Symmetry Breaking and Spectral Statistics of Acoustic Resonances in Quartz Blocks

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We study experimentally the spectral statistics of acoustic resonances in quartz blocks. About 1400 well resolved resonances are measured. The short range fluctuations are well described by random matrix theory. The properties of quartz allow us to measure the gradual breaking of a point-group symmetry. This is statistically fully equivalent to the breaking of a symmetrylike parity or isospin in a quantum system. [S0031-9007(96)01858-3]

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The paramount importance of symmetries in quantum mechanics is made apparent by the fact that every eigenstate can be classified by a set of quantum numbers like angular momentum, parity, isospin, etc. A symmetry breaking is immediately reflected in the spectrum of eigenvalues, and its study can give rich information about the underlying interaction. The spectral fluctuation properties turn out to be particularly useful since the effect of the symmetry breaking is statistically enhanced. This was first observed in the context of parity breaking in nuclear physics [1], confirming very general considerations in random matrix theory [2]. Isospin in nuclear physics is much more strongly broken than parity. Mitchell *et al.* [3] measured and analyzed about 100 low lying states with known values of the isospin quantum number in the nucleus ^{26}Al. In Ref. [4] their results were interpreted in the framework of a full-fledged random matrix model. The relatively low number of levels, however, although truly impressive from a nuclear physics viewpoint, limits the statistical significance of this analysis. We present a measurement of about 1400 acoustic resonances in a quartz block with broken point-group symmetry. This study is statistically highly significant. Moreover, we measure the transition from fully conserved to strongly broken symmetry by externally tuning a parameter. It should be emphasized that our system, although physically very different from a quantum system, exhibits identical spectral fluctuation properties. Hence our quartz blocks are the ideal experimental system to study symmetry breaking.

Our investigation follows the concept of other demonstrative experiments. In particular, experiments with microwaves in metal cavities have been used to study level statistics and chaos in billiard systems [5–8]. Recently, using an experimental approach due to Weaver [9], we presented a measurement of the crossover from Poisson regularity to chaos in the spectral statistics of acoustic resonances in aluminum blocks [10]. This proved, for the first time, that random matrix theory is applicable to transitions of this type in systems for which the underlying wave equation is completely different from the Schrödinger equation. In this work we switch from aluminum to monocrystalline quartz, which can give an order of magnitude better resolution. We used rectangular blocks with dimensions $14 \times 25 \times 40$ mm^3.

In contrast to aluminum, quartz is anisotropic. Crystalline quartz exhibits D_3 point-group symmetry, which consists of a single threefold rotation symmetry about the crystal's Z ("optical") axis, and three twofold rotation symmetries about the crystal's three X ("piezoelectric") axes; the latter three axes lie in a plane orthogonal to the Z axis, subtending angles of 120 degrees with respect to one another [11]. The Z axis of the crystal lies parallel to the block's shortest edges, and one of the crystal's X axes lies parallel to the block's longest edges. Thus the only symmetry possessed by the quartz block is a twofold "flip" symmetry about this X axis.

The acoustic spectrum must therefore be a superposition of two independent, noninteracting spectra belonging to the two different representations of this flip symmetry. This coincides exactly with the situation in a quantum system when the superposition of two spectra is studied which belong to two different values of a quantum number, for example, isospin 0 and 1 as considered in Refs. [3,4]. The general theory for the superposition of noninteracting spectra is presented in Ref. [12], the formulas for our specific case can also be found in Ref. [4]. Note that our rectangular block effectively is skew because of the crystal properties. The geometry and the crystal structure determine the dynamics and thus the level statistics. After presenting our experimental data, we will discuss that our rectangular quartz block with conserved flip symmetry has much in common with a scalar pseudointegrable system [13].

