Anisotropic Surface Growth Model in Disordered Media

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We introduce a self-organized surface growth model in 2 + 1 dimensions with an anisotropic avalanche process, which is expected to be in the universality class of the anisotropic quenched Kardar-Parisi-Zhang (KPZ) equation with alternative signs of the nonlinear KPZ terms. It turns out that the surface height correlation function in each direction scales distinctively. The anisotropic behavior is attributed to the asymmetric behavior of the quenched KPZ equation in 1 + 1 dimensions with respect to the sign of the nonlinear KPZ term. [S0031-9007(96)01888-1]

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The subject of the pinning-depinning (PD) transition by an external driving force has been of much interest recently. The problems of the interface growth in porous media under external pressure [1], the dynamics of a domain wall under random field [2], the dynamics of a charge density wave under external field [3], and the vortex motion in superconductors under external current [4,5] are typical examples. In the PD transition, there exists a critical value F_c of the driving force F, such that when $F < F_c$, interface (or charge, or vortex) is pinned by disorder, while for $F > F_c$, it moves with constant velocity v. The velocity v plays the role of order parameter in the PD transition, which behaves as

$$v \sim (F - F_c)^{\theta}. \qquad (1)$$

Recently, several stochastic models for the PD transition of interface growth in disordered media have been introduced [6,7]. It is believed that the models in 1 + 1 dimensions are described by the quenched Kardar-Parisi-Zhang (QKPZ) equation

$$\partial_t h = \nu \partial_x^2 h + \frac{\lambda}{2}(\partial_x h)^2 + F + \eta(x,h), \qquad (2)$$

where noise η depends on position x and height h with the properties of $\langle \eta(x,h)\rangle = 0$ and $\langle \eta(x,h)\eta(x',h')\rangle = 2D\,\delta(x-x')\delta(h-h')$. The QKPZ equation exhibits the PD transition at F_c. The surface at F_c can be described by the directed percolation (DP) cluster spanned perpendicularly to the surface growth direction in 1 + 1 dimensions. The roughness exponent α of the interface is given as the ratio of the correlation length exponents of the DP cluster in perpendicular and parallel directions, that is, $\alpha = \nu_\perp/\nu_\parallel \approx 0.63$.

The origin of the nonlinear term in the QKPZ equation is different from that of the thermal KPZ equation with the noise $\eta(x,t)$ [8]. For the quenched case, the nonlinear term is induced by the anisotropic nature of disordered media, while for the thermal case, it is induced by lateral growth, and thus the coefficient λ is proportional to the velocity of the interface, which vanishes at the threshold of the PD transition. For surfaces belonging to the DP universality class, the positive nonlinear KPZ term is induced under coarse graining of the quenched random force with amplitudes $\Delta_h^{1/2}$ and $\Delta_x^{1/2}$ in the h direction and in the x direction, respectively, when $\Delta_h > \Delta_x$. On the other hand, one may consider the case in 2 + 1 dimensions that the amplitudes of random force are anisotropic on a substrate, that is, $\Delta_h > \Delta_\parallel$ in one direction of the substrate and $\Delta_h < \Delta_\perp$ in the other. In such a case, following [8], the coarse-grained Langevin equation is expected to take the form of the anisotropic QKPZ (AQKPZ) equation given by

$$\partial_t h = \nu_\parallel \partial_\parallel^2 h + \nu_\perp \partial_\perp^2 h + \frac{\lambda_\parallel}{2}(\partial_\parallel h)^2 + \frac{\lambda_\perp}{2}(\partial_\perp h)^2 + F + \eta(\mathbf{r},h), \qquad (3)$$

with $\lambda_\parallel > 0$ and $\lambda_\perp < 0$.

