New Mechanism for Three-Dimensional Current Dissipation/Reconnection in Astrophysical Plasmas

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(Received 23 May 1996)

A "magnetohydrodynamiclike" theory has been previously developed for chaotic nonintegrable proton orbits which occur in highly stressed magnetic configurations. In this paper we give the solution to the Vlasov equation to next order in expansion of the particle bounce motion. The new contribution, a Boltzmann-like operator, provides a collisionless dissipation mechanism which may destabilize drift or drift ballooning Alfvén waves in high β plasmas. We discuss a number of applications of this new, potentially reconnective, mechanism in the magnetosphere, in stellar wind formation, and in the galactic dynamo. [S0031-9007(96)01701-2]

PACS numbers: 95.30.Qd, 52.30.–q, 52.35.Py, 94.30.Lr

In high temperature collisionless toroidal laboratory plasmas, it is well known that electron Landau damping replaces the mechanical electron-ion friction (resistivity) and allows reconnection of the magnetic field lines. This reconnection of field lines through the equilibrium magnetic surfaces is commonly known as the tearing mode. The theory behind this process has been worked out in the past [1]. Thirty years ago a two-dimensional (2D) version of this theory was subsequently applied to the stability of the geomagnetic tail by Coppi *et al.* [2] as a theory of substorms. This idea became a subject of both theoretical and observational interest for many years, since it had the potential to explain the "substorm breakup" and auroras by leading to the formation of neutral lines in the Earth's magnetosphere.

This 2D line reconnection for antiparallel magnetic field lines was later shown to be a very unlikely scenario [3–5]. The reason is that the existence of a finite, but small, normal component of the magnetic field, B_z, induces a strong stabilizing compression of the electron gas moving in a tearing electromagnetic field. Meanwhile, *in situ* magnetospheric measurements of the ionospheric currents and satellite observations have demonstrated that the "substorm breakup" phenomena is highly localized in longitude and usually located in the vicinity of geosynchronous orbit in a narrow cusp field region [6–9]. These experimental features are in conflict with what is expected from a linear tearing mode.

The failure of the previous theoretical attempts and the observations led Pellat [5] to propose that magnetohydrodynamiclike (MHD-like) modes with short transverse wavelength could be responsible for three-dimensional (3D) magnetic field reconnection. These modes had already been proposed for the substorm breakup [6,10], but with a nonrelevant theory. A first, now obvious, step was still missing. *Before* reconnection occurs the proton motion becomes nonintegrable as a result of the breaking of the invariance of the magnetic moment. Recently, this gap has been closed by solving the linearized (in electromagnetic potential perturbation amplitude) Vlasov equation for the chaotic proton motion [11–15].

The nonintegrable proton orbits allow one to recover a MHD-like theory with a modification of the polytropic index, for waves with frequencies smaller than the average bounce frequency of protons [12,14,15]. Additionally, a net dropoff potential, Φ_0, along field lines was found by carefully analyzing the quasineutrality equation for low frequency waves [13,15], as a consequence of the fast electron bounce motion along the magnetic field line.

It is the purpose of this Letter to go further in this analysis of 2D magnetic field equilibria (with 3D perturbations) where the proton motion becomes *nonadiabatic.* As a result of our continued analysis we find that we are forced into accepting a new paradigm for dissipation in reconnection regions. This new mechanism results from a Boltzmannian type collision term which arises naturally from the expansion of the solution of Vlasov equation to second order in the wave frequency (ω) over the typical proton bounce frequency (ω_{b_i}).

Originally, we were motivated by method of characteristics solutions, via computer, of the Vlasov equation [11,12] in which proton density fluctuations clearly exhibit a Poisson distribution with probability of the order of $\omega\tau_b$ where $\tau_b = 2\int_0^{l_b} dl/v_\parallel$ is half the bounce time. Also, we were aware of the destabilizing effect, on low frequency waves, of high frequency turbulence [16].

When a proton flows along a field line that has a small region where the radius of curvature is comparable to the proton Larmor orbit radius (a cusp), it can suffer an *effective* collision. That is, in such a region the proton's magnetic moment is not conserved and can take on any value with uniform probability (to very good accuracy)

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[11,12]. This suggests that *a solution of the Vlasov equation can be obtained by treating the proton motion as adiabatic in the strong magnetic field region and applying a random walk (in magnetic moments) in the narrow weak field region where the field radius of curvature is small.* Clearly, this picture suggests a strong collision operator of the Boltzmann type. From previous work [16], we already know that Fokker-Planck theory is adapted to a "weak nonadiabaticity."

