.6) [2.2]	···	7.3 (4.2) [2.6]	7.3 (4.1) [2.6]	···

$L_{\rm eff} = 50$ fb^{-1} upper bounds on $|c_5| = |bc|$ in models I and II are 0.65 and 0.55, respectively. The above results are all somewhat better than obtained for these same models using either of the observables ($\mathcal{O}$ or $\mathcal{O}_{\rm ropt}$) employed in Ref. [7].

In this Letter, we have outlined the optimal technique for extracting the coefficients that appear in a general observable which is a sum of model-dependent coefficients times known functions. Application of this technique to $e^+e^- \rightarrow t\bar{t}h$ results in good prospects for pinning down the *CP* nature of the *h* at a 1 TeV e^+e^- collider operating at an expected luminosity of $L = 200$ fb^{-1} per yr, provided that the *h* has a reasonable production cross section (roughly $\gtrsim 0.5$ fb) and that the $t\bar{t}h$ final state can be reconstructed with reasonable efficiency (roughly $\epsilon \gtrsim 0.1$). The precision with which both the *CP*-odd and *CP*-even $t\bar{t}$ Higgs couplings can be determined is somewhat improved for a negatively polarized electron beam, assuming there is no loss of luminosity. Most importantly, the coefficients of the various terms in the $e^+e^- \rightarrow t\bar{t}h$ cross section can be determined well enough that Higgs *CP* mixtures that are significantly different from one another can generally be distinguished at a substantial (sometimes very substantial) level of statistical significance.

We have implicitly assumed that the systematic error in the overall normalization of the $t\bar{t}h$ cross section will be relatively small, e.g., $\lesssim \pm 5\%$. If this is not the case, then one can focus on the ratios of the different cross section coefficients to one another. Our technique is easily adapted to this situation.

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[1] We believe that this technique may be new. A seemingly distant relative to our procedure that has been pointed out to us is the Karhunen-Loeve theory reviewed in W. L. Root, Proc. IEEE **75**, 1446 (1987) and employed recently in H. Abarbanel *et al.*, Phys. Rev. A **41**, 1782 (1990).

[2] J. F. Gunion, A. Stange, and S. Willenbrock, Report No. UCD-95-28, 1995 [in "Electroweak Physics and Beyond the Standard Model," World Scientific Publishing Co., edited by T. Barklow, S. Dawson, H. Haber, and J. Siegrist (Singapore, to be published)].

[3] B. Grzadkowski and J. F. Gunion, Phys. Lett. B **294**, 361 (1992); J. F. Gunion and J. G. Kelly, Phys. Lett. B **333**, 110 (1994).

[4] B. Grzadkowski and J. F. Gunion, Phys. Lett. B **350**, 218 (1995).

[5] D. Atwood and A. Soni, Phys. Rev. D **52**, 6271 (1995).

[6] J. F. Gunion and X.-G. He, Phys. Rev. Lett. **76**, 4468 (1996).

[7] S. Bar-Shalom, D. Atwood, G. Eilam, R. Mendel, and A. Soni, Phys. Rev. D **53**, 1162 (1996).

[8] D. Atwood and A. Soni, Phys. Rev. D **45**, 2405 (1992).

[9] The appropriate final state depends on the nature of the *h* and its mass. Even if $m_h > 2m_W$, $b\bar{b}$ could be the dominant decay if the *h* is mainly *CP* odd or has enhanced $b\bar{b}$ coupling.

[10] For $e^+e^- \rightarrow t\bar{t}$ (no Higgs) events, a reconstruction efficiency in the mixed final state mode of $\epsilon \sim 0.15$ is expected [11]. Since m_h will be known, the efficiency for reconstructing the *h* should be good unless $m_h \sim m_W$ so that the two jets from the *W* of one *t* decay could be confused with those from the *h* (a problem that is alleviated if *b* tagging is employed).

[11] See, e.g., P. Igo-Kemenes, in *Workshop on Physics and Experimentation with Linear e^+e^- Colliders*, Waikoloa, Hawaii, April 26–30 (World Scientific 1993) p. 95 .

[12] See J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunters Guide* (Addison-Wesley Publishing, Redwood City, CA, 1989), and references therein.

[13] We refer to a parameter location on the $\chi^2 = s^2$ surface as an *s*-sigma deviation in the sense that the relative probability or likelihood compared to $s = 0$ is given by $\exp[-s^2/2]$, just as for a one-dimensional parameter space. Thus, $\chi^2 = 36$ corresponds to relative probability of 1.52×10^{-8}. This differs from the integrated probability for the parameters to lie outside the $\chi^2 = s^2$ surface, which for $\chi^2 = 36$ is 7.49×10^{-8} for 3 parameters, i.e., degrees of freedom.