# Exact Shape of the Lowest Landau Level in a Double-Layer System and a Superlattice with Uncorrelated Disorder

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We extend Wegner's exact solution for the 2D density of states at the lowest Landau level with a short-range disorder to the cases of a double-layer system and a superlattice. For the double-layer system, an analytical expression for the density of states, illustrating the interplay between the tunnel splitting of Landau levels and the disorder-induced broadening, is obtained. For the superlattice, we derive an integral equation, the eigenvalue of which determines the exact density of states. By solving this equation numerically, we trace the disappearance of the miniband with increasing disorder. [S0031-9007(96)01846-7]

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The shape of the Landau levels (LL) in a 2D system in the presence of a disorder has been studied intensively during the last two decades [1–17]. The complexity of the problem arises from the fact that in the absence of the disorder the energy spectrum is discrete. As a result, the self-energy of an electron appears to be real in any finite order of the perturbation theory. Therefore obtaining a finite LL width requires summation of the entire diagram expansion. It was demonstrated [9,14] that such a summation is possible when the LL number is large. The simplifications in this limit are different for a short-range and a smooth disorder. In the former case only a subsequence of diagrams without self-intersections contributes to the self-energy, or, in other words, the self-consistent Born approximation [1,3] becomes asymptotically exact [10], resulting in the semielliptical shape of the LL. For a smooth disorder, with correlation length larger than the magnetic length, all diagrams are of the same order of magnitude, but magnetic phases, caused by self-intersections, become small. The origin of these phases lies in an uncertainty in the position of the center of the Larmour orbit. With phases dropped, the entire perturbation series can be summed up, leading to the Gaussian shape of the LL [14].

For low LL numbers and short-range disorder, the magnetic phases in diagrams are of the order of unity. A small parameter appears in the problem only if the energy $\varepsilon$ is much larger than the LL width $\Gamma$, making possible a calculation of the density of states (DOS) in the LL tails. Such calculations were carried out in the framework of the instanton approach [4,5,8,10–12,17], and the tails were shown to be Gaussian. In the domain $\varepsilon \sim \Gamma$ the problem has no small parameter and no simplifications are possible. However, for the lowest LL the exact DOS was found by Wegner [6] for arbitrary $\varepsilon/\Gamma$. Wegner has shown that diagrammatic expansion of the disorder-averaged Green function $G(\varepsilon)$ can be mapped onto that of the zero-dimensional complex $\varphi^4$ model with partition function $Z_0^{(1)}$ given by a simple integral

$$Z_0^{(1)}(\varepsilon,\Gamma) = \int d\varphi^* d\varphi \, \exp\left[ i\varepsilon\varphi^*\varphi - \frac{\Gamma^2}{4}(\varphi^*\varphi)^2 \right]. \tag{1}$$

The crucial observation made by Wegner was that the number of diagrams for the disordered system, which are mapped onto a single graph of the $\varphi^4$ model, is proportional to the inverse value of the diagram itself. The electron Green function is then given by $G = -(2\pi l^2)^{-1}\partial \ln Z_0^{(1)}/\partial\varepsilon$ ($l$ is the magnetic length) in the sense that coefficients in front of $\Gamma^n$ in each side of this equation coincide. Having a closed expression for $G(\varepsilon)$, Wegner obtained the following formula for the DOS in the lowest LL:

$$g(\epsilon) = \frac{1}{2\pi^2 l^2}\,\frac{2}{\sqrt{\pi}}\,\frac{e^{\varepsilon^2/\Gamma^2}}{[(2/\sqrt{\pi})\int_0^{\varepsilon/\Gamma} dx e^{x^2}]^2 + 1}. \tag{2}$$

The magnetic field dependence of the width $\Gamma$ is $\Gamma \propto \sqrt{B}$. More precisely, for the correlator of the random potential $V(\mathbf{r})$ of the form $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \gamma\delta(\mathbf{r}-\mathbf{r}')$, one has $\Gamma = (\gamma/2\pi l^2)^{1/2}$. An alternative derivation of Wegner's result was given by Brézin, Gross, and Itzykson [7] in the framework of functional-integral approach.