One has to solve the linearized Vlasov equation for the plasma distribution function, f, along the proton motion parallel to the equilibrium magnetic field. The equation reads

$$\left[i(\omega + \omega_d) + v_\parallel \frac{\partial}{\partial l}\right] g = i(\omega + \omega^*) H, \tag{1}$$

where l is the measure of length along the magnetic field line and $v_\parallel$ is the velocity of the guiding center of the proton projected along the magnetic field. The plasma distribution function is related to g through the equation $f = f_0 + q(\partial f_0/\partial E)[\Phi - (1 + \omega^*/\omega)\lambda e^{-iS} - g]$ where $\lambda = i(\omega/c)\int^l J_0 A_\parallel\, dl$, $A_\parallel$ is the component of the magnetic vector potential projected along the equilibrium magnetic field, Φ is the electrostatic potential, $S = \mathbf{k} \cdot \hat{b} \times \mathbf{v}/\Omega$ is the eikonal, Ω is the proton gyrofrequency, E is the energy, and f_0 is the equilibrium distribution function. The magnetic curvature-gradient drift frequency is given by

$$\omega_d = \frac{k_y c}{qB}\, \hat{b} \times [\nabla(\mu B) + m v_\parallel^2 \hat{b} \cdot \nabla \hat{b}], \tag{2}$$

where $\hat{b}$ is the unit vector along the magnetic field line, m is the particle mass, μ is the magnetic moment. H, the "Lagrangian density," is given by

$$H = J_0 \Phi - J_1 \frac{|v_\perp|}{k_\perp c} k_y A_\psi + \frac{\omega + \omega_d}{\omega}\lambda, \tag{3}$$

where $k_\perp$ is the component of the wave vector normal to the equilibrium magnetic field, A_ψ is the component of the perturbing magnetic vector potential along the $\nabla\psi$ direction, and $|v_\perp|$ is the magnitude of the velocity normal to the equilibrium magnetic field. Here $J_n = J_n(k_y \rho(l))$ is a Bessel function of nth order and $\rho(l)$ is the proton Larmor radius. The Bessel functions exhibit the beneficial property of reducing the proton response in the region of very large Larmor radius (which is the nonadiabatic motion region). This point helps to justify our treatment of the small field region as an effective scatterer. Further, this effect eliminates the singular behavior of the $\mathbf{B} \times \nabla\Phi$ drift velocity of pure MHD. Note, k_y is the wave number antiparallel to the direction of the equilibrium plasma current (the y direction). The short wavelength in the y direction (large k_y) yields a Doppler shifted frequency, $\omega + \omega_i^*$, where $\omega_i^* = k_y(cT_i/q_i)\partial \ln(n_i T_i)/\partial\psi$ is the diamagnetic drift frequency, c is the speed of light, T_i is the bulk proton temperature (in energy units), q_i is the proton charge, n_i is the proton number density, and ψ is the y component of the equilibrium magnetic vector potential (i.e., the flux function).

Now, the motion of the proton is adiabatic except where it crosses the field reversal plane (see Fig. 1) at $l = 0$. To obtain the solution for g, we have to solve a random walk problem, each "collision" with the midplane being independent. To lowest order the result is a phase mixed response [11,12]. Using the symmetry of the proton motion on the adiabatic part of its trajectory with the formal solution of Eq. (1) for streaming and antistreaming protons to first order in $\omega\tau_b$ we obtain the integration constant of (1). Retaining terms up to second order in $\omega\tau_b$ we find $g = g_1 + g_2$ where

$$g_1 = \left(\frac{\omega + \omega^*}{\omega\langle\bar{1}\rangle + \langle\bar{\omega}_d\rangle}\right)\langle\bar{H}\rangle \tag{4}$$

is the lowest order contribution found in our previous work [11,12]. We have made the operator definitions $\bar{\bullet} = 2\int_0^{l_b} \bullet\, dl/v_\parallel$ and $\langle\bullet\rangle = \int_0^{E/B} \bullet\, d\mu$. The average gradient-curvature drift frequency has been computed [11,12,15], $\langle\bar{\omega}_d\rangle/\langle\bar{1}\rangle = -(2/3)(k_y c/q)E\partial(\ln \oint dl/B)/\partial\psi$. The next order, is

$$g_2 = \frac{i}{2}(\omega + \omega^*) \times \left(\langle\bar{H}\rangle \frac{\langle\bar{\alpha}^2\rangle}{\langle\bar{\alpha}\rangle^2} + \bar{H} - \frac{\bar{\alpha}}{\langle\bar{\alpha}\rangle}\langle\bar{H}\rangle - \frac{\langle\bar{H}\bar{\alpha}\rangle}{\langle\bar{\alpha}\rangle}\right), \tag{5}$$

with $\alpha = 1 + \omega_d/\omega$. This is a "dissipative" collision-like contribution which vanishes exactly in the adiabatic case (since the μ integration is removed). From this result we can understand why computer calculations of g yield Poisson statistic fluctuations [11] with a mean square value $\sim O(\langle\omega\rangle/\langle\omega_b\rangle)$.