In this Letter, we study the surface of Eq. (3) by introducing a self-organized stochastic model. The universality class of the stochastic model is checked by comparing surface properties with those obtained from direct numerical integration of Eq. (3). As shown in Fig. 1, the surface of Eq. (3) at F_c forms the shape of a mountain range with a steep inclination in one direction, whereas it is gently sloping in the other direction. Accordingly, the roughness exponents of the height-height correlation functions in each direction scale distinctively, which leads to a new universality class. This result is remarkable as compared with the case of the thermal noise. For the thermal case, the height-height correlation function is isotropic, and the anisotropic KPZ equation in 2 + 1 dimensions renormalizes into the Edwards-Wilkinson equation [9]. This is because for the case described by the KPZ equation with thermal noise, the sign of the nonlinear terms is irrelevant in determining the universality class [10], and the two nonlinear terms with different signs are canceled out effectively [11]. However, for the quenched case, the anisotropic surface morphologies of Fig. 1 imply that such cancellation does not occur and the QKPZ equation is asymmetric with respect to the sign of the nonlinear KPZ terms. Accordingly, in this Letter, we also study

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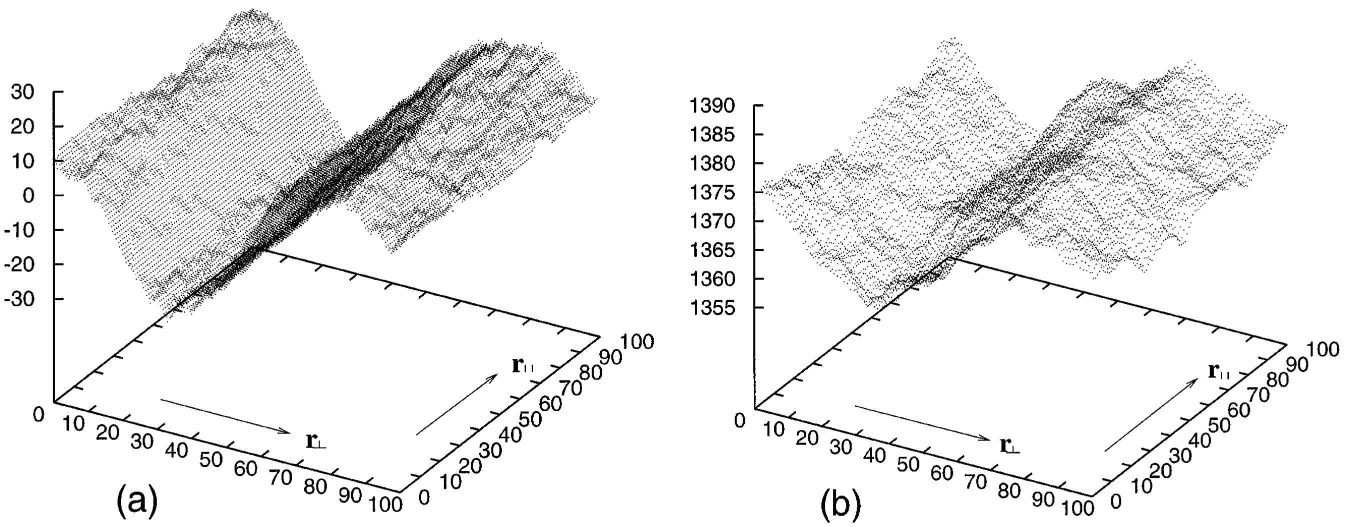

FIG. 1. A typical surface configuration of the AQKPZ equation generated by (a) the stochastic model and (b) direct numerical integration at the transition point.

the QKPZ equation with negative λ in $1 + 1$ dimensions, which enables one to understand the anisotropic nature of the AQKPZ equation in $2 + 1$ dimensions.

First, we study the QKPZ equation with $\lambda < 0$ in $1 + 1$ dimensions by direct numerical integration with the discretized version,

$$h(x, t + \Delta t) = h(x,t) + \Delta t\left\{h(x-1,t) + h(x+1,t) - 2h(x,t) + \frac{\lambda}{8}[h(x+1,t) - h(x-1,t)]^2 + F\right\} + (\Delta t)^{2/3}\xi(x,[h(x,t)]), \quad (4)$$

where $[\cdots]$ denotes the integer part and ξ is uniformly distributed in $[-\frac{1}{2}, \frac{1}{2}]$. The prefactor $(\Delta t)^{2/3}$ of the noise term arises from approximately coarse graining the noise $\eta(x,h)$ during the time interval Δt. Numerical integration using Eq. (4) for $\lambda = 1$ and $\Delta t = 0.01$ yields, even for a modest system size of $L = 10^3$, the roughness exponent $\alpha \approx 0.63$, which is consistent with the value of the DP universality. Note that the use of the usual prefactor (Δt) requires a much larger computational cost to obtain the DP value of α [12,13]. Details for the derivations of Eq. (4) and the result of the direct numerical integration for $\lambda > 0$ will be published elsewhere [14].

