Quantum Reservoir Engineering with Laser Cooled Trapped Ions

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We show how to design different couplings between a single ion trapped in a harmonic potential and an environment. The coupling is due to the absorption of a laser photon and subsequent spontaneous emission. The variation of the laser frequencies and intensities allows one to "engineer" the coupling and select the master equation describing the motion of the ion. [S0031-9007(96)01762-0]

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According to quantum mechanics [1], a system can exist in a superposition of distinct states, whereas these superpositions seem not to appear in the macroscopic world. One possible explanation of this paradox [2] is based on the fact that systems are never completely isolated but interact with the surrounding environment, which contains a large number of degrees of freedom. The environment influences the system evolution which continuously decoheres and transforms system superpositions into statistical mixtures which behave classically [2,3]. This subject is directly related to the problem of measurement in quantum theory [4,5] where the system to be measured is described by quantum mechanics and the measurement apparatus is assumed to behave classically. Apart from this fundamental point of view a more practical aspect is the question to what extent one can preserve quantum superpositions, which is the basis of potential applications of quantum mechanics, such as quantum cryptography and computation [6,7].

In this Letter we will show how to "engineer" the system-environment coupling in a situation that is experimentally accessible with existing technology. The system of interest will be an ion confined in a electromagnetic trap, and the environment will be the vacuum modes of the electromagnetic field. This corresponds to an experimental realization of a harmonic oscillator coupled to a reservoir of oscillators. The coupling between our system and the environment takes place through the recoil experienced by the ion when it interchanges photons with the electromagnetic field. As we will show below, this coupling can be manipulated by laser radiation. Variations of the laser frequency and intensity allow one to engineer such a coupling.

Laser cooled trapped ions [8] are a unique experimental system: unwanted dissipation can be made negligible for very long times, much longer than typical times in which an experiment takes place. Furthermore, arbitrary quantum states of the ion's motion can be synthesized and coherently manipulated using laser radiation [9]. In addition, the state of motion can be completely determined in the sense of tomographic measurements [10]. In a series of remarkable experiments, Wineland and collaborators have generated a variety of nonclassical states of ion motion [11,12]. In particular, they have been able to produce [12] a so-called "Schrödinger cat state" [13] corresponding to

$$|\Psi\rangle \propto |\alpha\rangle + |-\alpha\rangle, \quad (1)$$

with $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\alpha^n/\sqrt{n!}\,|n\rangle$ a coherent (quasiclassical) state. In Fig. 1(a) we have plotted the density operator for such a state in the position representation, i.e. (the real part of) $\rho(x,x') = \langle x|\rho|x'\rangle$. The peaks near the diagonal correspond to two possible localizations of the particle, whereas the other two peaks are related to the coherences that are responsible from the quantum behavior [2].

The interaction of a Schrödinger cat state with the environment has been the paradigm of decoherence of superposition states. As first argued by Zurek [2] (see also Refs. [2,3,14–16]), for a coupling which is linear in the system coordinates, a macroscopic superposition of the form (1) decays to a statistical mixture $\rho \propto |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$, on a short time scale (decoherence time) which is related to the size of the cat ($|\alpha|^2$) and is much faster than the energy dissipation time: this provides an explanation for the absence of superpositions in the macroscopic world [2]. We emphasize that the decoherence process of (1) is sensitive to the form of the reservoir coupling. For some quadratic couplings, for example, the decoherence and energy dissipation time can become identical [15]; moreover, there exist interactions which allow Schrödinger cat states to be stable, and, what is more surprising, dissipation can drive a system into a steady state of the form (1) [15]. For example, in Figs. 1(b) and 1(c) the decay of a Schrödinger cat under linear and quadratic coupling is illustrated: for a linear coupling [Fig. 1(b)] the nondiagonal peaks (coherences) of the density matrix decay much faster than for the quadratic couplings [Fig. 1(c). We will show that all these theoretical predictions can be tested experimentally for the case of a trapped ion.

The process of decoherence can be analyzed in detail under very general assumptions invoking the so-called Markov approximation, which considers the correlation time for the environment to be much shorter than the evolution time of the system due to the coupling [17]. In this case the interaction of a system with an environment is described in terms of a master equation. For a single