Consider now a system consisting of two parallel 2D layers. In the absence of a disorder and magnetic field, a tunnel coupling between the layers would cause a splitting of size quantization levels by an amount of $2t$, $t$ being the tunnel integral. In a perpendicular field, the spectrum of the system represents two staircases of LL shifted in energy by $2t$. Assume that the field is strong, so that the cyclotron energy is much larger than $t$. If a disorder is present in the layers, the shape of two adjacent LL's depends on the ratio $\Gamma/t$. If this ratio is large, then the tunneling does not play any role, so that the DOS is twice the DOS in an individual layer (2). In the opposite case, $t \gg \Gamma$, the peaks in the DOS, corresponding to the symmetric and antisymmetric states, are broadened independently; they are distanced by $2t$ and their shape is described by (2) with the width $\Gamma/\sqrt{2}$ [18]. Factor

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$1/\sqrt{2}$ appears because the effective random potential for the symmetric (antisymmetric) state is $[V_1(\mathbf{r}) \pm V_2(\mathbf{r})]/2$, where $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ are the potentials in the layers. If $\langle V_1(\mathbf{r})V_2(\mathbf{r}')\rangle = 0$, the correlator for each effective potential appears to be twice as small as that for an individual layer.

For $\Gamma \sim t$, calculation of the DOS in a double-layer system seems to pose an even harder problem than for a single layer, since here the DOS represents a two-parametric function, $g_{\mathrm{dl}}(\varepsilon/\Gamma, t/\Gamma)$, with both arguments of the order of unity. Nevertheless, as we demonstrate below, for the lowest LL the exact DOS can be obtained in a closed form by generalizing Wegner's approach. Moreover, such a generalization can be carried out for an arbitrary number of layers, and, in particular, we consider the case when the number of layers is infinite (superlattice). In the absence of a disorder, each LL in a superlattice gives rise to a miniband of a width $2t$. Gradual switching on a disorder first smears out the singularities in DOS at the edges of the miniband and then, as $\Gamma$ exceeds $t$, transforms the DOS into a single peak corresponding to an individual layer. We derive an integral equation the eigenvalue of which determines the DOS in a superlattice, and trace this transformation by solving it numerically.

Consider first a double-layer system. The free Green function represents a $2 \times 2$ matrix which, after projecting onto the lowest LL, takes the form

$$\hat{G}^0(\mathbf{r}, \mathbf{r}') = \frac{\hat{Q}}{2\pi l^2} \exp\left[ -\frac{(\mathbf{r} - \mathbf{r}')^2}{4l^2} + \frac{i}{2l^2}(\mathbf{r} \times \mathbf{r}') \right], \tag{3}$$

$$\hat{Q} = (\varepsilon - \hat{t})^{-1}, \qquad \hat{t} = \begin{pmatrix} 0 & t \\ t & 0 \end{pmatrix}. \tag{4}$$

The perturbation expansion for $\hat{G}$, averaged over $V_1$ and $V_2$, has the same diagrammatic representation as for a single layer. Lowest order diagrams are shown in Figs. 1(a) and 1(b). Solid lines correspond to $\hat{G}^0$ and dashed lines correspond to the correlator of the random potential. Solid lines carry indices, reflecting the fact that an electron can tunnel from one layer to another between two successive scattering acts. Since the scattering retains an electron in the same layer, the indices at the ends of each dashed line coincide. Introducing projecting operators $\hat{\tau}_i$ as

$$\hat{\tau}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \hat{\tau}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{5}$$

the expressions corresponding to diagrams (a) and (b) can be written in the form

$$\begin{aligned} \hat{G}^{(1)} &= F^{(1)}\Gamma^2 \sum_i (\hat{Q}\hat{\tau}_i\hat{Q}\hat{\tau}_i\hat{Q}), \\ \hat{G}^{(2)} &= F^{(2)}\Gamma^4 \sum_{ij} (\hat{Q}\hat{\tau}_i\hat{Q}\hat{\tau}_j\hat{Q}\hat{\tau}_i\hat{Q}\hat{\tau}_j\hat{Q}), \end{aligned} \tag{6}$$

where $F^{(1)}$ and $F^{(2)}$ are spatial integrals. Similarly, spatial integrals in higher order diagrams are separated out as factors in front of products of matrices $\hat{Q}$ and $\hat{\tau}_i$. Important is that all $F^{(n)}$ are *exactly the same as those for a single-layer*.

