

Reentrance of the Metallic Conductance in a Mesoscopic Proximity Superconductor

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We present an experimental study of the diffusive transport in a normal metal near a superconducting interface, showing the reentrance of the metallic conductance at very low temperature. This new mesoscopic regime comes in when the thermal coherence length of the electron pairs exceeds the sample size. The reentrance is suppressed by a bias voltage given by the Thouless energy and can be strongly enhanced by an Aharonov-Bohm flux. Experimental results are well described by the linearized quasiclassical theory. [S0031-9007(96)01817-0]

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During the past few years, the proximity effect between a superconductor (S) and a normal (N) metal has met a noticeable revival, thanks to spectacular progress in the fabrication of samples of mesoscopic size [1]. Experimental study of the transport near a S-N interface has shown that the proximity effect strongly affects electron transport in mesoscopic S-N systems: The deviation ΔG of the conductance from its normal-state value depends strongly on temperature T and oscillates in an applied magnetic field H if a N loop is present [2–4]. Various theoretical approaches were suggested to explain this behavior. A scattering matrix method based upon the Landauer formula [5] as well as a numerical solution of the Bogolubov–de Gennes equations [6] were used. These studies demonstrated that superconductivity does not affect the charge transfer in the N metal if the temperature T and the voltage V are zero, i.e., ΔG is zero at zero energy. A more powerful method based on the equations for the quasiclassical Green's functions [7–10] was used to obtain the dependence of ΔG on T and V . It has been established [9] that at $V = 0$ the deviation of the conductance ΔG increases from zero at $T = 0$ (if electron-electron interaction in N is negligible) with increasing T , reaches a maximum at approximately the Thouless temperature $\epsilon_c/k_B = \hbar D/k_B L^2$, and decreases to zero at $T \gg \epsilon_c/k_B$. This constitutes the reentrance effect for the metallic conductance of the N metal. Similar dependence of $\Delta G(V)$ at $T = 0$ has been found in [10] both in a numerical solution of the Bogoliubov–de Gennes equations and in an analytical solution of the equations for the quasiclassical Green's functions.

The physics behind this reentrance effect involves nonequilibrium effects between quasiparticles injected by the N reservoirs and electron pairs leaking from S. At the N-S interface, an incident electron is reflected into a hole of the same energy ϵ compared to the Fermi level E_F , but with a slight change in wave vector δk due to the branch crossing: $\delta k/k_F = \epsilon/E_F$, k_F being the Fermi

wave vector. The phase conjugation between the electron and the hole results in a finite pair amplitude involving states $(k_F + \epsilon/\hbar v_F, -k_F + \epsilon/\hbar v_F)$, v_F being the Fermi velocity. Such a pair maintains coherence in N up to the energy-dependent diffusion length $L_\epsilon = \sqrt{\hbar D/\epsilon}$ [7,11] which coincides with the well-known thermal length $L_T = \sqrt{\hbar D/k_B T}$ at $\epsilon = 2\pi k_B T$.

In the high-temperature regime $L_T < L$ or, equivalently, $\epsilon_c < k_B T$, it is well known that the proximity effect results in the subtraction of a length L_T of N metal from the resistance of a S-N junction. In the low temperature $L_T > L$ or $\epsilon_c > k_B T$ and low voltage $eV < \epsilon_c$, electron pairs are coherent over the whole sample. The proximity effect on the N metal resistance is still predicted to be zero. In this Letter, we report the experimental realization of both limits ($L < L_T$ and $L > L_T$) and the observation of the reentrance of the metallic conductance in a mesoscopic proximity superconductor. The low-temperature reentrant regime is destroyed by increasing the temperature [9] or the voltage [10]. As will be discussed below, an Aharonov-Bohm flux modifies the effective length of the sample and therefore shifts the energy crossover of the reentrant regime.

Figure 1 shows a micrograph of the sample made of a square copper (Cu) loop in contact with a single aluminum (Al) island. The loop, although not essential for the occurrence of the reentrance effect, allows one to control boundary conditions for the pair amplitude. The Cu wire width is 150 nm and its thickness is about 40 nm. The distance between the Cu loop and the Al island is about 100 nm, whereas the perimeter of the loop is $2 \mu\text{m}$. One should note that the sample geometry differs from all previous sample geometries with two superconducting contacts [2–4] in that there is a single superconducting phase and therefore no possible Josephson contribution. Two voltage probes measure the distribution at the outflows of the reservoirs, which are the wide contact pads at both ends of the Cu wire. The Cu surface is *in situ*

