

Four-Dimensional Greybody Factors and the Effective String

Steven S. Gubser* and Igor R. Klebanov†

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544

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Recently Maldacena and Strominger found that the calculation of greybody factors for $D = 5$ black holes carrying three U(1) charges gives striking new evidence for their description as multiply wound effective strings. Here we show that a similar result holds for $D = 4$ black holes with four U(1) charges. In this case the effective string may be thought of as the triple intersection of the 5-branes in M theory compactified on T^7 . [S0031-9007(96)01587-6]

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Recently remarkable progress has been achieved in modeling the Hawking radiation within the string theory or M -theory context. There exist supersymmetric $D = 5$ black holes with a nonvanishing horizon area which may be embedded into string theory using intersecting D -branes [1]. Their low-energy dynamics is described by small fluctuations of a long intersection string [2,3]. In [2] it was shown how Hawking emission from the near-extremal black holes takes place in the stringy description. The model involves n_1 1-branes marginally bound to n_5 5-branes, with some longitudinal momentum along the 1-branes carried by left-moving open strings. In the nonextremal case, right movers are also present, so that a left-moving and a right-moving open string may collide to produce an outgoing closed string [2,4]. This mechanism leads to a thermal distribution for the massless outgoing particles [2], as expected of Hawking radiation. The inverse of this process, which gives the leading order contribution to the absorption of closed strings, was also found to be in agreement with the semiclassical gravity, up to an overall normalization [5]. More recently, Das and Mathur [6] carefully normalized the leading emission and absorption rates, both in semiclassical gravity and in the D -brane picture, and found perfect agreement! The specific picture used in [6] follows that suggested in [3,7]: The low-energy dynamics of the D -brane configuration is captured by a single string with winding number $n_1 n_5$ which is free to vibrate only within the 5-brane hyperplane.

The calculation of Das and Mathur was carried out in the near-extremal regime

$$r_0 \ll r_n \ll r_1, r_5, \quad (1)$$

where r_0 is the radius of the horizon, and r_n, r_1, r_5 are three other radii determined by the charges. Very recently, Maldacena and Strominger [8] generalized the calculation to a less restrictive choice,

$$r_0, r_n \ll r_1, r_5. \quad (2)$$

The large difference in the scales is necessary to suppress the antibranes [9]. Dropping the restriction $r_0 \ll r_n$ allows the left- and right-moving temperatures of the effective string to be comparable. The calculation of greybody factors in this regime reveals a dependence on

T_L and T_R which provides striking new evidence in favor of the effective string model of $D = 5$ black holes.

The purpose of this Letter is to present similar evidence for supersymmetric $D = 4$ black holes with regular horizons [10,11]. Such black holes may be embedded into string theory (for a treatment of the nonextremal case, see [12]). In [13–15] it was argued, however, that it is advantageous to view these $D = 4$ black holes as dimensionally reduced configurations of intersecting branes in M theory. A specific configuration useful for explaining the Bekenstein-Hawking entropy is the $5 \perp 5 \perp 5$ intersection [14]: There are n_1 5-branes in the (12345) hyperplane, n_2 5-branes in the (12367) hyperplane, and n_3 5-branes in the (14567) hyperplane. One also introduces a left-moving momentum along the intersection string (in the $\hat{1}$ direction). If the length of this direction is L_1 , then the momentum is quantized as $2\pi n_K/L_1$, so that n_K plays the role of the fourth U(1) charge. Upon compactification on T^7 the metric of the $5 \perp 5 \perp 5$ configuration reduces to that of the $D = 4$ black hole with four charges. Just like in the D -brane description of the $D = 5$ black hole, the low-energy excitations are signals propagating along the intersection string. In M theory the relevant states are likely to be small 2-branes with three holes glued into the three different hyperplanes [14]. As a result, the effective length of the intersection string is $L_{\text{eff}} = n_1 n_2 n_3 L_1$. This fact, together with the assumption that these modes carry central charge $c = 6$, is enough to reproduce the extremal Bekenstein-Hawking entropy $S = 2\pi \sqrt{n_1 n_2 n_3 n_K}$ [14]. In our previous paper [16] we showed that this “multiply-wound string” model of the four-charge $D = 4$ black hole correctly reproduces the Hawking radiation of both neutral and Kaluza-Klein charged scalars. The calculation of [16] was performed in the near-extremal regime $r_0 \ll r_4 \ll r_1, r_2, r_3$, where r_0 is the horizon radius while r_1, \dots, r_4 are four other radii related to the charges. Here we generalize it to a less restrictive choice of parameters