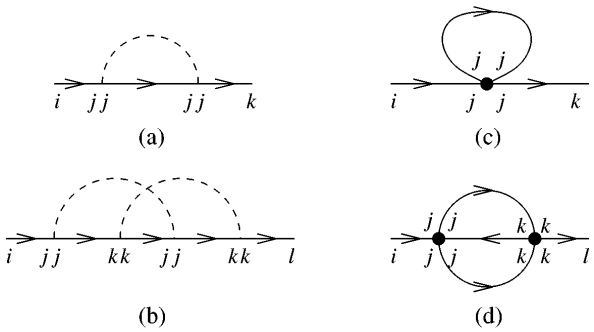


FIG. 1. First (a) and second (b) order diagrams for the Green function mapped on graphs (c) and (d), respectively.

The mapping is carried out following Wegner's prescription: One identifies pairs of points in a diagram connected by dashed lines, and one gets a graph with four lines entering each vertex [see Figs. 1(c) and 1(d)]. In doing so, one obtains, in general, a set of diagrams yielding the same graph. It is clear, however, that since assigning indices to the lines does not alter in any way the topology of diagrams or graphs, the number of diagrams in a set is the same for both single- and double-layer cases. Moreover, one observes that the contractions of matrices $\hat{\tau}_i$ precisely follow the identification of points [as it can be seen, e.g., in Figs. 1(b) and 1(d)], so that all diagrams in such a set are equal. The fundamental relation, established by Wegner, is that for each diagram in the set one has $F^{(n)} = 1/\mathcal{N}s$, where $\mathcal{N}$ is the number of diagrams in the set and $1/s$ is the symmetry factor of the graph ($s$ is the number of permutations leaving graph invariant). This factor is also unchanged by assigning indices to the graph; for example, permutation of upper and lower lines leaves graph 1(d) invariant in both cases. Thus the contribution of the set, being proportional to $\mathcal{N}F^{(n)}$, is $\mathcal{N}$ independent, and the problem again reduces to a zero-dimensional field theory. The remaining question is whether matrix products of type (6) can be generated in the perturbation expansion of some generalized $\varphi^4$ model. Our main observation is that the model with the partition function

$$Z_0^{(2)} = \int d\Phi^* d\Phi \exp\left[ i\Phi^*\hat{Q}^{-1}\Phi - \frac{\Gamma^2}{4} \sum_i (\Phi^*\tau_i\Phi)^2 \right] \tag{7}$$

accomplishes this task. Here $\hat{Q}$ and $\hat{\tau_i}$ are matrices defined by (4) and $\Phi$ is a *two-component* complex field $\Phi = (\varphi_1, \varphi_2)$. Indeed the $n$th order term in the expansion of exponent (7) in terms of $\Gamma^2$ represents a product of $2n$ matrices $\hat{\tau}_i$ (with all pairwise contractions) separated by $2n$ products of the form $\Phi\Phi^*$. Then the Gaussian integral over $\Phi$ inserts the "Green function" $\hat{Q} = -i\langle\Phi\Phi^*\rangle$ in place of each pair of fields $\Phi$ and $\Phi^*$, with all possible contractions between them yielding all the $n$th order graphs with appropriate symmetry factors.