We describe the experimental setup only briefly; a more detailed presentation is given in Ref. [14]. A Hewlett-Packard HP-3589A network analyzer provides a sine wave of a known frequency and voltage. A high-frequency power amplifier is used to raise the signal to approximately 200 V peak-to-peak in order to drive a piezoelectric transducer made of Ferroperm Pz34 (modified lead-titanate) with dimensions $5 \times 2.5 \times 0.43$ mm^3, which converts the alternating voltage into a mechanical vibration. This vibration is coupled to the quartz block via a sapphire stylus. The subsequent motion of the quartz block is measured by an identical transducer-stylus combination, which converts the displacement of the block at a point of its surface into an electrical signal. The outgoing signal is pre-amplified close to the receiving transducer, then further amplified and fed back into the network analyzer. We use the analyzer in swept-spectrum mode to measure the response of the block in transmission. The quartz block is supported by the two styli plus a third, passive stylus. Thus the acoustic coupling between the transducer and the quartz block is made through tiny point contacts to minimize the damping of the resonances. To reduce the acoustic coupling between the quartz block and the surrounding air, the pressure is held below 10^{-2} Torr. Any further reduction of the air pressure does not lower the damping significantly. Our block is thus as near as possible an elastomechanical system with free boundary conditions. The quality of the spectra is characterized by the ratio $Q = f/\Delta f$, where f is the frequency and Δf the width of a given resonance. A section of a typical spectrum is displayed in Fig. 1. The average Q was roughly 10^5, which is about ten times better than the experiments with aluminum blocks [9,10]. In billiard systems, this is second only to the experiments with superconducting microwave cavities, for which a better resolution was obtained [7].

To accumulate data, we found it useful to measure in the frequency range between 600 and 900 kHz in which about 1400 well resolved resonances were found. For the determination of the eigenmodes, three spectra were measured with different positions of the styli, so as to reduce the chance of missing a peak when a stylus is located close to a node. The extracted set of 1424 eigenfrequencies was analyzed as described in Ref. [10]. The cumulative eigenfrequency density shown in Fig. 1 behaves as a third-order polynomial since our system is three dimensional. For the unfolding procedure, we used a fit of a third order polynomial to the data. To the best of our knowledge, no Weyl-type formula exists for our system. Such a formula is extremely difficult to obtain since our system is, first, elastomechanical, second, anisotropic, and, third, since mode conversion takes place at the boundaries and corners.

In Fig. 2(a), the nearest neighbor spacing distribution is plotted. The theoretical predictions on the figure are, first,

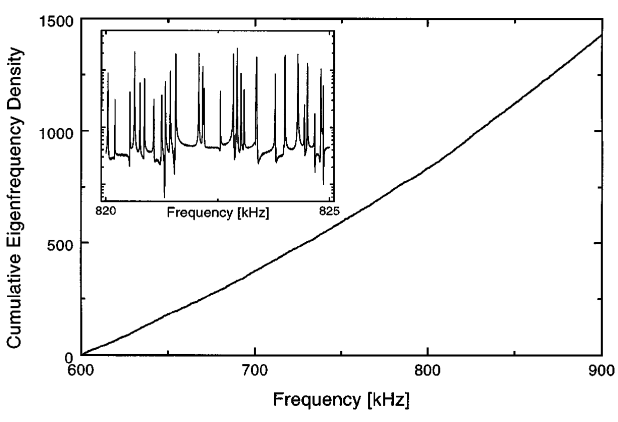

FIG. 1. Cumulative eigenfrequency density for the rectangular block. On this figure the curves corresponding to each of the other five systems measured would be indistinguishable from the curve shown. The inset displays on a linear-logarithmic plot the section of the spectrum between 820 and 825 kHz.