$$r_0, r_4 \ll r_1, r_2, r_3. \quad (3)$$

As in the work of [8], this relaxes the condition $T_R \ll T_L$. The dependence of the greybody factors on T_L and T_R is characteristic of the effective string model.

Semiclassical gravity analysis.—We start with the 11-dimensional configuration of three sets of 5-branes intersecting along a common 1-brane and carrying momentum along it. The nonextremal metric of this configuration was constructed in [17]. (Our notation differs from that in [17] by the replacements $\mathcal{P}_i \rightarrow r_i$, $\tilde{Q} \rightarrow r_4$, and $\mu \rightarrow r_0$.) Compactifying this metric on T^6 we arrive at the string in $D = 5$ which has a constant dilaton and the following metric:

$$ds_{(5)}^2 = (f_1 f_2 f_3)^{-1/3} \times \left[-f_4^{-1} h dt^2 + f_4 \left(dy - \frac{Q}{r+r_4} dt \right)^2 \right] + (f_1 f_2 f_3)^{2/3} (h^{-1} dr^2 + r^2 d\Omega^2), \quad (4)$$

where

$$f_i = 1 + \frac{r_i}{r}, \quad h = 1 - \frac{r_0}{r}. \quad (5)$$

It is useful to define a hyperbolic angle σ such that

$$r_4 = r_0 \sinh^2 \sigma, \quad Q = r_0 \sinh \sigma \cosh \sigma. \quad (6)$$

The quantities r_4 and Q are comparable to the horizon radius r_0 . Therefore σ is of order 1. Dimensional reduction on S^1 from $D = 5$ to $D = 4$ gives a black hole with four U(1) charges described by the following metric and Kaluza-Klein gauge potential A_0 :

$$ds_{(4)}^2 = -f^{-1/2} h dt^2 + f^{1/2} (h^{-1} dr^2 + r^2 d\Omega^2),$$

$$f = \prod_{i=1}^4 \left(1 + \frac{r_i}{r} \right), \quad (7)$$

$$A_0 = Q / (r_4 + r).$$

For the region of parameters (3), the Bekenstein-Hawking entropy of this black hole is [17,18]

$$S_{\text{BH}} = \frac{2\pi}{\kappa_4^2} A_h = \frac{8\pi}{\kappa_4^2} \sqrt{r_1 r_2 r_3 r_0} \cosh \sigma. \quad (8)$$

Calculation of the Hawking temperature gives [17,18]

$$\frac{1}{T_H} = 4\pi \sqrt{\frac{r_1 r_2 r_3}{r_0}} \cosh \sigma. \quad (9)$$

We may write $\frac{2}{T_H} = \frac{1}{T_L} + \frac{1}{T_R}$, where

$$\frac{1}{T_L} = 4\pi \sqrt{\frac{r_1 r_2 r_3}{r_0}} e^{-\sigma}, \quad \frac{1}{T_R} = 4\pi \sqrt{\frac{r_1 r_2 r_3}{r_0}} e^{\sigma}. \quad (10)$$

In the effective string model T_L and T_R have the meaning of left- and right-moving temperatures. Indeed it may be shown that the entropy (8) equals $\pi L_{\text{eff}}(T_L + T_R)$, which is the entropy of a string of length L_{eff} , carrying massless modes of central charge $c = 6$.