Having the mapping established, the DOS in a double-layer system can be calculated directly from (7). It is also instructive to rewrite $Z_0^{(2)}$ in a different form. First, we decouple the quartic term in the exponent of (7),

$$iS = \sum_j \left[ i\varepsilon\varphi_j^*\varphi_j - \frac{\Gamma^2}{4}(\varphi_j^*\varphi_j)^2 \right] - it(\varphi_1^*\varphi_2 + \varphi_2^*\varphi_1), \tag{8}$$

with the help of the Gaussian integral over a pair of auxiliary variables. Performing the remaining integral over $\varphi_i$ we then obtain

$$Z_0^{(2)} = \frac{(i\pi)^2}{\pi\Gamma^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{d\lambda_1\, d\lambda_2 e^{-(\lambda_1^2+\lambda_2^2)/\Gamma^2}}{(\varepsilon + \lambda_1)(\varepsilon + \lambda_2) - t^2}. \tag{9}$$

From the form (9) both limiting cases of large and small $t$ are evident. For small $t$, the partition function factorizes, $Z_0^{(2)} = (Z_0^{(1)})^2$, yielding twice the DOS (2). For $t \gg \Gamma$ the characteristic values of $\lambda_1$, $\lambda_2$ in (9), being of the order of $\Gamma$, are much smaller than $t$. This allows one to neglect the product $\lambda_1\lambda_2$ in the denominator; $Z_0^{(2)}$ is not small only if $(\varepsilon \pm t) \sim \Gamma$. Introducing new variables $\mu_\pm = \lambda_1 \pm \lambda_2$, the integral over $\mu_-$ contributes a factor $\sqrt{2\pi}\Gamma$, and the integral over $\mu_+$ reproduces Wegner's result with the width $\Gamma/\sqrt{2}$, as discussed above. Evolution of the DOS between two limits is shown in Fig. 2(a).

Let us now turn to a superlattice. The partition function (7) can be straightforwardly generalized to $n$ layers with nearest-neighbor tunneling

$$Z_0^{(n)} = \int \prod_{i=1}^{n} d^2\varphi_i \exp\left[ i\varepsilon \sum_{j=1}^{n} \varphi_j^*\varphi_j - \frac{\Gamma^2}{4}\sum_{j=1}^{n}(\varphi_j^*\varphi_j)^2 - it\sum_{j=1}^{n-1}(\varphi_j^*\varphi_{j+1} + \varphi_{j+1}^*\varphi_j)\right]. \tag{10}$$

We are interested in the asymptotic behavior of $Z_0^{(n)}(\varepsilon, \Gamma)$ as $n \to \infty$. For this purpose we employ a method similar to the transfer-matrix method in the theory of 1D spin chains. Note that the expression (10) for $Z_0^{(n)}$ can be rewritten as

$$Z_0^{(n)} = \int d^2\varphi \exp\left[ i\varepsilon\varphi^*\varphi - \frac{\Gamma^2}{4}(\varphi^*\varphi)^2 \right] I_n(\varphi^*, \varphi), \tag{11}$$

where $I_1 = 1$ and the functions $I_n(\varphi^*, \varphi)$ satisfy the following recurrence relation

$$I_{n+1}(\varphi^*, \varphi) = \hat{T}_{\varepsilon,\Gamma} I_n \equiv \int d^2\varphi_1 \exp\left[ i\varepsilon\varphi_1^*\varphi_1 - \frac{\Gamma^2}{4}(\varphi_1^*\varphi_1)^2 - it(\varphi^*\varphi_1 + \varphi_1^*\varphi)\right] I_n(\varphi_1^*, \varphi_1). \tag{12}$$

Consider now the eigenvalues $\lambda_{\varepsilon,\Gamma}^{(k)}$ and eigenfunctions $\Omega_{\varepsilon,\Gamma}^{(k)}(\varphi^*, \varphi)$ of the operator $\hat{T}_{\varepsilon,\Gamma}$: $\hat{T}_{\varepsilon,\Gamma}\Omega^{(k)} = \lambda^{(k)}\Omega^{(k)}$. Assume that $\lambda^{(0)}$ has the maximal absolute value. Then in the limit $n \to \infty$, $Z_0^{(n)}$ will behave as $(\lambda^{(0)})^n$. Hence the DOS *per layer* in a superlattice can be expressed through $\lambda_{\varepsilon,\Gamma}^{(0)}$ as follows: $g_{sl}(\varepsilon, \Gamma) = (2\pi^2 l^2)^{-1}\partial \ln \lambda^0/\partial\varepsilon$.

