Angular Momentum of Supersymmetric Cold Rydberg Atoms

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Semiunitary transformation is applied to discuss supersymmetrization of cold Rydberg atoms. In the limit of vanishing kinetic energy the lowest angular momentum of the supersymmetric cold Rydberg atom is $3\hbar/2$. A possible experimental verification is suggested. [S0031-9007(96)00109-3]

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Recently Baxter [1] showed that cold Rydberg atoms can play an interesting role of realizable analogs of Chern-Simons theory [2,3]. By choosing an atomic dipole of a cold Rydberg atom in electric and magnetic field, and by an appropriate experimental arrangement the motion of the dipole is constrained to be planar and rotationally symmetric, the Röntgen interaction takes on the form of a Chern-Simons term [3]. By placing the dipole in a strong magnetic field and in an appropriate optical trapping field, the elimination of the kinetic energy term in the Lagrangian could be achieved physically and Baxter [1] showed that the canonical angular momentum spectrum changes from one consisting of integers to one consisting of positive half integers; thus in principle an experimental verification of the Chern-Simons feature of fractional angular momentum is allowed.

Reference [4] showed evidence for a phenomenological supersymmetry in atomic physics. In SS QM the term "supersymmetry" has nothing to do with spins at all, its meaning is just as that SS QM is represented by a pair of bosonic Hamiltonians H_- and H_+ which are superpartners of the SS Hamiltonian $H_s = H_- \oplus H_+$, and SS charge Q_s, $Q_s^\dagger$, and H_s satisfy SS algebra [5].

In SS QM the form of the problem must be truly one dimensional. For the one-dimensional system there is no room to define the angular momentum; one cannot discuss the relation of spectra of angular momenta between the system and its superpartner. For the case of two-dimensional Rydberg atoms angular momentum can be defined as an exterior product. This system can be reduced into two uncoupled one-dimensional systems, thus the supersymmetrization of its Hamiltonian and angular momentum can be simultaneously performed by a semiunitary transformation (SUT) which will be defined later. This for the first time allows in principle a verification of the relation of spectra of angular momentum between a system and its superpartner. The results show that in the limit of vanishing kinetic energy the angular momentum spectrum of the SS cold Rydberg atom changes from one consisting of integers to one consisting of positive half integers, with the lowest angular momentum $3\hbar/2$; SUT destroys the lowest angular momentum state with $\hbar/2$ of the cold Rydberg atom.

Cold Rydberg atom.—The Lagrangian of the cold Rydberg atom with an atomic dipole d in electric and magnetic field is [1]

$$L = \tfrac{1}{2}M\dot{R}_i\dot{R}_i + \tfrac{1}{2}g\epsilon_{ij}R_i\dot{R}_j - \tfrac{1}{2}KR_iR_i\,, \qquad (1)$$

where M is the mass of the dipole, the electric field $\vec{E}$ acts radially in the x-y plane, $E_i \sim -R_i$ $(i = 1, 2)$, and the constant magnetic field $\vec{B}$ aligns the z axis. The constant parameters g and K are proportional to the magnitude of the dipole moment and, respectively, magnetic and electric field dependent. $\epsilon_{12} = -\epsilon_{21} = 1$; $\epsilon_{11} = \epsilon_{22} = 0$. In (1) the second term $\frac{1}{2}g\epsilon_{ij}R_i\dot{R}_j$ takes on the form of the Chern-Simons interaction. The Hamiltonian obtained from (1) is

$$H = \frac{1}{2M}\left(P_i + \frac{g}{2}\epsilon_{ij}R_j\right)^2 + \frac{K}{2}R_iR_i$$
$$= \frac{1}{2M}P_iP_i + \frac{1}{2M}g\epsilon_{ij}P_iR_j + \frac{1}{2}M\Omega^2R_iR_i\,, \qquad (2)$$

where the canonical momentum $P_i = M\dot{R}_i - \frac{1}{2}g\epsilon_{ij}R_j$, and the frequency

$$\Omega = (g^2/4M^2 + K/M)^{1/2}. \qquad (3)$$

In (3) the dispersive "mass" term $g/2M$ comes from the presence of the Chern-Simons term. By changing the variables R_i, P_i to