By constructing moments for the perturbed density and currents from the Vlasov solution (i.e., $\delta n = \int d^3v\, f_1$ and $\delta\mathbf{j} = q\int d^3v\, \mathbf{v} f_1$) and using the Poisson equation

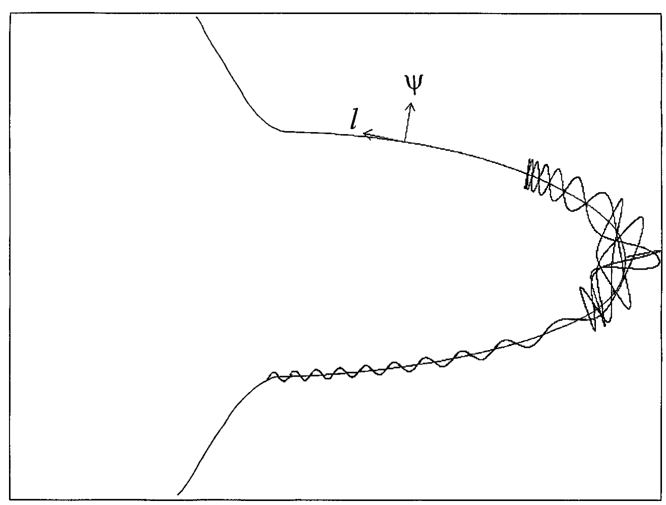

FIG. 1. A typical cusplike magnetic field line is shown with a stochastic proton trajectory. The y coordinate is into the page. The Earthward direction is to the left and the tailward direction is to the right.

and Ampère's law we find that the quadratic form of Pellat [5] and its quasi-MHD application [14] are modified by the dissipative contribution of g_2. The new contribution is

$$-\frac{i}{2}(\omega + \omega^*)\left\langle\left(\bar{H} - \frac{\bar{\alpha}}{\langle\bar{\alpha}\rangle}\langle\bar{H}\rangle\right)^2\right\rangle, \tag{6}$$

i.e., a Boltzmann type result.

Extending our previous work [12,14,15] we find the variational principle (neglecting T_e/T_i corrections),

$$\frac{1}{n_i m_i}\oint\frac{dl}{B}\left\{\left(\frac{\partial X}{\partial l}\right)^2 - \frac{p'}{RB}\left[X - \frac{\oint(dl/RB^2)X}{\oint(dl/RB^2)}\right]^2\right\}$$
$$= \omega(\omega + \omega^*)\oint\frac{dl}{B^3}\left[X - \frac{\oint(dl/B^3)X}{\oint(dl/B^3)}\right]^2$$
$$-\frac{i}{2}(\omega + \omega_2^*)\sqrt{\frac{2T}{m_i}}\left\langle\left(\bar{H}_m - \frac{\bar{\omega}_d}{\langle\bar{\omega}_d\rangle}\langle\bar{H}_m\rangle\right)^2\right\rangle, \tag{7}$$

where R is the local radius of curvature of the magnetic field line, $X = k_y c\lambda/\omega$, $p' = \partial p/\partial\psi$, $\omega_2^* = \omega^* + (ck_y/q)\partial T/\partial\psi$, and in the integrations on the last term μ has been replaced by $\mu B/E$ so as to remove the energy dependence. In the last term of (7), α has been replaced by ω_d after substituting the drop-off potential [15], $\Phi_0 = \omega\langle\bar{H}_m\rangle/\langle\bar{\omega}_d\rangle$ (computed from quasineutrality) into H. Minimization has simplified (6), leaving only the difference between the adiabatic and stochastic compressibilities. The resulting minimized Lagrangian density is found to be [14,15]

$$H_m = \left[\frac{\omega_d}{k_y c}X + \frac{4\pi p'\mu}{qB}\left(X - \frac{\oint(dl/RB^2)X}{\oint(dl/RB^2)}\right)\right]\frac{q}{E}. \tag{8}$$

The first term of (7) is the δW of Hurricane *et al.* [15], the second term is related to the kinetic energy, and the last term is the dissipation. The fact that (7) identically vanishes for constant X is a result of quasineutrality; i.e., there is no effect without a finite parallel wavelength. Physically the dissipative term is essentially a type of parallel Landau damping: a stochastic proton being scattered in μ as it transits the high curvature region of the field line will have its bounce frequency take on a continuum of values (the adiabatic case has ω_b fixed). *Thus a particle-wave resonance occurs along the parallel motion for a segment of the proton trajectory.* Depending upon the sign of the Doppler shift, the resonance can lead to either wave damping or wave growth.