We are interested in studying the propagation of scalars carrying Kaluza-Klein charge in the background of this charged black hole. This is best viewed as propagation of scalars in the background of the $D = 5$ string, with the role of the charge being played by the momentum along

the string k_4 . Thus we substitute the ansatz $\phi(t, y, r) = e^{-i\omega t} e^{-ik_4 y} R(r)$ into the $D = 5$ scalar equation

$$\frac{1}{\sqrt{-g^{(5)}}} \partial_M \left(\sqrt{-g^{(5)}} g^{MN} \partial_N \phi \right) = 0. \quad (11)$$

The resulting radial equation is

$$\prod_{i=1}^3 \left(1 + \frac{r_i}{r} \right) \left[\omega^2 - k_4^2 + (\omega \sinh \sigma - k_4 \cosh \sigma)^2 \frac{r_0}{r} \right] R + \frac{h}{r^2} \frac{d}{dr} h r^2 \frac{dR}{dr} = 0. \quad (12)$$

This is very similar to the radial equation found for the $D = 5$ black hole [8]. It is again possible to define new variables

$$\omega'^2 = \omega^2 - k_4^2, \quad e^{\pm\sigma'} = e^{\pm\sigma} \frac{(\omega \mp k_4)}{\omega'}, \quad (13)$$

such that (12) becomes

$$\omega'^2 \left(1 + \frac{r_0 \sinh^2 \sigma'}{r} \right) \prod_{i=1}^3 \left(1 + \frac{r_i}{r} \right) R + \frac{h}{r^2} \frac{d}{dr} h r^2 \frac{dR}{dr} = 0, \quad (14)$$

which describes propagation of a neutral particle of energy ω' near a black hole with the hyperbolic angle parameter σ replaced by σ' . The absorption cross section may be calculated using the matching method in a manner similar to [8].

Rather than run through the beautiful matching calculation of [8] step by step, we will give only a summary designed to provide some continuity with the analysis of previous papers [5,6,16]. We will restrict our attention to a neutral scalar of energy ω propagating in the black hole background described by (7), that is to say by unprimed variables. At the end we will recover the general charged case using the primed variables defined in (13).

A variety of radial coordinates are useful in different contexts. The ones we will employ are related as follows:

$$z = 1 - \frac{r_0}{r} = e^{-r_0/u}. \quad (15)$$

This equation can be taken as the definition of z and u . The coordinate u is useful because it brings Eq. (14) into the simple form

$$\left(\frac{1}{u^2} \frac{d}{du} u^2 \frac{d}{du} + \frac{r^4}{u^4} f \omega'^2 \right) R = 0, \quad (16)$$

and because the radial flux per unit solid angle is just

$$\mathcal{F} = \frac{1}{2i} \left(R^* u^2 \frac{d}{du} R - \text{c.c.} \right). \quad (17)$$

The matching calculations that produced the results of [5,6,16] amount essentially to the following: We simplify (16) in the regions near the horizon (I) and far from the black hole (III) by keeping only the leading term in a small u or large u expansion of the term $f r^4 / u^4$. Modulo

some technical assumptions, the near and far solutions can then be matched directly onto one another. In the present instance, one obtains

$$R_I = Ae^{i\sqrt{P}/u}, \quad \text{where } P = \omega^2 \prod_{i=1}^4 (r_i + r_0), \quad (18)$$

$$R_{III} = \alpha \frac{\sin \omega u}{\omega u} - \beta \frac{\cos \omega u}{\omega u}, \quad (19)$$

and these can be matched by making expansions of R_I and R_{III} for large and small u , respectively. The S -matrix element S_0 for reflection of the s wave can be read off from comparison of (19) to the standard form

$$R \sim \frac{S_0 e^{i\omega u} - e^{-i\omega u}}{\omega u}, \quad (20)$$

but a simpler method employed by [8] is to note that the absorption probability is the ratio of two fluxes

$$1 - |S_0|^2 = \frac{\mathcal{F}_{\text{absorbed}}}{\mathcal{F}_{\text{incoming}}}. \quad (21)$$