$$X_a = (M\Omega/2\omega_a)^{1/2}R_1 - (1/2M\Omega\omega_a)^{1/2}P_2,$$
$$X_b = (M\Omega/2\omega_b)^{1/2}R_1 + (1/2M\Omega\omega_b)^{1/2}P_2\,, \qquad (4a)$$
$$P_a = (\omega_a/2M\Omega)^{1/2}P_1 + (M\Omega\omega_a/2)^{1/2}R_2,$$
$$P_b = (\omega_b/2M\Omega)^{1/2}P_1 - (M\Omega\omega_b/2)^{1/2}R_2\,, \qquad (4b)$$

where

$$\omega_a = \Omega + g/2M, \quad \omega_b = \Omega - g/2M\,, \qquad (5)$$

the Hamiltonian is rewritten in the form of two uncoupled harmonic oscillators of unit mass and of frequencies ω_a and ω_b. Define

$$A_\alpha = (\hbar/2\omega_\alpha)^{1/2}d/dX_\alpha + (\omega_\alpha/2\hbar)^{1/2}X_\alpha,$$
$$A_\alpha^\dagger = -(\hbar/2\omega_\alpha)^{1/2}d/dX_\alpha + (\omega_\alpha/2\hbar)^{1/2}X_\alpha$$
$$(\alpha = a, b)\,; \qquad (6)$$

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the Hamiltonian (2) becomes

$$H = H_a + H_b, \quad H_\alpha = \hbar\omega_\alpha(A_\alpha^\dagger A_\alpha + \tfrac{1}{2}) \quad (\alpha = a, b). \tag{7}$$

If $|n_\alpha\rangle$ is an eigenfunction of H_α with eigenvalue $E_{n_\alpha} = \hbar\omega_\alpha(n_\alpha + 1/2)$ $(\alpha = a, b)$, $|n_a, n_b\rangle \equiv |n_a\rangle|n_b\rangle$ is an eigenfunction of H with eigenvalue

$$E_{n_a,n_b} = \hbar\omega_a(n_a + \tfrac{1}{2}) + \hbar\omega_b(n_b + \tfrac{1}{2}), \quad (n_a, n_b = 0, 1, 2, \ldots). \tag{8}$$

In two dimensions the canonical angular momentum is defined as an exterior product which is a scalar

$$J = \epsilon_{ij} R_i P_j = J_a - J_b, \quad J_\alpha = \hbar(A_\alpha^\dagger A_\alpha + \tfrac{1}{2}) \quad (\alpha = a, b). \tag{9}$$

Because $[H, J] = 0$, J is a constant of motion, and J and H have common eigenfunctions $|n_a, n_b\rangle$. Notice that J_a and J_b have zero-point angular momentum $\hbar/2$. Because of the cancellation of $\hbar/2$ between modes a and b, the eigenvalues of J are integer multiples of $\hbar$,

$$j_{n_a,n_b} = \hbar(n_a - n_b), \quad (n_a, n_b = 0, 1, 2, \ldots). \tag{10}$$

SS QM.—In one-dimensional SS QM a quantum system is described by a pair of related bosonic Hamiltonians H_- and H_+ [5]:

$$H_\pm = -\frac{\hbar^2}{2}\frac{d^2}{dx^2} + V_\pm(x). \tag{11}$$

Suppose the ground-state wave function $\psi_0^{(-)}(x) \equiv \psi_0(x)$ of H_- is known, consider only unbroken supersymmetry corresponding to a normalizable ground state ψ_0 and adjust the ground-state energy $E_0^{(-)} = 0$. Introducing the superpotential $W(x) = -(\hbar/\sqrt{2})\psi_0'/\psi_0$ (a prime denotes d/dx), we have $V_\pm(x) = W^2(x) \pm (1/\sqrt{2})W'(x)$. The Hamiltonians $H_\pm$ can be factorized as

$$H_- = A^\dagger A, \quad H_+ = AA^\dagger, \tag{12}$$

where

$$A = \frac{\hbar}{\sqrt{2}}\frac{d}{dx} + W(x), \quad A^\dagger = -\frac{\hbar}{\sqrt{2}}\frac{d}{dx} + W(x),$$

$$[A, A^\dagger] = \sqrt{2}\hbar W'(x). \tag{13}$$

The eigenfunctions $\psi_n^{(-)}$ and $\psi_n^{(+)}$ of H_- and H_+, respectively, with eigenvalues $E_n^{(-)}$ and $E_n^{(+)}$ are related by

$$E_n^{(+)} = E_{n+1}^{(-)}, \tag{14a}$$

$$\psi_n^{(+)} = [E_{n+1}^{(-)}]^{-1/2} A \psi_{n+1}^{(-)} \quad (n = 0, 1, 2, \ldots). \tag{14b}$$