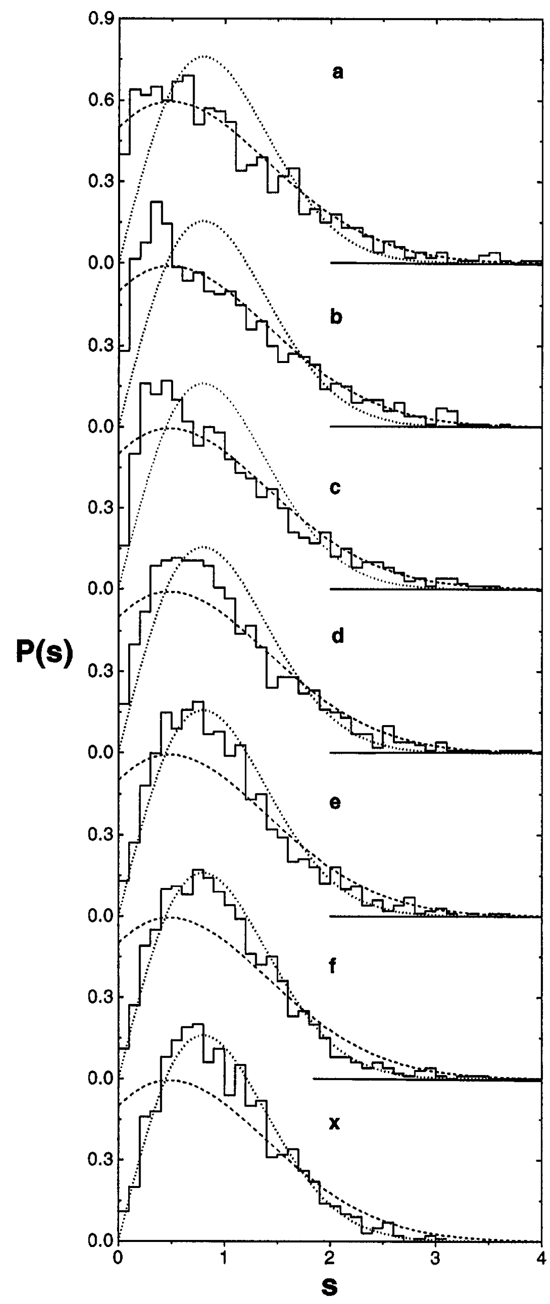

FIG. 2. The nearest neighbor spacing distributions $P(s)$ for the different radii r of the removed octant: (a) $r = 0$, the flip symmetry is fully conserved, (b) $r = 0.5$ mm, (c) $r = 0.8$ mm, (d) $r = 1.1$ mm, (e) $r = 1.4$ mm, (f) $r = 1.7$ mm, (x) the block with the huge defocusing structures, these data were derived from a spectrum ranging from 720 to 920 kHz. The dotted and the dashed curves are the theoretical predictions for a chaotic system containing no or one symmetry, respectively.

the Wigner distribution and, second, the nearest neighbor spacing distribution for two noninteracting spectra constructed from two Wigner distributions [4,12]. The data follow the latter distribution closely. Notice that the distribution differs from zero for zero spacing because the eigenmodes belonging to different representations of the flip symmetry do not repel each other. Since we expect an equal number of eigenmodes in each of the two noninteracting spectra, the numerical value of the spacing distribution at zero spacing should be $\frac{1}{2}$, which is in agreement with the experiment. The spectral rigidity displayed in Fig. 3(a) follows the prediction [4,12] for a chaotic system for smaller interval lengths but deviates for larger ones. We will come back to this point later.

In order to break the flip symmetry gradually, we used an ISEL Automation *XYZ* milling machine equipped with high-quality dental tools. An octant of a sphere with a successively larger radius was removed from one corner of the rectangular quartz block, thereby making a three dimensional Sinai billiard out of it. We performed measurements as described above for radii of 0.5, 0.8, 1.1, 1.4, and 1.7 mm. We found 1414, 1424, 1414, 1424, and 1419 eigenmodes, respectively. The resulting nearest neighbor spacing distributions and spectral rigidities are shown in Figs. 2 and 3 from (a) to (f).

The most spectacular result is the spacing distribution for the smallest radius of 0.5 mm in Fig. 2(b), in which the three following features are seen: the distribution dips at zero spacing, for slightly larger spacings it "overshoots" the prediction, and for larger spacings no change is seen. The breaking of the flip symmetry causes the eigenmodes belonging to the two different representations to interact. Thus the probability of finding degeneracies, i.e., the value of the spacing distribution at zero spacing, decreases sharply. However, since the symmetry breaking is only very weak, this "gap" in the distribution for very small spacings is immediately compensated through a considerably higher probability of finding slightly larger spacings. This "overshoot" can be viewed as the restoration of the normalization. The weakness of the symmetry breaking implies that larger spacings are not affected. This has never been observed experimentally before.

As the radius is made bigger, the gap and the overshoot broaden, and the spacing distribution makes a fast transition towards the distribution of one chaotic system without symmetries. The volume of the removed octant is 5×10^{-6} and 2×10^{-4} of the total volume for $r = 0.5$ and 1.7 mm, respectively. Because of the statistical enhancement effect [1,2], a very small symmetry breaking is sufficient to change the fluctuation properties considerably. Our experimental results confirm the numerical simulations of Ref. [4] very well. As can be seen from Fig. 3, the spectral rigidity, unlike the spacing distribution for zero spacing, changes continuously and smoothly for all interval lengths as the symmetry is broken. The spectral rigidity