$\mathcal{F}_{\text{incoming}}$ is computed by applying (17) to the incoming wave $(\alpha + i\beta)ie^{-i\omega u}/(2\omega u)$ at infinity. We consider only perturbative scattering processes where $|\beta| \ll |\alpha|$, so the leading order result depends only on α . Similarly, $\mathcal{F}_{\text{absorbed}}$ is computed by applying (17) to $R_I = Ae^{i\sqrt{P}/u}$ at the horizon. The result for the absorption probability is

$$1 - |S_0|^2 = 4\omega\sqrt{P} \frac{|A|^2}{|\alpha|^2}. \quad (22)$$

The optical theorem converts this probability into a cross section

$$\sigma_{\text{abs}} = \frac{\pi}{\omega^2} (1 - |S_0|^2) = A_h |E|^{-2}, \quad (23)$$

where we have defined the greybody factor $E = \alpha/A$.

The simplicity of the ratio of fluxes method outlined above is that E can be obtained by matching the limiting value of R_I at large u with the limiting value of R_{III} at small u . Of course, one loses all phase information for the reflected wave in this approach, and one must still check that a full matching is possible.

Matching (18) and (19) in the manner described yields $E = 1$ and hence $\sigma_{\text{abs}} = A_h$. This most naive matching scheme fails when one gives up the condition $r_0 \ll r_n$ because there are neglected terms in the near region equation which are of order r_0/r_n . Following the approach of [8], we retain all such terms and exclude only terms of order r_0/r_i for $i = 1, 2, 3$. The effect on the infalling solution R_I is to introduce a modulating factor $F(u)$: Instead of (18) we now have

$$R_I = Ae^{i\sqrt{P}/u} F(u). \quad (24)$$

The boundary conditions $F(u) = 1$ and $u^2 F'(u) = 0$ are imposed at $u = 0$, so $\mathcal{F}_{\text{absorbed}}$ is the same as one would calculate from the bare exponential (18). E is read off as the limiting value of $F(u)$ for large u . Starting from (16) and neglecting r_0/r_i for $i = 1, 2, 3$, we find that $F(u)$

satisfies a hypergeometric equation in the variable z :

$$\left[z(1-z) \frac{d^2}{dz^2} + (1-z)(1-i[a+b]) \frac{d}{dz} + ab \right] F = 0, \quad (25)$$

where the parameters a and b are given by

$$a = \frac{\omega}{4\pi T_R}, \quad b = \frac{\omega}{4\pi T_L}, \quad (26)$$

and the temperatures T_R and T_L are given by (10). Interestingly, (25) and (26) are identical to the inner region equations for the $D = 5$ black hole [8], although z , T_L , and T_R are defined differently.

Once it is established that $F(z)$ is a hypergeometric function, E can be read off directly from the asymptotics of $F(z)$ near $z = 1$ [8]:

$$E = \frac{\Gamma(1-ia-ib)}{\Gamma(1-ia)\Gamma(1-ib)}. \quad (27)$$

The formula for the absorption cross section then becomes

$$\sigma_{\text{abs}} = A_h \frac{\omega}{2(T_L + T_R)} \frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\omega}{2T_L}} - 1)(e^{\frac{\omega}{2T_R}} - 1)}. \quad (28)$$

The cross section for the five-dimensional case can also be written in precisely this form. As $\omega \rightarrow 0$, $\sigma_{\text{abs}} \rightarrow A_h$ in accord with the general result of [19]. Equation (28) contains even more universal information: It captures the behavior of the cross section as ω is increased to values that are comparable with the temperatures.

Restoring primes, we obtain for the charged case

$$\sigma_{\text{abs}} = 4\pi^2 r_1 r_2 r_3 \omega' \frac{e^{\frac{\omega'}{T_H}} - 1}{(e^{\frac{\omega'}{2T_L'}} - 1)(e^{\frac{\omega'}{2T_R'}} - 1)}. \quad (29)$$