For the case of $\lambda < 0$, we performed the numerical integration with $\lambda = -1$ and $\Delta t = 0.01$ for convenience. Figure 2 shows typical surface configurations evolved temporally at the PD transition point, $F_c \approx 1.98$, which exhibits the shape of a mountain with a flat inclination in the pinned state. The surface in the pinned state looks similar to that of model A by Sneppen [15], and the shape of the surface determines the roughness exponent to be $\alpha = 1$. Note that the RSOS restriction before deposition in model A of Sneppen does not allow the particle to deposit in every site, which makes the growth velocity reduced, and results in $\lambda < 0$ [16]. Accordingly, it is reasonable to have the surface morphology as shown in Fig. 2 for the QKPZ equation with negative λ, which is different from the one for $\lambda > 0$. In addition, we also examined the noise distribution on perimeter sites. It reveals that the pinning is caused by relatively large pinning strengths around the site where the height is minimum. The flatness on the inclination makes the term $(\partial_x h)^2$ large, which with the large pinning strengths compensates the external driving force. Thus the growth velocity becomes zero, and the surface is pinned. Since the surface pinning is caused mainly by the barrier of the pinning strengths around the site of minimum height, the growth velocity exhibits a sudden jump as the barrier is overcome

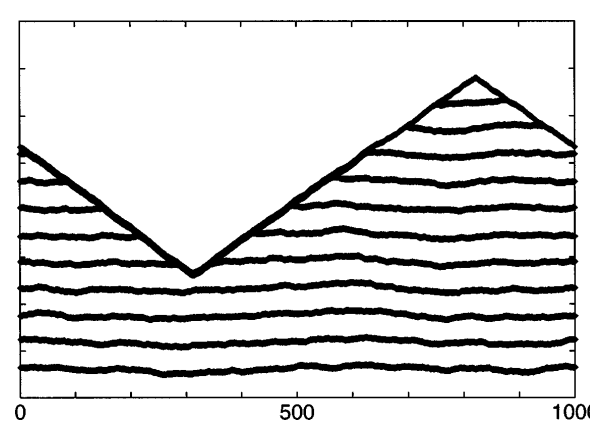

FIG. 2. Temporally evolved surface configurations of the QKPZ equation with $\lambda < 0$ in $1 + 1$ dimensions at the pinning-depinning transition point. Successive height profiles are shown at constant time intervals.

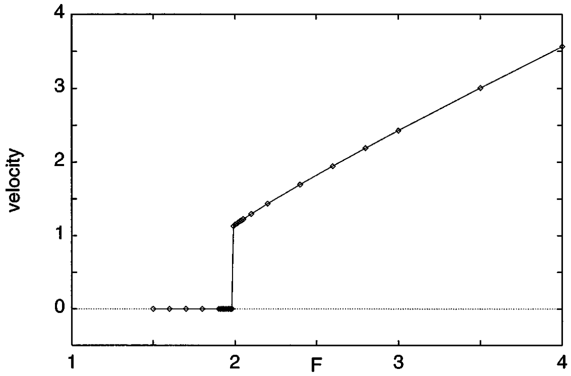

FIG. 3. The growth velocity versus force for the QKPZ equation with negative λ in $1 + 1$ dimensions.

by increasing the external driving force F. Accordingly, the PD transition is of first order as depicted in Fig. 3. On the other hand, for $\lambda > 0$, pinned sites are not localized, but scattered, so that the PD transition is continuous. Therefore the surface of the QKPZ equation in $1 + 1$ dimensions is asymmetric with respect to the sign of the nonlinear term.