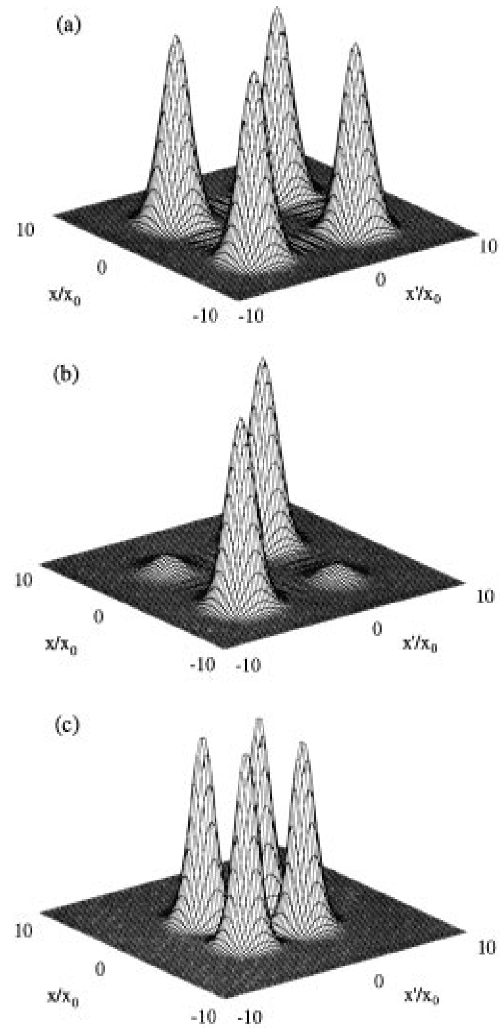

FIG. 1. (a) $\rho(x, x')$ for the state (1) ($\alpha = 3$). (b),(c) Numerical simulation of the interaction with a laser for a time $\tau = 0.06\gamma^{-1}$ and $\eta = 0.03$: (b) $\omega_L = \omega_0 - \nu$, ($f = a$); (c) $\omega_L = \omega_0 - 2\nu$, ($f = a^2$).

decay channel this equation has the form ($\hbar = 1$)

$$\dot{\rho} = \gamma(2f\rho f^\dagger - f^\dagger f\rho - \rho f^\dagger f). \tag{2}$$

Here ρ is the reduced density operator for the system in the interaction picture after tracing over the reservoir. The operator f and the parameter γ reflect the system-environment coupling. For a harmonic oscillator f will be a function of the creation and annihilation operators a and $a^\dagger$, which are defined as usual $X = 1/(2M\nu)^{1/2}(a^\dagger + a)$, $P = i(M\nu/2)^{1/2}(a^\dagger - a)$, where X and P are the position and momentum operators and M the particle's mass. According to Zurek [2], the coupling with the environment singles out in a quantum system a preferred set of states, sometimes called "the pointer basis." This basis depends on the form of the coupling f. For example, for $f = X$ the pointer basis is the position eigenstates. The density operator describing the system evolves in such a way that it rapidly becomes diagonal in this preferred basis, which is usually connected to the disappearance of quantum interferences. Our goal is now to find an experimental realization of the master equation (2) for different system-reservoir couplings $f \equiv f(a, a^\dagger)$.

Let us consider a single ion moving in a one-dimensional harmonic potential. The ion interacts with a laser in a standing wave configuration of frequency ω_L, close to the transition frequency ω_0 of two internal levels $|g\rangle$ and $|e\rangle$. Using standard methods in quantum optics based on the the dipole, Born-Markov, and rotating wave approximations, the master equation that describes this situation can be written in the general form

$$\dot{\rho} = -iH_{\rm eff}\rho + i\rho H_{\rm eff}^\dagger + J\rho, \tag{3}$$

where $H_{\rm eff} = \nu a^\dagger a + \frac{1}{2}\omega_0\sigma_z + H_{\rm cou} - i\frac{\Gamma}{2}|e\rangle\langle e|$, with

$$H_{\rm cou} = \frac{\Omega}{2}\sin[\eta(a + a^\dagger) + \phi] \times (\sigma_+ e^{-i\omega_L t} + \sigma_- e^{i\omega_L t}), \tag{4}$$

and

$$J\rho = \Gamma\int_{-1}^{1} du\, N(u) e^{-i\eta u(a+a^\dagger)} \times \sigma_-\rho\sigma_+ e^{i\eta u(a+a^\dagger)}. \tag{5}$$

Here, $\sigma_+ = |e\rangle\langle g| = (\sigma_-)^\dagger$ and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ are the usual spin-$\frac{1}{2}$ operators describing the internal transition, ν is the trap frequency, Γ the spontaneous emission rate, $\eta = (k_L^2/2M\nu)^{(1/2)}$ the Lamb-Dicke parameter, and $N(u)$ the normalized dipole pattern. In the Hamiltonian describing the coupling with the lasers, Ω is the Rabi frequency, and ϕ characterizes the relative position of the trap center with respect to the node of the laser standing wave. Here we will assume that either $\phi = 0$ (excitation at the node of the standing wave) or $\phi = \pi/2$ (excitation at the antinode).