In computing (7) we took the bulk plasma to be stochastic. Since peak the contribution to the moment integrals comes from protons with energy $E \sim 2T - 3T$ depending upon the term [due to moment integrals of the form $\int d^3v\, E^\alpha e^{-E/T}$], (7) is valid in a regime with a partially adiabatic population as long as $2T$ corresponds to the stochastic regime. If the bulk plasma is adiabatic, one must recompute the dispersion relation using a separate perturbed distribution function for both the bulk and energetic population.

Recall that, up to now, all attempts to destabilize this configuration within the framework of standard MHD have failed [17]. Further, the more precise adiabatic theory appears to be even more stabilizing [14]. Now we have a new mechanism of instability. Consider equation (7) in the following notationally simplified form

$$\omega^2 + \omega(\omega^* - i\nu) - i\nu\omega_2^* - \omega_{\mathrm{MHD}}^2 = 0, \tag{9}$$

where ν (the effective collision frequency with the weak field region) and ω_{MHD}^2 (the MHD frequency) can be identified by comparison with (7). Considering the MHD marginal case ($\omega_{\mathrm{MHD}}^2 = 0$) and assuming ν is small compared to the diamagnetic drift frequencies we find the two roots

$$\omega_+ = i\nu\left(\frac{\omega^* - \omega_2^*}{\omega^*}\right), \quad \omega_- = -\omega^* + i\nu\frac{\omega_2^*}{\omega^*}. \tag{10}$$

The first root is an absolute growing (or damping) mode only if $\partial T/\partial\psi \neq 0$. The second root has the character of drift wave with growth, if $\omega_2^*/\omega^* < 0$, and with damping in the opposite case.

Initially, our "quasi-MHD theory" has been motivated by observations made in the Earth's magnetosphere. We have proposed elsewhere [18] that interchange-ballooning modes should be the basic wave polarization involved in a substorm breakup, flux transfer events, and bursty bulk flows. Including proton stochasticity appeared necessary to obtain instability and cover the eventual 3D reconnection observed. This new theory builds upon the older, but similar, idea of Hagège *et al.* [16], but this case is fully electromagnetic, does not involve extra turbulence, and has explicitly solved quasineutrality [13,15].

It is worthwhile to give a preliminary attempt at a nonlinear theory involving the same waves. Proceeding in a way similar to Hagège *et al.* [16] we can compute the quasilinear effect of a turbulent spectrum of our MHD-like "high" frequency ($\omega \sim \omega_i^*$) large k_y waves during quasisteady reconnection. The resulting Ohm's law is

$$\oint\frac{dl}{B}\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)\cdot\hat{y}$$
$$= \left(\frac{1}{n_i T_i}\oint\frac{dl}{B}\int dk_y\,\frac{\Gamma_{k_y}}{|\omega|^2}|\delta E_{k_y}|^2\right)j_y, \tag{11}$$

where $\vec{E}$ is the equilibrium electric field, δE_{k_y} is the Fourier component of the perturbed electric field, and $j_y \propto p'$ is the confinement current (the "Chapman-Ferraro current" in a magnetosphere). It is important to note that the "growth rate," Γ_{k_y}, is really a correlation time when working in the context of steady reconnection (the reconnection length should be related to the correlation length of the turbulence). One expects this "α dynamo-like" resistivity to remove the local magnetic field stress, increasing the length of the field line, and as usual contribute to an inverse cascade of MHD turbulence. By the same notion, magnetic loops on the surface of the sun may utilize this mechanism to open forming the solar wind.

At galactic scales, standard dynamo theory needs a primordial seed, according to Kulsrud *et al.* [19]. Unfortunately, no scenario whatever during the inflation period or in the pre-recombination relativistic plasma, as considered by Tajima *et al.* [20], can directly produce sufficient power in the long wavelength range to seed the dynamo [21]. More careful investigation remains for the so-called "pre-big-bang scenario" which is at a time when large quantum fluctuations of the electromagnetic field are present and the relevant Lagrangian is nonlinear. The nonlinear coupling can provide an interaction between short and long wavelengths which is necessary if we are to apply our classical plasma finding. In any case, the cosmic ray pressure and the magnetic field pressure are comparable which satisfies one basic requirement of our picture.

We thank F. V. Coroniti, C. F. Kennel, and J. Cornwall from UCLA and A. Roux and his colleagues from CEPT for helpful discussions.

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