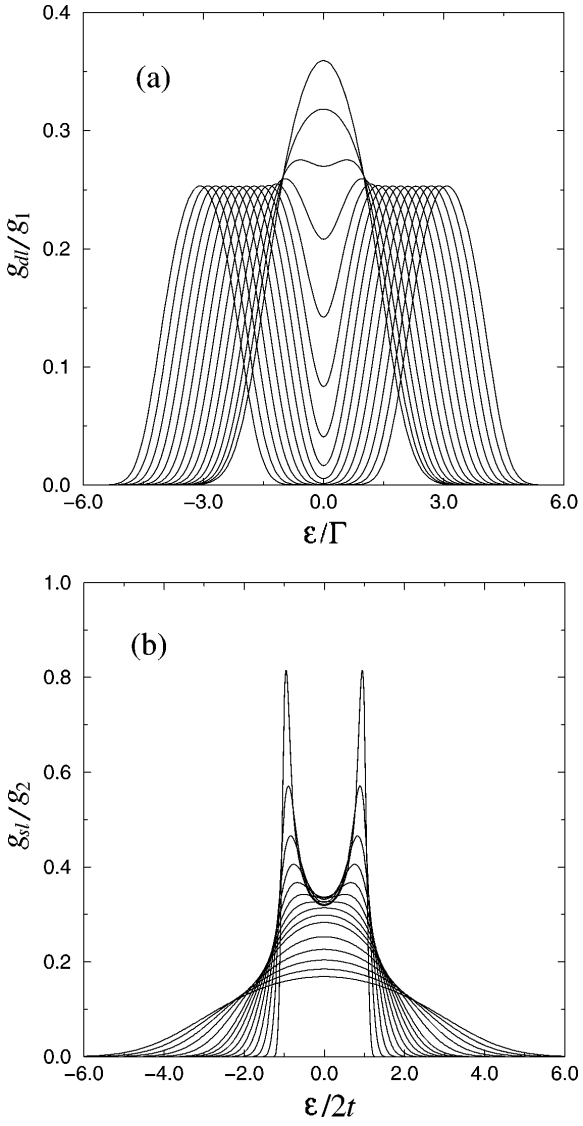


FIG. 2. (a) DOS per layer for a double-layer system in units of $g_1 = (2\pi l^2)^{-1}\Gamma^{-1}$ for values of $t/\Gamma = 0.0$ (highest curve), 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, and 3.0, respectively. (b) DOS per layer for a superlattice in units of $g_2 = (2\pi l^2)^{-1}(2t)^{-1}$ for values of $\Gamma/2t = 0.1$ (highest curve), 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.4, 1.6, 1.8, and 2.0, respectively.

Consider first the case of a weak disorder $\Gamma \to 0$. One can check that eigenfunctions of $\hat{T}_{\varepsilon,0}$ have the form

$$\Omega_{\varepsilon,0}^{(p,m)} = e^{im\alpha - (i\varepsilon + \sqrt{4t^2-\varepsilon^2})R^2/2}\left(R^2\sqrt{4t^2 - \varepsilon^2}\right)^{m/2} \times L_p^m\left(R^2\sqrt{4t^2 - \varepsilon^2}\right), \tag{13}$$

where $R$ and $\alpha$ are, respectively, the absolute value and the phase of $\varphi$, and $L_p^m(x)$ is the Laguerre polynomial. The corresponding eigenvalues $\lambda_{\varepsilon,0}^{(p,m)}$ equal

$$\lambda_{\varepsilon,0}^{(p,m)} = \frac{\pi i}{t}\left(\frac{2it}{i\varepsilon - \sqrt{4t^2 - \varepsilon^2}}\right)^{2p+m+1}, \tag{14}$$

where square root is defined as $i^{-1}\,\mathrm{sgn}(\varepsilon)\sqrt{\varepsilon^2 - 4t^2}$ for $|\varepsilon| > 2t$. Outside the interval $|\varepsilon| < 2t$, the phases of eigenvalues (14) have no energy dependence, supporting the obvious observation that the DOS is zero outside the miniband. Within the miniband, all $\lambda^{(p,m)}(\varepsilon, 0)$ have the same absolute value. This is a manifestation of the fact that for a large but finite number of layers the DOS in the absence of disorder represents a set of delta peaks. However, with arbitrary weak disorder present, only the eigenvalue $\lambda^{(0,0)}(\varepsilon, 0)$ will survive in the limit $n \to \infty$, yielding the familiar result

$$2\pi^2 l^2 g_{sl}(\varepsilon, 0) = \mathrm{Im}\frac{\partial \ln \lambda_{\varepsilon,0}^{(0,0)}}{\partial \varepsilon} = \frac{1}{\sqrt{4t^2 - \varepsilon^2}}. \quad (15)$$