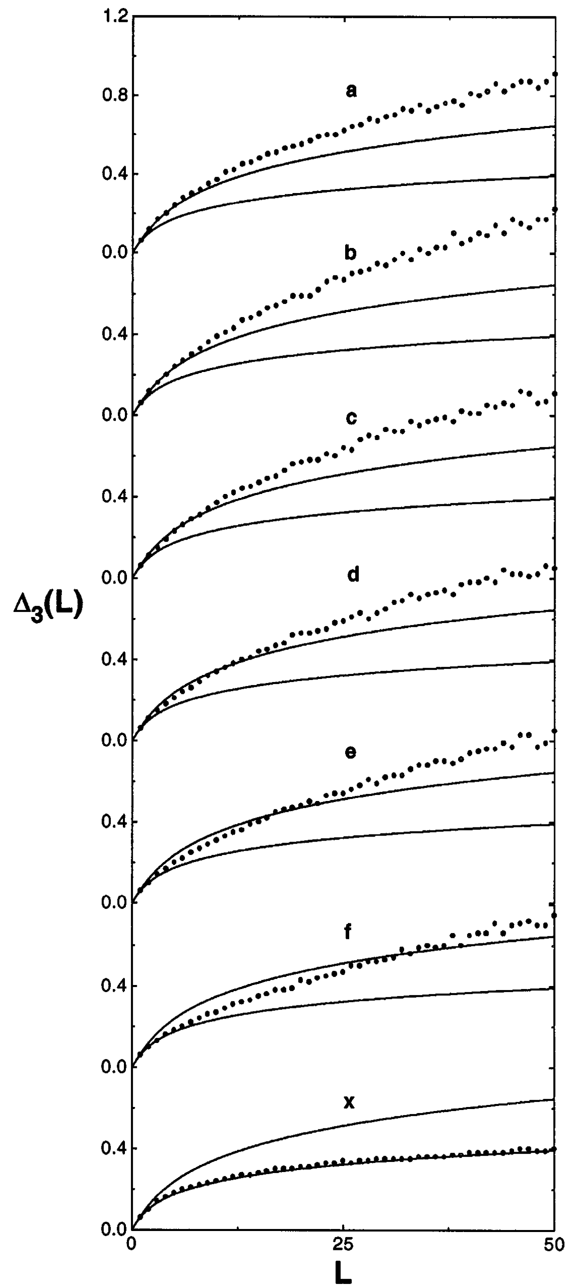

FIG. 3. The spectral rigidity $\Delta_3(L)$ for the different radii r of the removed octant, letters as in Fig. 2. The lower and the upper solid curves are the theoretical predictions for a chaotic system containing no or one symmetry, respectively.

is a smoothing double integral [2] over the spectral two-point correlation function, which does behave discontinuously [4]. Thus for the spectral rigidity, it takes larger radii to make the symmetry breaking visible than for the spacing distribution. However, for both observables, shorter scales are more strongly affected than larger ones. The parameter for the transitions of the spectral fluctuations on the unfolded energy scale is the root mean square strength of the symmetry breaking normalized to the mean level spacing. This parameter has a relatively weak frequency dependence [15] which was found to have a negligible effect in the frequency interval measured.

We will now comment on the striking rise of the spectral rigidity for larger interval lengths over the theoretical prediction [4,12] in the case of conserved flip symmetry, and on the continuation of this when the symmetry is broken. A subtraction of “bouncing ball” orbits as in Ref. [7] does not remove this effect. Thus our findings indicate that the block with conserved flip symmetry has much in common with a scalar pseudointegrable system [13]. Note that, first, mode mixing due to mode conversion is a strong effect [16] and, second, due to the crystal structure and the anisotropy, one cannot simply deduce the dynamics from the shape of the block alone. A further measurement supports these considerations. A sphere with a radius of 10 mm was removed from a quartz block which was originally identical to the one mentioned above. The center of this sphere lies inside the block, but close to one of the corners. The resulting geometry contains huge defocusing structures and thus should yield fully chaotic statistics. Indeed, this is confirmed by the experimental results displayed in Figs. 2(x) and 3(x). Hence the special properties of acoustic waves in an anisotropic crystal cannot cause deviations from chaotic behavior of the type discussed above. Therefore it is fair to say that we measured symmetry breaking in a system which is similar to a scalar pseudointegrable one. A study of this from the viewpoint of periodic orbit theory is in progress. Since on scales of only a few mean level spacings, a system with pseudointegrable features is known to behave like a chaotic one, our findings are on those scales indistinguishable from the symmetry breaking in a chaotic system.

In conclusion, we have presented a measurement of symmetry breaking in acoustic experiments. This is the first time that the whole transition has been studied by externally tuning the symmetry breaking. Moreover, this is also the first time that this transition has been measured in a three dimensional billiard geometry.

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