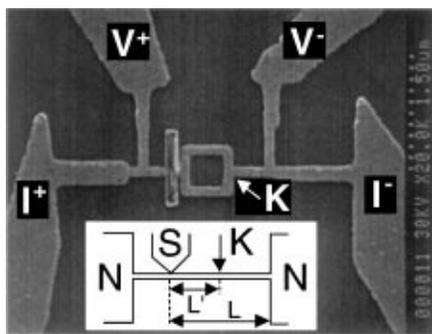


FIG. 1. Micrograph of the sample made of a continuous Cu (= N) loop in contact with a single Al (= S) island. Inset: simple model. At half-integer magnetic flux, a condition $F = 0$ is enforced at the point K at $L' = 1 \mu\text{m}$ from S. The distances between S and the right and left N reservoirs are, respectively, $L = 2 \mu\text{m}$ and $0.5 \mu\text{m}$

cleaned before Al deposition in order to ensure an optimum transparency of the Cu/Al interface [4].

We performed transport measurements in a μ -metal-shielded dilution refrigerator down to 20 mK. From the normal-state conductance $G_N = 0.0937 \text{ S}$ we find a diffusion coefficient $D = 70 \text{ cm}^2/\text{s}$, an elastic mean free path $l_e = 13 \text{ nm}$, and a thermal coherence length $L_T = 92 \text{ nm}/\sqrt{T}$. Aluminum islands become superconducting below $T_c \approx 1.4 \text{ K}$. The behavior of the conductance in the high-temperature regime $L_T < L$ (i.e., above 500 mK) is very similar to the two-island case [4]. At lower temperatures, so that $L_T \approx L$, and zero magnetic field, we observe a *decrease* of the low-voltage conductance [Fig. 2(inset)]. This occurs below 50 mK, at the temperature where a Josephson coupling would be expected in a two-island geometry. The voltage dependence of the measured conductance shows the most striking behavior, i.e., an *increase* of the conductance when the bias voltage is *increased* (Fig. 2). This non-linear behavior discards an interpretation in terms of weak localization, which is known to be insensitive to voltage. The conductance peak is observed at a bias voltage (about $1.7 \mu\text{V}$) of the order of the calculated Thouless energy $\epsilon_c = 1.1 \mu\text{V}$ related with a sample length $L = 2 \mu\text{m}$. In the Fig. 2 inset, the peak position is also consistent with the Thouless temperature $\epsilon_c/k_B = 13 \text{ mK}$. One can note that the discussed energies are much smaller than Al energy gap Δ .

Let us now analyze the effect of the magnetic field. Figure 3 shows oscillations of the magnetoconductance with a periodicity of one flux quantum $\Phi_0 = h/2e$ in the loop area. Here the reentrance effect can be seen very clearly at $\phi = \Phi_0/2, 3\Phi_0/2$, and at higher field. As previously observed in two-island samples, the oscillation amplitude decays slowly with a $1/T$ power law down to 200 mK [Fig. 3(inset)]. Figure 4 shows the temperature dependence of the conductance for various values of the magnetic flux in the loop. On this scale, the reentrance at

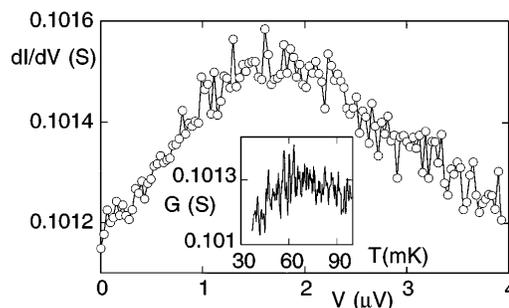


FIG. 2. Non-monotonous voltage dependence of the sample conductance at $T = 22 \text{ mK}$; the normal-state conductance G_N is 0.0937 S . Inset: temperature dependence. Measurement current of 70 nA in both curves.

zero field is hardly distinguishable [12]. At $\phi = \Phi_0/2$, the conductance maximum is near 500 mK and the reentrance has a much larger amplitude. At $\phi = \Phi_0$, the curve is close to the zero-field case, and, at $\phi = 3\Phi_0/2$, close to the $\Phi_0/2$ case. Increasing further the magnetic field [Fig. 4(left part)], the conductance peak is displaced to higher temperature and the Φ_0 periodic modulation is suppressed. Figure 5 shows the conductance peaks energies obtained from both the current-voltage characteristics and temperature dependence of the conductance as a function of the magnetic flux. Hence the magnetic field has two effects on the reentrance: (i) a large Φ_0 oscillation of the peak position at low field; (ii) a monotonous shift at higher fields.