We find that the superpartner Hamiltonians H_- and H_+ can be related by a SUT [6]. Consider the operator $AA^\dagger$. Because $AA^\dagger\psi_n^{(+)} = E_n^{(+)}\psi_n^{(+)}$, $E_n^{(+)} > 0$ $(n = 0, 1, 2, \ldots)$, $AA^\dagger$ is positive definite, we can define

$$U = (AA^\dagger)^{-1/2} A, \quad U^\dagger = A^\dagger(AA^\dagger)^{-1/2}. \tag{15}$$

U and $U^\dagger$ satisfy

$$UU^\dagger = I, \quad U^\dagger U = A^\dagger(AA^\dagger)^{-1}A \equiv Q. \tag{16}$$

Notice that $A^+A\psi_n^{(-)} = E_n^{(-)}\psi_n^{(-)}$, $E_0^{(-)} = 0$, $E_n^{(-)} > 0$ for $n \geq 1$, so $A^\dagger A$ is semipositive definite and $(A^\dagger A)^{-1}$ is singular. Thus we cannot use the operator identities $f(AA^\dagger)A = Af(A^\dagger A)$ and $A^\dagger f(AA^\dagger) = f(A^\dagger A)A^\dagger$; the operator Q defined in (16) is not a unit operator. The operator U defined in (15) which satisfies (16) is called semiunitary [7]. Under this SUT H_- is transformed into H_+,

$$H_+ = UH_-U^\dagger. \tag{17}$$

Correspondingly, $\psi_n^{(-)}$ is transformed into $U\psi_n^{(-)}$. Notice that $U\psi_0 = 0$, $U\psi_{n+1}^{(-)} = [E_{n+1}^{(-)}]^{-1/2}A\psi_{n+1}^{(-)} = \psi_n^{(+)}$, with eigenvalues $E_n^{(+)} = E_{n+1}^{(-)}$ $(n = 0, 1, 2, \ldots)$. The ground energy $E_0^{(+)} = E_1^{(-)} > 0$; here $E_0^{(-)} = 0$ disappears in the spectrum of H_+. Thus the SUT transforms the $(-)$ system into its superpartner system $(+)$, which fully covers the results of SS QM.

The operator Q defined in (16) satisfies

$$Q^2 = Q, \quad [Q, H_-] = 0. \tag{18}$$

Q is a projection operator, and its eigenvalues q $(q = 0, 1)$ are good quantum numbers. Q and H_- have common eigenfunctions. Because $Q\psi_0 = 0$, $Q\psi_{n+1}^{(-)} = \psi_{n+1}^{(-)}$, we denote $|\psi_0\rangle = |0, 0\rangle$, $|\psi_{n+1}^{(-)}\rangle = |n + 1, 1\rangle$ $(n = 0, 1, 2, \ldots)$, which are common eigenstates of H_- and Q. The Hilbert space $\mathcal{H}$ is divided into two subspaces $\mathcal{H}_0$ and $\mathcal{H}_1$, consisting of eigenstates $|0, 0\rangle$ and $|n + 1, 1\rangle$ with, respectively, eigenvalues $q = 0$ and 1, $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1$. In the basis $|0, 0\rangle$ and $|n + 1, 1\rangle$ $Q = I - |0, 0\rangle\langle 0, 0|$, thus in the $\mathcal{H}_1$ subspace the operator Q reduces to a unit operator and the SUT reduces to a unitary transformation.