Assume now that the disorder is finite but $\Gamma \ll t$. It is convenient to formally present the operator $\hat{T}_{\varepsilon,\Gamma}$ as

$$\lambda_{\varepsilon,\Gamma}^{(0)}\Omega_{\varepsilon,\Gamma}^{(0)} = \hat{T}_{\varepsilon,\Gamma}\Omega_{\varepsilon,\Gamma}^{(0)} = \int \frac{dE}{\sqrt{\pi}\Gamma} e^{-(E-\varepsilon)^2/\Gamma^2}\hat{T}_{E,0}\Omega_{\varepsilon,\Gamma}^{(0)}. \quad (16)$$

For small $\Gamma$, only $E$ close to $\varepsilon$ contribute to the integral (16). This suggests starting the iteration procedure by substituting, as a zero approximation, the $m = p = 0$ eigenfunction $\Omega_{E,0}^{(0,0)}$ of $\hat{T}_{E,0}$ into the right-hand side. This generates the first approximation for the function $\Omega_{\varepsilon,\Gamma}^{(0)}$,

$$\tilde{\Omega}_{\varepsilon,\Gamma}^{(0)} = \frac{1}{\sqrt{\pi}\Gamma\lambda_{\varepsilon,\Gamma}^{(0)}} \int dE e^{-(E-\varepsilon)^2/\Gamma^2}\lambda_{E,0}^{(0,0)}\Omega_{E,0}^{(0,0)}. \quad (17)$$

Substituting this function back into (16), we obtain

$$\hat{T}_{\varepsilon,\Gamma}\tilde{\Omega}_{\varepsilon,\Gamma}^{(0)} = \frac{1}{\pi\Gamma^2\lambda_{\varepsilon,\Gamma}^{(0)}} \int dE \int dE' \times e^{-(E-\varepsilon)^2/\Gamma^2 - (E'-\varepsilon)^2/\Gamma^2}\lambda_{E',0}^{(0,0)}\hat{T}_{E,0}\Omega_{E',0}^{(0,0)}. \quad (18)$$

Note now that $\Omega_{E',0}^{(0,0)}$ as a function of $E'$ changes significantly on the scale $E' \sim t$. On the other hand, exponential factors in (18) enforce the difference between $E$ and $E'$ to be of the order of $\Gamma$. This allows one to replace $\Omega_{E',0}^{(0,0)}$ by $\Omega_{E,0}^{(0,0)}$ under the integral. Then we immediately observe that the right-hand side takes the form $\tilde{\lambda}_{\varepsilon,\Gamma}^{(0)}\tilde{\Omega}_{\varepsilon,\Gamma}^{(0)}$ with

$$\tilde{\lambda}_{\varepsilon,\Gamma}^{(0)} = \frac{2\pi}{\sqrt{\pi}\Gamma} \int_{-\infty}^{\infty} dE \frac{e^{-(E-\varepsilon)^2/\Gamma^2}}{\sqrt{4t^2 - E^2} - iE}. \quad (19)$$