To write this expression in terms of physical quantities we use the formulae [8]

$$\begin{aligned} \frac{\omega'}{T_L'} &= \frac{\omega + k_4}{T_L}, & \frac{\omega'}{T_R'} &= \frac{\omega - k_4}{T_R}, \\ \frac{\omega'}{T_H'} &= \frac{\omega - \phi k_4}{T_H}, \end{aligned} \quad (30)$$

where $\phi = A_0(r_0) = \tanh \sigma$ is the $U(1)$ potential on the horizon. Thus for a particle of energy ω and charge k_4 , the absorption cross section is

$$\sigma_{\text{abs}} = 4\pi^2 r_1 r_2 r_3 \sqrt{\omega^2 - k_4^2} \frac{e^{\frac{\omega - \phi k_4}{T_H}} - 1}{(e^{\frac{\omega + k_4}{2T_L}} - 1)(e^{\frac{\omega - k_4}{2T_R}} - 1)}. \quad (31)$$

From this we may obtain the differential Hawking emission rate

$$\begin{aligned} \Gamma(\vec{k}) \frac{d^3 k}{(2\pi)^3} &= \frac{\sqrt{\omega^2 - k_4^2}}{\omega} \sigma_{\text{abs}} \frac{1}{e^{\frac{\omega - \phi k_4}{T_H}} - 1} \frac{d^3 k}{(2\pi)^3} \\ &= 4\pi^2 r_1 r_2 r_3 \frac{\omega^2 - k_4^2}{\omega} \\ &\quad \times \frac{1}{(e^{\frac{\omega + k_4}{2T_L}} - 1)(e^{\frac{\omega - k_4}{2T_R}} - 1)} \frac{d^3 k}{(2\pi)^3}. \end{aligned} \quad (32)$$

Remarkably, this formula is in precise agreement with the effective string model. Indeed, in [16] it was shown that this model predicts the leading order Hawking rate

$$\Gamma(\vec{k}) \frac{d^3k}{(2\pi)^3} = \frac{\kappa_4^2 L_{\text{eff}}}{4} \frac{\omega^2 - k_4^2}{\omega} \times \frac{1}{(e^{\frac{\omega+k_4}{2r_L}} - 1)(e^{\frac{\omega-k_4}{2r_R}} - 1)} \frac{d^3k}{(2\pi)^3}, \quad (33)$$

where $L_{\text{eff}} = n_1 n_2 n_3 L_1$ and L_1 is the length of the circle around which the string is wound. This rate is due to a left-moving boson and a right-moving boson on the string producing an outgoing scalar, hence the presence of the two Bose-Einstein distribution factors. The string effective Lagrangian contains no cubic term coupling a left-moving fermion and a right-moving fermion to a scalar in the bulk; therefore, there is no additive contribution containing two Fermi-Dirac distributions.

The radii r_i (approximately equal to the charges Q_i) are related to the numbers of 5-branes [14]:

$$r_1 = \frac{n_1}{L_6 L_7} \left(\frac{\kappa_{11}}{4\pi} \right)^{2/3}, \quad r_2 = \frac{n_1}{L_4 L_5} \left(\frac{\kappa_{11}}{4\pi} \right)^{2/3}, \\ r_3 = \frac{n_1}{L_2 L_3} \left(\frac{\kappa_{11}}{4\pi} \right)^{2/3}, \quad (34)$$

where L_i is the range of the coordinate y_i . Thus we have

$$\frac{\kappa_4^2 L_{\text{eff}}}{4} = 4\pi^2 r_1 r_2 r_3, \quad (35)$$

where we have used $\kappa_4^2 = \kappa_{11}^2 / \prod_{i=1}^7 L_i$. Equation (35) establishes exact equality between (33), the Hawking rate for charged particles calculated in the effective string model, and (32), the corresponding rate calculated in semiclassical gravity.

In closing we would like to reflect on the significance of our result. Because the M theory description of four-dimensional black holes is not as developed as the D -brane description of five-dimensional ones, it seems to us very encouraging to find that the greybody factors in the four-dimensional case confirm the underlying effective string description which was previously used in [14,16,18]. We regard (32) as “new data” from semiclassical relativity, obtained in the region (3) of parameter space, that supports the claim that black holes in four dimensions admit an effective string description.

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*Electronic address: ssgubser@puhep1.princeton.edu

†Electronic address: klebanov@puhep1.princeton.edu

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