Now we compare a unitary transformation and a SUT. In quantum mechanics a unitary operator maintains all the physical properties of a quantum system. The situation is different for SUT. The main reason is that SUT is a singular operator which readily appears in the structure of the projector Q defined in (16). Because of such singularities, SUT only partly maintains physical properties of the $(-)$ system. For example, it maintains the complete relation of $\psi_n^{(-)}$, but does not maintain the orthogonality and normalization of $\psi_n^{(-)}$, and only partly maintains the eigenvalues of H_-. However, in the subspace $\mathcal{H}_1$ the SUT behaves just like a unitary transformation. SUT transforms the $(-)$ system defined in the full Hilbert space $\mathcal{H}$ into the $(+)$ system defined in the subspace $\mathcal{H}_1$, destroys the ground state of H_-, but maintains all other physical properties of the $(-)$ system in the subspace $\mathcal{H}_1$. This explains the legitimacy of the applications of SUT to SS QM.

Supersymmetrization of cold Rydberg atoms.—Using SUT we can thoroughly study the supersymmetrization of cold Rydberg atoms. The Hamiltonian (7) has two modes a and b. We define the potential $V_-(X) = \sum_{\alpha=a,b}(\frac{1}{2}\omega_\alpha^2 X_\alpha^2 - \frac{1}{2}\hbar\omega_\alpha)$, with the Hamiltonian H_- and

the angular momentum J_- represented by A_α and $A_\alpha^\dagger$ defined in (6),

$$H_- = H_{-a} + H_{-b}, \quad H_{-a} = \hbar\omega_a A_a^\dagger A_a \otimes I_b,$$
$$H_{-b} = \hbar\omega_b I_a \otimes A_b^\dagger A_b\,, \tag{19}$$

$$J_- = J_{-a} - J_{-b}, \quad J_{-a} = \hbar(A_a^\dagger A_a + \tfrac{1}{2}) \otimes I_b,$$
$$J_{-b} = \hbar I_a \otimes (A_b^\dagger A_b + \tfrac{1}{2})\,, \tag{20}$$

where I_a and I_b are unit operators in the Hilbert spaces $\mathcal{H}_a$ and $\mathcal{H}_b$, respectively, corresponding to modes a and b. The common eigenstates of H_- and J_- are $|n_a, n_b\rangle_- = |n_a\rangle\,|n_b\rangle$ with, respectively, eigenvalues

$$E^{(-)}_{n_a,n_b} = \hbar\omega_a n_a + \hbar\omega_b n_b,$$
$$j^{(-)}_{n_a,n_b} = \hbar(n_a + \tfrac{1}{2}) - \hbar(n_b + \tfrac{1}{2})$$
$$(n_a, n_b = 0, 1, 2, \ldots)\,. \tag{21}$$

SUT is defined as

$$U = U_a \otimes U_b, \quad U^\dagger = U_a^\dagger \otimes U_b^\dagger,$$
$$U_\alpha = (A_\alpha A_\alpha^\dagger)^{-1/2} A_\alpha, \quad U_\alpha^\dagger = A_\alpha^\dagger (A_\alpha A_\alpha^\dagger)^{-1/2}$$
$$(\alpha = a, b)\,, \tag{22}$$

with

$$UU^\dagger = I, \quad U^\dagger U = Q\,, \tag{23}$$

where

$$Q = Q_a \otimes Q_b, \quad Q_\alpha = A_\alpha^\dagger (A_\alpha A_\alpha^\dagger)^{-1} A_\alpha \neq I_\alpha$$
$$(\alpha = a, b)\,. \tag{24}$$

Under SUT (22), H_- and J_- are transformed into

$$H_+ = UH_-U^\dagger = H_{+a} + H_{+b}, \quad H_{+a} = \hbar\omega_a A_a A_a^\dagger \otimes I_b,$$
$$H_{+b} = \hbar\omega_b I_a \otimes A_b A_b^\dagger\,, \tag{25}$$

$$J_+ = UJ_-U^\dagger = J_{+a} - J_{+b},$$
$$J_{+a} = \hbar(A_a A_a^\dagger \otimes I_b + \tfrac{1}{2} I_a \otimes I_b)\,,$$
$$J_{+b} = \hbar(I_a \otimes A_b A_b^\dagger + \tfrac{1}{2} I_a \otimes I_b)\,, \tag{26}$$