We will proceed now by simplifying the master equation for the ion in a regime defined by three limits which are typically fulfilled in experiments [11,12]: (i) Lamb-Dicke, (ii) strong confinement, and (iii) low intensity. The first one allows us to expand the above master equation in terms of the Lamb-Dicke parameter $\eta \ll 1$, retaining only the orders that contribute to the dynamics. The second one assumes $\Gamma \ll \nu$ and together with the third one allows us to include in the coupling Hamiltonian only on-resonance terms (secular approximation). Finally, the third one assumes a sufficiently low laser intensity (the specific form of this limit will be given later), and will serve us to eliminate the internal excited level $|e\rangle$.

Let us start by simplifying the coupling Hamiltonian under the above limits. To do that, we move to a rotating frame defined by the unitary operator $\mathcal{U} = e^{-i(\nu a^\dagger a + \frac{1}{2}\omega_0\sigma_z)t}$. Following Ref. [9] we assume the following: (i) For excitation at the node ($\phi = 0$), $\delta = \omega_L -$

$\omega_0 = (2k + 1)\nu$ ($k = 0, \pm 1, \ldots$). (ii) For excitation at the antinode ($\phi = \pi/2$), $\delta = 2k\nu$ ($k = 0, \pm 1, \ldots$). In this rotating frame, after performing the rotating wave approximation and the Lamb-Dicke expansion, we obtain $H_{\rm cou} = \frac{\Omega'}{2}(\sigma_+ f + f^\dagger \sigma_-)$, where both Ω' and the form of the operator f depend on the frequency of the laser. For example, for $\delta = -\nu$, we have $f = a$, and $\Omega' = \Omega\eta$, whereas for $\delta = -2\nu$, $f = a^2$, and $\Omega' = -\Omega\eta^2/2$. Apart from the strong confinement, in the first case, the secular approximation can be performed for $\Omega'/2 \ll \nu$, whereas in the second case it is needed $\Omega^2/8\nu \ll \Omega'$. These two conditions can always be fulfilled for low enough laser intensity, and together with $\Omega' \ll \Gamma$ define the low intensity limit.

In the next step we eliminate the internal excited state using standard procedures of quantum optics [17]. The resulting master equation is of the form (3), with $H_{\rm eff} = -i\gamma f^\dagger f$, and

$$J\rho = 2\gamma \int_{-1}^{1} du\, N(u) e^{-i\eta u(a+a^\dagger)} \times f\rho f^\dagger e^{i\eta u(a+a^\dagger)}, \qquad (6)$$

where $\gamma = \Omega'^2/2\Gamma$. Finally, expanding in powers of η we find the desired master equation (2), with corrections of the order η^2. The master equation will be valid for times such that these corrections are not important, that is, for times $t \ll (\gamma\eta^2\bar{n})^{-1}$, where $\bar{n}$ is the typical phonon number of the state of the ion. Nevertheless in the Lamb-Dicke limit this time can be much longer than the time required to reach the steady state using the approximated master equation. Note that the most important relaxation parameter is the effective rate γ. For attainable experimental parameters [$\Gamma = 40$ kHz (see Ref. [18]), $\nu = 30$ MHz, $\eta = 0.15$, $\Omega = 1$ MHz [12]] we have that for $f = a^2$ all the conditions are fulfilled and $\gamma = 2$ KHz. Obviously, for $f = a$ higher values of γ can be more easily reached.

According to our analysis, by varying the laser frequency we obtain the master equation (2) with different coupling operators f. In Fig. 2 we have illustrated the laser configurations which produce several f operators. In Fig. 2(a), for example, the laser is tuned to the so-called "lower motional sideband," $\delta = -\nu$, and the ion is located at the node of the standing wave field which leads to a coupling operator $f = a$. This can be easily understood by noting that in each absorption and spontaneous emission cycle one phonon is annihilated on a time scale given by the optical pumping time. Mathematically, this phonon annihilation is represented by the action of the operator $f = a$, which therefore defines the coupling. Similarly, in Fig. 2(b) the laser is tuned to the "second lower sideband" $\delta = -2\nu$ at the antinode of the laser standing wave which gives the two-phonon coupling $f = a^2$. These two cases of linear and quadratic coupling correspond to the two examples discussed in Figs. 1(b) and 1(c). In fact, these figures were obtained by a numerical solution of the full master equation (3) with quantum Monte Carlo wave function simulations [19]. As noted before, the decoherence acts in a different way depending on the coupling operator, according to our previous discussion.