Most of our observations can be analyzed in the framework of the quasiclassical theory for inhomogeneous superconductors [7,9–11]. We present here a simplified version that, however, keeps the essential physical features. We consider the mesoscopic regime where the inelastic scattering length is larger than the sample length L between the reservoirs. The flow of electrons at a particular energy ϵ is then uniform over the sample, and one has to consider transport through independent channels at the energy ϵ . In a perfect reservoir, electrons follow the Fermi

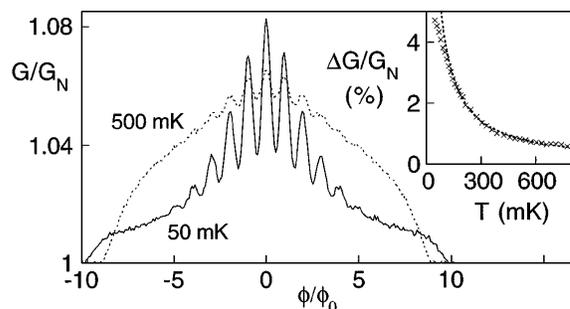


FIG. 3. Magnetoconductance oscillations showing $\Phi_0 = h/2e$ flux periodicity at $T = 50 \text{ mK}$ (solid line) and $T = 500 \text{ mK}$ (dashed line). Reentrance of the resistance is visible at half-integer magnetic flux and at high field. Inset: oscillations amplitude together with a $1/T$ power law fit.

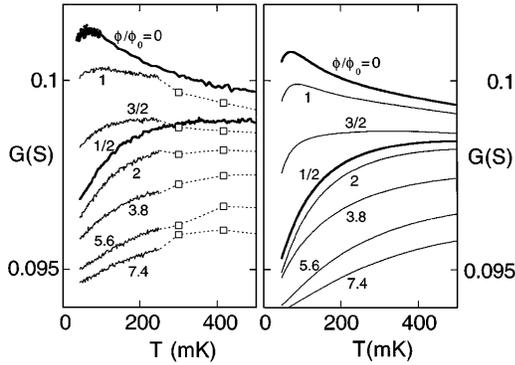


FIG. 4. Left: measured temperature dependence of the conductance at different values of the magnetic flux Φ in units of the flux quantum: $\Phi/\Phi_0 = 0; 1/2; 1; 3/2; 2; 3.8; 5.6; 7.4$. Measurement current is 200 nA. Right: calculated conductance in the linear approximation for the sample model of the Fig. 1 inset and at the same values of the flux. The only adjustable parameter is the effective width of the wires.

equilibrium distribution at temperature T and chemical potential μ . Charges are injected in the system from one reservoir at $\mu = eV$ and transferred to the other, so that the current is carried by electrons within an energy window $[0, \mu = eV]$ with a thermal broadening $k_B T$. Let us assume that the proximity effect can be accounted for as a conductivity enhancement $\delta\sigma(\epsilon, x)$ depending on both the energy ϵ and the distance x from the S interface. From the behavior of $\delta\sigma(\epsilon, x)$ it is then straightforward to calculate the excess conductance $\delta g(\epsilon)$ for the precise geometry of the sample. The excess conductance $\delta G(V, T)$ at voltage V and temperature T writes as

$$\delta G(V, T) = \int_{-\infty}^{\infty} \delta g(\epsilon) P(V - \epsilon) d\epsilon, \quad (1)$$

where $P(\epsilon) = [4k_B T \cosh^2(\epsilon/2k_B T)]^{-1}$ is a thermal kernel which reduces to the Dirac function at $T = 0$. Hence, the low-temperature differential conductance $dI/dV = G_N + \delta G$ probes the proximity-induced excess conductance $\delta g(\epsilon)$ at energy $\epsilon = eV$ with a thermal broadening

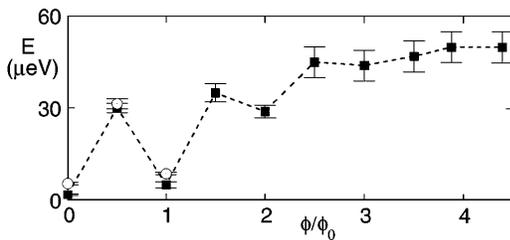


FIG. 5. Energy of the conductance maximum as a function of the magnetic flux ϕ in units of Φ_0 . The black square dots are obtained from the voltage-conductance $dI/dV(V)$ characteristics at $T = 100$ mK, except for $\Phi/\Phi_0 = 0$ and 1 ($T = 22$ and 45 mK). The white circles are obtained from Fig. 4. The discrepancies in between reflect the imperfections of the reservoirs at low energies.