Correspondingly, under SUT (22) $|n_a, n_b\rangle_-$ is transformed into $U|n_a, n_b\rangle_-$. Notice that $U|0,0\rangle_- = 0$, $U|0, n_b\rangle_- = 0$, and $U|n_a, 0\rangle_- = 0$ which shows that the states $|0,0\rangle_-$, $|0, n_b\rangle_-$, and $|n_a, 0\rangle_-$ are destroyed by SUT. Thus $|n_a, n_b\rangle_+ = U|n_a + 1, n_b + 1\rangle_- = [E^{(-)}_{n_a+1} E^{(-)}_{n_b+1}]^{-1/2} A_a|n_a + 1\rangle_- \otimes A_b|n_b + 1\rangle_-$ $(n_a, n_b = 0, 1, 2, \ldots)$. The common eigenstates of H_+ and J_+ are $|n_a, n_b\rangle_+ = U|n_a + 1, n_b + 1\rangle_-$ with, respectively, eigenvalues

$$E^{(+)}_{n_a,n_b} = \hbar\omega_a(n_a + 1) + \hbar\omega_b(n_b + 1)\,,$$
$$j^{(+)}_{n_a,n_b} = \hbar(n_a + \tfrac{3}{2}) - \hbar(n_b + \tfrac{3}{2})$$
$$(n_a, n_b = 0, 1, 2, \ldots)\,. \tag{27}$$

For the $(+)$ system the ground state $|0,0\rangle_+$ has energy $E^{(+)}_{0,0} = \hbar(\omega_a + \omega_b) = 2\hbar\Omega$. Here the states $|0,0\rangle_-$, $|0, n_b\rangle_-$, and $|n_a, 0\rangle_-$ of H_- with energy $E^{(-)}_{0,0} = 0$, $E^{(-)}_{0,n_b} = \hbar\omega_b n_b$, and $E^{(-)}_{n_a,0} = \hbar\omega_a n_a$ disappear.

Because of the cancellation of the angular momenta between modes a and b the angular momenta J_- and J_+ have the same spectrum $j^{(-)}_{n_a,n_b} = \hbar(n_a - n_b) = j^{(+)}_{n_a,n_b} =$ any integer multiple of $\hbar$. The $(+)$ system still has the same lowest eigenvalue as the $(-)$ system $j^{(+)}_{0,0} = j^{(-)}_{0,0} = 0$.

Now we consider the interesting limit case $M \to 0$ discussed by Baxter [1]. In this case there are constraints which should be carefully considered. The first equation of (2) shows that the limit case $M \to 0$ requires the constraints

$$C_i \equiv P_i + g\epsilon_{ij} R_j/2 = 0\,. \tag{28}$$

We observe that the Poisson brackets $\{C_i, C_j\} = g\epsilon_{ij} \neq 0$ [8], so that the Dirac brackets can be determined [9]

$$\{R_1, P_1\}_D = \{R_2, P_2\}_D = \tfrac{1}{2}\,,$$
$$\{R_1, R_2\}_D = -1/g\,, \quad \{P_1, P_2\}_D = -g/4\,. \tag{29}$$

Other Dirac brackets of R_i and P_i are zero. The Dirac brackets of C_i with any variables R_i and P_i are zero; thus (28) are strong conditions in the sense of Dirac which can be used to eliminate the dependent variables. Choosing the independent variables (R_1, P_1), (28) fixes the dependent variables $R_2 = -2P_1/g$, $P_2 = gR_1/2$. In the reduced phase space of the independent variables (R_1, P_1) the Hamiltonian (2) has the structure of a one-dimensional harmonic oscillator

$$H = \frac{2K}{g^2} P_1^2 + \frac{1}{2} K R_1^2\,. \tag{30}$$

According to (29) the quantization condition of the independent variables is $[R_1, P_1] = i\hbar/2$. Set $R_1 = q/\sqrt{2}, P_1 = p/\sqrt{2}$ which leads to $[q, p] = i\hbar$. Introduce $V_-(q) = \frac{1}{2} m^* \omega^2 q^2 - \frac{1}{2}\hbar\omega$ where the effective mass m^* and the frequency ω are

$$m^* = g^2/2K, \quad \omega = K/g\,. \tag{31}$$

Define

$$A = (\hbar/2m^*\omega)^{1/2} d/dq + (m^*\omega/2\hbar)^{1/2} q,$$
$$A^\dagger = -(\hbar/2m^*\omega)^{1/2} d/dq + (m^*\omega/2\hbar)^{1/2} q\,; \tag{32}$$