Next, we study the AQKPZ equation, Eq. (3), in $2 + 1$ dimensions by introducing a stochastic model. It is a natural extension of the RSOS model introduced previously by the current authors to study the anisotropic thermal KPZ equation [17]. The stochastic model is based on a combination of Sneppen's model A for $\lambda < 0$ and model B for $\lambda > 0$, which is realized by assigning an anisotropic avalanche process. The advantage of studying such a stochastic model is twofold: first, there is no need for fine-tuning of F to get the critical state, and second, asymptotic states can be readily reached for small system size. The model is defined on the checkerboard lattice, square lattice rotated by 45°. Initially, we begin with a flat surface characterized by the height 0 on one sublattice and 1 on the other (see Fig. 4). Random numbers are assigned to each site. At each time step, selected is the site with the minimum random number among the sites (i, j) of which the two nearest neighbors at $(i + \frac{1}{2}, j - \frac{1}{2})$ and $(i - \frac{1}{2}, j - \frac{1}{2})$ are higher. The site is updated by increasing the height by 2. Next, the anisotropic avalanche process may occur on the neighboring sites, $(i + \frac{1}{2}, j + \frac{1}{2})$ and $(i - \frac{1}{2}, j + \frac{1}{2})$. If their height is lower by 3 than that of (i, j), the height is increased by 2. The avalanche rule is then applied successively to the next rows in the $\hat{j}$ direction until there is no change. The sites with increased height are updated by new random numbers. The avalanche direction, $\hat{j}$ direction, corresponds to the $r_\parallel$ direction in Eq. (3). The anisotropic avalanche process along the positive $\hat{j}$ direction is an interesting aspect of our model, which is a generalization of the model by Maslov and Zhang [18] into $2 + 1$ dimensions. Such an anisotropic avalanche process is to be distinguished from the isotropic avalanche process on tilted substrates, in which the roughness exponent along the tilt direction ($r_\perp$ direction) is $\frac{1}{3}$ [8]. Using the tilt argument, it can be shown that our model includes alternative signs of the nonlinear terms, that is, $\lambda_\parallel > 0$ and $\lambda_\perp < 0$ [17]. A typical surface morphology is shown in Fig. 1(a). We measured the roughness exponents for the height-height correlation functions, $C_\parallel(r_\parallel) \equiv \langle \frac{1}{L^2} \sum_x [h(\mathbf{x}) - h(\mathbf{x} + \mathbf{r}_\parallel)]^2 \rangle \sim r_\parallel^{2\alpha_\parallel}$ and $C_\perp(r_\perp) \sim r_\perp^{2\alpha_\perp}$. The roughness exponents for each direction are obtained as $\alpha_\parallel = 0.25(1)$ and $\alpha_\perp = 0.75(1)$ as shown in Fig. 5. We also measured the height fluctuation width, $W^2 \equiv \frac{1}{L^2} \sum_r (h_r - \bar{h})^2 \sim L^{2\alpha}$ for $t \gg L^z$ and $\sim t^{2\beta}$ for $t \ll L^z$, where the exponents α, β, and z are the roughness, the growth, and the dynamic exponents, respectively, and L is the system size. It is obtained that $\alpha = 0.87(1)$, $\beta = 0.80(1)$, and z is given as $z = \alpha/\beta$. The values of the exponents are different from those of the isotropic case in $2 + 1$ dimensions, where $\alpha \approx 0.48$

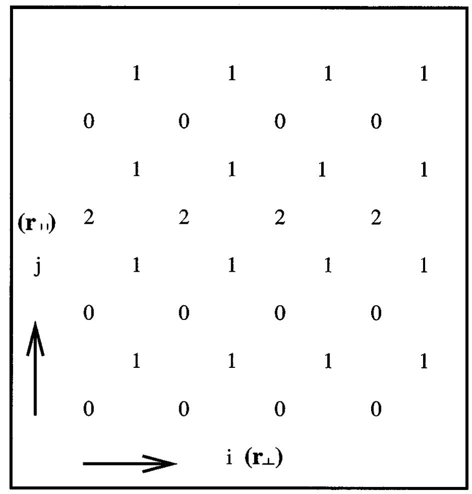

FIG. 4. The configuration of a substrate with one step along a row for linear size $L = 4$. The avalanche process occurs in the $\hat{j}$ direction.

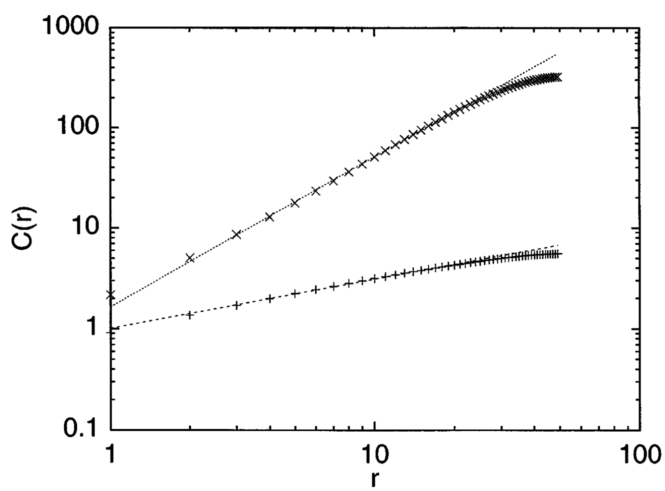

FIG. 5. The height-height correlation functions in parallel (lower data) and perpendicular (upper ones) directions for the AQKPZ stochastic model in $2 + 1$ dimensions.