$k_B T$. Independent measurements of the excess conductance as a function of T and V agree with Eq. (1) for not too low temperatures and voltages, provided the chemical potential in the reservoirs is taken as equal to the measured voltage times a geometrical factor of 1.05. This correspondence, however, fails for energies less than about $5 \mu\text{eV}$: we believe that, at these energies, the inelastic collision rate is too small to ensure a good thermalization in the reservoirs [13].

In N and at zero magnetic field, $F(\epsilon, x)$ follows the Usadel equation that for small F can be linearized as

$$\hbar D \partial_x^2 F + \left(2i\epsilon - \frac{\hbar D}{L_\varphi^2} \right) F = 0. \quad (2)$$

The linearization used above is a very crude approximation, since near the interface, which is believed to be clean, the pair amplitude should be large [in this approximation, we have $F(\epsilon \ll \Delta, 0) = -i\pi/2$ for a perfectly transparent interface [13]]. At the contact with a N reservoir, F is assumed to be zero. However, this simple formulation enables a straightforward understanding of the physical root of the conductance enhancement in a proximity system. Indeed, Eq. (2) features simply a diffusion equation for the pair amplitude F at the energy ϵ with a decay length L_ϵ and a cutoff at L_φ . At a particular energy ϵ , the real part of the pair amplitude is zero at the S interface, maximum at a distance L_ϵ if $L_\epsilon \ll L_\varphi, L$, and then decays in an oscillating way. The pair amplitude F is responsible for the local enhancement of the conductivity $\delta\sigma(\epsilon, x) = \sigma_N (\text{Re}[F(\epsilon, x)])^2$ for small F , σ_N being the normal-state conductivity [7,9–11]. It involves two contributions: a positive and dominant one which is similar to the Maki-Thompson fluctuation term in superconductors above T_c [14] and a negative one related to the decrease of the density of states. The two contributions cancel each other at zero energy [15].

We model the sample as two independent S-N circuits in series as shown in the inset of Fig. 1. This is our main approximation. It describes the main physics of our particular geometry and illustrates more general situations. Both circuits consist of a N wire between a superconductor S and a normal reservoir N. Along the wire, the excess conductivity $\delta\sigma(\epsilon, x)$ at a given energy ϵ increases from zero at the N-S interface to a maximum of about $0.3\sigma_N$ at a distance L_ϵ from the interface (if $L \ll L_\varphi$) and then decays exponentially with x . The integrated excess conductance $\delta g(\epsilon)$ of the whole sample rises from zero with an ϵ^2 law at low energy, reaches a maximum of $0.15G_N$ at about $5\epsilon_c$, and goes back to zero at higher energy with a $1/\sqrt{\epsilon}$ law. This behavior is indeed confirmed in the experimental results in Fig. 2. We observe a conductance peak as a function of both a temperature and voltage. The conductance is maximum for a temperature (50 mK) close to the calculated crossover temperature ($5\epsilon_c/k_B = 65$ mK), see the Fig. 2 inset. Only qualitative agreement between

the observed ($1.7 \mu\text{V}$) and calculated energy ($5\epsilon_c = 5.5 \mu\text{V}$) is obtained. This discrepancy is believed to be due to insufficient energy relaxation efficiency at low energy in the Cu reservoirs.

Because of the loop geometry, a magnetic field induces an Aharonov-Bohm flux, which changes the boundary conditions on the pair amplitude $F(\epsilon)$. At zero magnetic flux, $F(\epsilon)$ is zero only at contact with the normal reservoirs. At half-magnetic flux, destructive interference of the pair functions in the two branches enforces a zero in $F(\epsilon)$ at the node K [see Fig. 1(inset)]. Consequently, the pair amplitude is also zero between the loop and the N reservoir. Half a flux quantum then reduces the effective sample size to the length L' between the S interface and the point K. In the intermediate temperature regime $k_B T > \epsilon_c$ (or $L_T < L$), this modulates the conductance with a relative amplitude of the order of $\epsilon_c/k_B T$ [13], in qualitative agreement with the experiment.