In other words, for small $\Gamma$ the function $\tilde{\Omega}_{\varepsilon,\Gamma}^{(0)}$ satisfies (16) yielding the eigenvalue (19). In principle, to assess the region of large $\Gamma$ one should keep iterating Eq. (16). However, as we have established numerically, the function $\tilde{\Omega}^{(0)}$ is already a very good approximation for $\Omega^{(0)}$ and $\tilde{\lambda}^{(0)}$ is a very good approximation for $\lambda^{(0)}$ not only for a small, but also for an *arbitrary* ratio $\Gamma/t$. Indeed, as $\Gamma/t$ increases, one should reproduce Wegner's result, which corresponds to $t = 0$ and $\Omega^{(0)} = \mathrm{const}$. On the other hand, it is easy to see that $\tilde{\Omega}^{(0)}$ turns to constant as $t \to 0$, and that in this limit $\tilde{\lambda}^{(0)}$ turns to $Z_0^{(1)}$. Thus Eq. (19) is exact in both limits. The numerical results for the DOS, obtained with the use of $\tilde{\lambda}^{(0)}$ (19), are shown in Fig. 2(b). We see that the miniband is completely destroyed as $\Gamma/t$ exceeds 1.4.

Note in conclusion, that a decade ago there was a significant interest in transport in multilayer systems in a strong magnetic field [19,20]. Recently, this interest was renewed [18,21]. The focus of the study is a transition from a purely 2D to the 3D behavior of the conductivity with increasing $t$. As was shown in [22], the structure of electronic states in a multilayer system can be tuned by tilting magnetic field. For the lowest LL, the role of $B_\parallel$ (the parallel component of $B$) reduces to the renormalization of $t$. The renormalized tunneling $\tilde{t}$ differs from $t$ by the overlap integral of two oscillator wave functions with the centers displaced by an amount proportional to $B_\parallel$. Concerning our results, one can check that they apply in the presence of $B_\parallel$ after replacing $t$ by $\tilde{t}$.


[1] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).
[2] É. M. Baskin, L. N. Magarill, and M. V. Éntin, Zh. Eksp. Teor. Fiz. **75**, 723 (1978) [Sov. Phys. JETP **48**, 365 (1978)].
[3] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[4] L. B. Ioffe and A. I. Larkin, Zh. Eksp. Teor. Fiz. **81**, 1048 (1981) [Sov. Phys. JETP **54**, 556 (1981)].
[5] I. Affleck, J. Phys. C **16**, 5839 (1983).
[6] F. Wegner, Z. Phys. B **51**, 279 (1983).
[7] E. Brézin, D. Gross, and C. Itzykson, Nucl. Phys. **B235**, 24 (1984).
[8] I. Affleck, J. Phys. C **17**, 2323 (1984).
[9] K. A. Benedict and J. T. Chalker, J. Phys. C **18**, 3981 (1985); *ibid.* **19**, 3587 (1986).
[10] K. A. Benedict, Nucl. Phys. **B280**, 549 (1987).
[11] W. Apel, J. Phys. C **20**, L577 (1987).
[12] K. B. Efetov and V. G. Marikhin, Phys. Rev. B **40**, 12 126 (1989).
[13] V. A. Geiler and V. A. Margulis, Zh. Eksp. Teor. Fiz. **95**, 1134 (1989) [Sov. Phys. JETP **68**, 654 (1989)].
[14] M. E. Raikh and T. V. Shahbazyan, Phys. Rev. B **47**, 1522 (1993).
[15] S. A. Gredeskul, Y. Avishai, and M. Ya Azbel', Europhys. Lett. **21**, 489 (1993).
[16] Y. Avishai, M. Ya. Azbel', and S. A. Gredeskul, Phys. Rev. B **48**, 17 280 (1993).
[17] M. Zusman, Y. Avishai, and S. A. Gredeskul, Phys. Rev. B **48**, 17 922 (1993).
[18] E. S. Sørensen and A. H. MacDonald, Phys. Rev. B **54**, 10 675 (1996); T. Ohtsuki, B. Kramer, and Y. Ono, J. Phys. Soc. Jpn. **62**, 224 (1993).
[19] H. L. Störmer *et al.*, Phys. Rev. Lett. **56**, 85 (1986).
[20] S. E. Ulloa and G. Kirczenow, Phys. Rev. Lett. **57**, 2991 (1986).
[21] J. T. Chalker and A. Dohmen, Phys. Rev. Lett. **75**, 4496 (1995).
[22] J. Hu and A. H. MacDonald, Phys. Rev B **46**, 12 554 (1992).