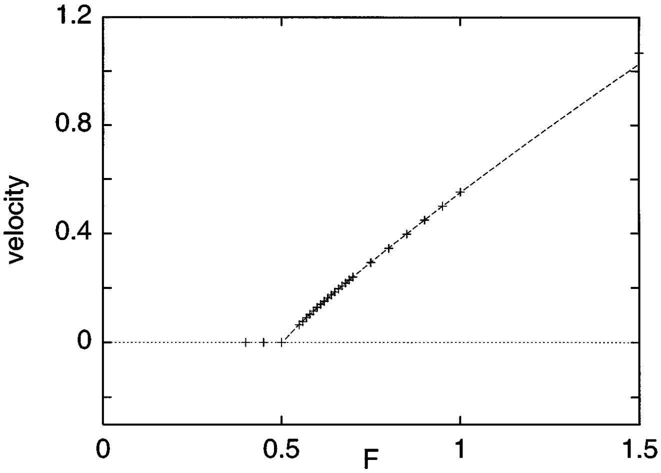

FIG. 6. The growth velocity versus force for the AQKPZ equation in 2 + 1 dimensions.

and $\beta \approx 0.41$ [19]. We also measured the avalanche size distribution, $P(s) \sim s^{-\tau}$. The exponent τ is obtained as $\approx 1.35(3)$.

In order to check if the stochastic model reduces to the AQKPZ universality, we considered the surface of Eq. (3) by carrying out the direct numerical integration using the two-dimensional version of Eq. (4). We used the numerical values of $\Delta t = 0.01$, $\lambda_{\parallel} = 1$, and $\lambda_{\perp} = -1$ for convenience. Figure 1(b) shows the surface morphology obtained by the direct numerical integration at the threshold of the PD transition, $F_c \approx 0.50$, which looks similar to the one in Fig. 1(a). However, we could not measure the roughness exponents $\alpha_{\parallel}$ and $\alpha_{\perp}$ precisely, because their precise measurement requires a relatively large system size and huge computing times. Nevertheless, since the morphologies of Figs. 1(a) and 1(b) are similar to each other and that of $\lambda_{\parallel} > 0$ and $\lambda_{\perp} < 0$ can be proven using the tilt argument for the stochastic model [17], we believe that the stochastic model belongs to the AQKPZ universality. The PD transition turns out to be continuous as depicted in Fig. 6. The velocity exponent θ defined in Eq. (1) is obtained as $\theta = 0.9(1)$, which is somewhat larger than $\theta \approx 0.8$ for the isotropic case [19]. The numerical results, $\tau \approx 1.35$, $z \approx 1.09$, and $\theta \approx 0.9$ seem to represent the characteristics of the self-organized critical depinning transition [18,20], but the relation of those exponents to the anisotropic roughness exponents in 2 + 1 dimensions is not clear yet. Further study is required about this point. We have also examined the surface in a moving state, $F > F_c$. In this regime, the surface is no longer anisotropic, and reduces to the AKPZ equation with thermal noise for $F \gg F_c$.

In summary, we have studied the AQKPZ equation with alternative signs of the nonlinear terms in 2 + 1 dimensions and have found that it leads to a new universality class. The surface exhibits anisotropic scaling behavior, which is due to the asymmetric behavior of the QKPZ equation with respect to the sign of the nonlinear term. The QKPZ equation with $\lambda < 0$ in 1 + 1 dimensions has also been studied. We have obtained that the surface forms the shape of a mountain with a flat inclination and the PD transition is of first order. Since the anisotropic KPZ equation with thermal noise has been applied to the flux line dynamics [21], the AQKPZ equation considered in this Letter may also be relevant to the flux line depinning problem in disordered media. Further details will be published elsewhere.

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