As an additional effect of the magnetic field H , the phase-memory length L_φ is renormalized due to the finite width w of the Cu wire [16]:

$$L_\varphi^{-2}(H) = L_\varphi^{-2}(0) + \frac{\pi^2}{3} \frac{H^2 w^2}{\Phi_0^2}. \quad (3)$$

When smaller than the sample length L , the phase-memory length $L_\varphi(H)$ plays the role of an effective length for the sample. As a result, the conductance peak is shifted to higher temperatures and energies when the magnetic field increases, see Figs. 4 and 5. At high magnetic field, the position of the conductance maxima does not increase as rapidly as would be expected because of the field-induced depletion of the gap Δ . In the right part of Fig. 4, we show the calculated conductance using Eqs. (1)–(3) in the modeled geometry of the Fig. 1 inset in the case of a fully transparent interface. The only free parameter is the width of the wires which has been adjusted so that the experimental damping of the amplitude of the magnetoconductance oscillations by the magnetic field, see Fig. 3, is well described by the calculation. The discrepancy between the fitted value $w = 65 \text{ nm}$ and the measured value is attributed to deviations of sample geometry from our simple model. Our calculation accounts for both the global shape and amplitude of the curves, and for their behavior as a function of the magnetic flux. This is particularly remarkable with respect to the strong assumptions of the model. One should note that the qualitative shape and amplitude of the curves are conserved if nonlinearized Usadel equations or slightly different geometrical parameters are used.

In conclusion, we have measured the energy dependence of the proximity effect on the conductance near a N-S junction. As predicted in recent works [9,10], we have observed the reentrance of the metallic conductance

when all energies involved are below the Thouless energy of the sample. In contrast with a very recent similar observation [17], the energy crossover has been tracked as a function of temperature, voltage, and magnetic field. Our experimental results are well described by the linearized Usadel equations from the quasiclassical theory.

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- [1] See references in “Mesoscopic Superconductivity”, ed. by F. Hekking, G. Schön, and D. Averin, *Physica* (Amsterdam) **203B** (1994).
 - [2] V. T. Petrashov, V. N. Antonov, P. Delsing, and T. Claeson, *Phys. Rev. Lett.* **70**, 347 (1993); *Phys. Rev. Lett.* **74**, 5268 (1995).
 - [3] A. Dimoulas, J. P. Heida, B. J. van Wees, T. M. Klapwijk, W. v. d. Graaf, and G. Borghs, *Phys. Rev. Lett.* **74**, 602 (1995).
 - [4] H. Courtois, Ph. Gandit, D. Mailly, and B. Pannetier, *Phys. Rev. Lett.* **76**, 130 (1996).
 - [5] C. W. J. Beenakker, *Phys. Rev. B* **46**, 12 841 (1992).
 - [6] C. J. Lambert, *J. Phys. Condens. Matter* **3**, 6579 (1991).
 - [7] A. F. Volkov, A. V. Zaitsev, and T. M. Klapwijk, *Physica* (Amsterdam) **210C**, 21 (1993); A. F. Volkov and A. V. Zaitsev, *Phys. Rev. B* **53**, 9267 (1996); A. V. Zaitsev, *JETP Lett.* **51**, 35 (1990); *Phys. Lett. A* **194**, 315 (1994).
 - [8] A. A. Golubov, F. Wilhelm, and A. D. Zaikin (to be published).
 - [9] Y. V. Nazarov and T. H. Stoof, *Phys. Rev. Lett.* **76**, 823 (1996).
 - [10] A. F. Volkov, N. Allsopp, and C. J. Lambert, *J. Phys. Condens. Matter* **8**, L45 (1996).
 - [11] F. Zhou, B. Spivak, and A. Zyuzin, *Phys. Rev. B* **52**, 4467 (1995).
 - [12] We observed recently a reentrance effect with a much larger amplitude in smaller samples; see P. Charlat, H. Courtois, Ph. Gandit, D. Mailly, A. Volkov, and B. Pannetier, *Czech. J. Phys.* **46**, S6 (1996).
 - [13] P. Charlat, H. Courtois, Ph. Gandit, D. Mailly, and B. Pannetier (unpublished).
 - [14] B. R. Patton, *Proceedings of the 13th International Conference on Low Temp. Phys., 1972* edited by W. S. O’Sullivan (Plenum, New York, 1974).
 - [15] A. F. Volkov and V. V. Pavlovsky (to be published).
 - [16] B. Pannetier, J. Chaussy, and R. Rammal, *Phys. Scripta* **T13**, 245–251 (1986).
 - [17] S. G. den Hartog, C. M. A. Kapteyn, B. J. van Wees, and T. M. Klapwijk, *Phys. Rev. Lett.* **76**, 4592 (1996).