we obtain from (30)

$$H_- = \frac{1}{2m^*}[p^2 + (m^{*2}\omega^2 q^2 - m^*\hbar\omega)] = \hbar\omega A^\dagger A \tag{33}$$

with eigenvalues

$$E_n^{(-)} = n\hbar\omega \ (n = 0, 1, 2, \ldots)\,. \tag{34}$$

Observe that if we rewrite (30) as $H = (P_1^2 + m_1^2\omega_1^2 R_1^2)/2m_1$ where the effective mass $m_1 = g^2/4K$ and the frequency $\omega_1 = 2K/g$, we find that the frequency ω of (31) differs from that of the conventional harmonic oscillator by a factor of $1/2$. This is the representation of the well-known fact that reduction to the reduced phase space alters the symplectic structure [9].

In the limit $M \to 0$ the angular momentum is

$$J_- = gR_iR_i/2 = \hbar(A^\dagger A + \tfrac{1}{2}). \quad (35)$$

The spectrum of J_- is a positive half-integer multiple of $\hbar$,

$$j_n^{(-)} = \hbar(n + \tfrac{1}{2}), \quad (n = 0, 1, 2, \ldots). \quad (36)$$

Comparing (33), (35) with (19), (20), we see that in the limit $M \to 0$ when the mode b disappears, only the mode a is maintained [1]. Using A and $A^\dagger$ defined in (32) we construct $U = (AA^\dagger)^{-1/2}A$ which is a SUT. With such a SUT, H_- in (33) and J_- in (35) are simultaneously transformed into their superpartners

$$H_+ = UH_-U^\dagger = \hbar\omega AA^\dagger,$$
$$J_+ = UJ_-U^\dagger = \hbar(AA^\dagger + \tfrac{1}{2}), \quad (37)$$

with, respectively, spectra

$$E_n^{(+)} = \hbar\omega(n + 1),$$
$$j_n^{(+)} = \hbar(n + 3/2)\ (n = 0, 1, 2, \ldots). \quad (38)$$

Comparing with the spectrum of J_-, the spectrum of J_+ is also a positive half integer of $\hbar$, but starting from $3\hbar/2$, the lowest angular momentum $\hbar/2$ of J_- is destroyed by SUT.

A possible experimental verification of the lowest angular momentum of SS cold Rydberg atoms is allowed in principle. In the present case the difference between V_+ and V_- is a constant, $V_+ - V_- = 2\hbar(g/4M^2 + K/M)^{1/2}$. Taking the limit of vanishing kinetic energy is achieved as follows [1]. If the magnetic field is strong enough, the second term in (2) is dominant. Further, in an appropriate optical trapping field the speed of the atom can be slowed to the extent that the kinetic energy term in (2) may be removed [10], which leads to the limit $M \to 0$ [11]. Assuming that the planar, confined dipole is prepared in its energy ground state and interacts with a radiation mode of a Laguerre-Gaussian form (since a Laguerre-Gaussian beam carries orbital angular momentum along its direction of propagation [12]). The expectation value of the angular momentum in the long time limit shows two distinct resonances at ω_a, ω_b. As a diminution in the kinetic energy term, the b resonance occurs at even greater values of frequency, until only the a resonance remains achievable at $\omega_a = K/g$. The supersymmetrization of the $(-)$ system can be achieved with a constant shift of the potential V_-. Thus the location and nature of possible angular momentum resonances allow in principle the experimental verification of the lowest angular momentum spectrum of SS cold Rydberg atoms.

To summarize, the main results obtained in this paper are as follows: (1) SUT is applied to discuss supersymmetrization of cold Rydberg atoms. The Hamiltonian and angular momentum of the SS Rydberg atom are simultaneously obtained. This allows the possibility of comparing the relation of angular momenta between the $(-)$ system and $(+)$ system. It is interesting to exploit further possible applications of SUT in physics. (2) In the limit of vanishing kinetic energy the lowest angular momentum of the SS cold Rydberg atom is $3\hbar/2$. The suggested possible experimental verification provides a crucial test of the idea of SS QM.

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