It is simple to generalize the above derivation to find situations with other interesting (and perhaps unusual) coupling operators f. For example, consider the case in which two lasers of frequency $\omega_0 + \nu$ and $\omega_0 - \nu$ interact with the ion [Fig. 2(c)]. This corresponds to a coherent excitation of the lower and upper motional sidebands [9]. In this case, following the same arguments, one can easily show that the operator is $f = \mu a + \nu a^\dagger$,

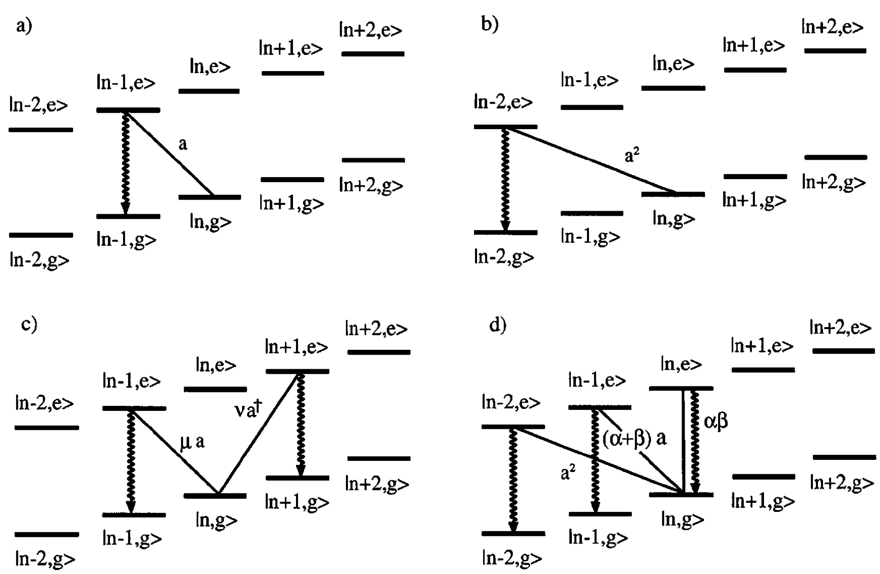

FIG. 2. Laser configurations for several coupling operators f. (a) Laser tuned to $|n, g\rangle \rightarrow |n-1, e\rangle$, which rapidly decays into the state $|n-1, g\rangle$ leading to $f = a$; (b) $f = a^2$; (c) $f = \mu a + \nu a^\dagger$; (d) $f = (a - \alpha)(a - \beta)$.

where $\mu^2 - \nu^2 = 1$ and μ/ν is the quotient of the Rabi frequencies. This operator corresponds to a squeezed vacuum coupling which has been the basis for numerous theoretical predictions in quantum optics [17]. In particular, choosing equal Rabi frequencies, the coupling is $f = a + a^\dagger \propto X$. This corresponds to the case analyzed theoretically by Unruh and Zurek, Caldeira and Leggett, and other authors [2,3] to describe the decoherence process in terms of the projection of the state of the system onto the pointer basis given, in this case, by the position eigenstates. Another interesting combination of lasers [Fig. 2(d)] yields $f = (a - \alpha)(a - \beta)$, where α and β are given complex numbers. For $\alpha = -\beta$ the Schrödinger cat state (1) is an eigenstate of this operator with zero eigenvalue, and thus this state does not decohere under this form of coupling. We note that, as pointed out by de Matos Filho and Vogel [20], one can employ this particular form of system–reservoir coupling to generate a cat state (1) by choosing as the initial state the ground level $|0\rangle$. Tuning a laser on resonance at the antinode of a standing light wave one can design the coupling in the form of a quantum nondemolition measurement of the phonon number, $f = a^\dagger a$, with the Fock states as the pointer basis. Using more complicated laser configurations involving tuning the laser to upper motional sidebands (cf. $\delta = -n_0\nu$) one can readily show that other f operators containing higher powers of a and $a^\dagger$ can be engineered. If the maximum power of f in a and $a^\dagger$ is n_0, then Ω' scales as $\Omega\eta^{n_0}$. Thus, in order to satisfy the low intensity limit, smaller Rabi frequencies are required (i.e., the evolution will become slow). From the practical point of view, this can be a serious restriction for large n_0, since other technical sources of noise can become important.

Obviously, there are numerous possibilities to generalize the concept of reservoir engineering in ion traps. First of all, decoherence of a two- or three-mode system can be studied by considering the two- or three-dimensional motion of a trapped ion, respectively. Furthermore, a master equation with more than one decoherence channel, i.e., an equation containing sums of damping terms of the form (2) with different operators f_i $(i = 1, \ldots, N)$ [17], can also be easily implemented. This can be accomplished by exciting transitions with several incoherent lasers. Another important generalization concerns the possibility of coupling a two-level system to a harmonic oscillator (Jaynes-Cummings model) which in turn is coupled to an environment. In particular, this will allow us to test experimentally one of the outstanding predictions of quantum optics [21], namely, the damping of a two-level system interacting with a squeezed reservoir. Finally, these ideas can be extended to linear ion traps [22] in order to study collective effects in an N-atom+ harmonic oscillator system.

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