Resonance of Quantum Noise in an Unstable Cavity Laser

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We have measured the quantum-limited lpinewidth of a hard-edged unstable cavity gas laser. Our results confirm the predicted resonant behavior of the quantum-noise strength as a function of equivalent Fresnel number. This behavior is due to the nonorthogonality of the transverse eigenmodes. [S0031-9007(96)01699-7]

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Unavoidable quantum noise sets a limit to the coherence of a laser. The phase of the laser field diffuses under the influence of spontaneous emission, leading to the so-called Schawlow-Townes laser linewidth. This has been the subject of many theoretical and experimental investigations and is well understood for lasers with stable cavities and small losses per cavity transit [1,2]. The eigenmodes of such a laser form a set of orthogonal modes. For stable cavity lasers which have large mirror transmission and for lasers which operate on an unstable cavity the eigenmodes are nonorthogonal; as a consequence the Schawlow-Townes linewidth is enhanced by the so-called K factor or excess-noise factor [3–12]. Strong enhancement of the quantum-limited linewidth of unstable cavity lasers has been predicted and observed for a solid-state laser with a variable reflectivity mirror (VRM) [10,11] and most recently also in a hard-edged-mirror solid-state laser [12]. It has been predicted [4–6], but not yet verified, that this enhancement shows resonant behavior as a function of the equivalent Fresnel number. This resonance will occur only in hard-edged resonators, since it is essential that the shape of the transverse mode profile is determined by the precise cavity dimensions. In our experiments on a hard-edged unstable cavity gas laser we have been able to confirm this most intriguing aspect of unstable-resonator quantum-noise theory: the resonance of the quantum noise with equivalent Fresnel number.

In a hard-edged unstable resonator a fraction of the reproducing mode spills, each round-trip, over a small feedback mirror. We call this mode the matched mode. It is biorthogonal to the so-called adjoint mode, which is a direction-reversed version of the matched mode [5,10,13]. It has been shown that the K factor is identical to the injected wave excitation factor, which is the factor by which the power of the mode, when excited by the adjoint mode, exceeds that when using matched-mode excitation [3,5]. This factor, and thus the K factor, depends strongly on the precise shape of the transverse mode profile. We emphasize that there exists no fundamental relation between large losses (or gain) and excess noise factors; two different transverse mode profiles that have identical diffraction losses may have K factors that differ by orders of magnitude if their nonorthogonality differs appreciably. Note that the quantum-noise properties of a hard-edged unstable-resonator laser are described by two parameters only: the round-trip linear magnification M and the equivalent Fresnel number $N_{\mathrm{eq}} = (M^2 - 1)a^2/2M\lambda B$ where λ is the wavelength, a is the radius of the small feedback mirror, and B is an effective cavity length [13].

The periodic variation of hard-edged unstable-resonator properties with the equivalent Fresnel number N_{eq} can be understood as follows. The distance from the outer edge of the small feedback mirror (or aperture) to the nearest point on the incident wave front, when that wave front just touches the center of the feedback mirror, is equal to $N_{\mathrm{eq}}\lambda/2$ [14]. The periodicity with N_{eq} results from the periodic variation of the phase of the wave that is diffracted by the feedback mirror. Hence the transverse profile of the laser mode shows a periodicity with N_{eq}, with periodic maxima in modal diffraction losses near integer values of N_{eq} [13]. Since the K factor strongly depends on the transverse mode profile, this periodicity must also occur in the excess-noise factor [4–6].

For the experimental demonstration we use a miniature HeXe laser with an unstable resonator; the gain medium is a HeXe gas discharge in a capillary tube, and the laser operates on the high-gain 3.51 μm Xe transition. The operating pressure of HeXe is 0.5 kPa which results in a FWHM gain linewidth of 152 MHz [15]. The unsaturated gain at this pressure was measured to be about 110 dB/m. The capillary tube is terminated by two 0.5 mm thick quartz windows with a single-pass transmission of 0.92. The 4.7 cm long cavity consists of a convex gold mirror with a 36 cm radius of curvature and a flat mirror with 93% reflectivity. This results in a round-trip magnification M of 2.03. As a limiting aperture we insert right in front of the 93% mirror a screen with a square aperture, which consists of four razor blades that can be adjusted to change the aperture size. The size is measured using a microscope. The capillary diameter is 5 mm; this value ensures that the mode profile remains clear of the capillary edges.

The excess-noise factor is revealed by comparing the laser linewidth of an unstable cavity to that of a stable cavity with the same loss per round-trip. Measuring the cavity loss rate of an unstable cavity laser is not a trivial

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matter [16]; we use an axial magnetic field and measure cavity mode pulling [17]. The Zeeman splitting of the gain transition produced by the magnetic field leads to pulling on the circularly polarized (σ_+ and σ_-) cavity modes; the pulling strength depends on the cavity loss rate. The frequency splitting of the σ_+ and σ_- modes (and thus the pulling strength) is reflected in the beat frequency that is recorded by a detector after a polarizer. The spectrum of the beat between the σ_+ and σ_- modes is used for the actual linewidth measurements [18]. Since the σ_+ and σ_- modes oscillate in the same laser cavity, both mode frequencies are disturbed by the same technical noise sources; therefore in the difference frequency the technical noise cancels, whereas the quantum noise remains. When one measures the frequency noise in the beat one finds that the spectral width shows the expected Schawlow-Townes inverse power dependence and that the spectral shape is Lorentzian, confirming the white noise character of quantum noise.

We prefer to operate the laser at modest values of $N_{\rm eq}$ (e.g., $N_{\rm eq} \sim 1$) to make the alignment dependence of K as small as possible. Misalignment leads to effectively different values of $N_{\rm eq}$ for the upper and lower halves of the resonator [19,20], which, for high values of $N_{\rm eq}$, can modify the position, width, and height of the resonances in K. The experimental demonstration of the resonance of the K factor is performed using a square aperture inside the laser (instead of a more conventional circular aperture). This is done to improve the signal-to-noise ratio in the quantum-noise measurements which otherwise turned out to remain somewhat marginal at values of $N_{\rm eq} \sim 1$ (see below).

So before we move on to present the results on the resonant behavior we will, as an intermezzo, discuss the results on the excess-noise factor for square versus circular resonator geometry. It is easily seen that the mode profile can be influenced by the transverse symmetry if one compares the diffraction patterns of a circular and a square aperture. For stable cavity lasers and for VRM unstable cavity lasers the mode profiles are always Gaussian, so that no change in K factor is expected when going from square to circular geometry. For a hard-edged unstable cavity laser the transverse mode profiles have to be calculated numerically. This can be done most efficiently by means of virtual-source theory, where diffraction is modeled by an edge wave produced by virtual sources that represent the diffracting aperture or mirror [21]. The virtual-source theory is used to calculate both the transverse mode profile and the corresponding complex eigenvalue, α, of the specific transverse mode. The absolute value of α is related to the loss of that mode compared to the geometrical loss [13]. We have performed numerical calculations of the mode profiles for various values of the cavity parameters $N_{\rm eq}$ and M, and found that the transverse mode profile is generally different in cavities with square and circular transverse geometry. The modes of a cavity with circular mirrors generally have their intensity more concentrated along the resonator axis than the modes in a cavity with square mirrors [5,6,13,21]. This can be understood by noting that the virtual sources on the edge of a circular aperture add up in phase on the resonator axis, whereas they do not for a square aperture. Therefore the square modes suffer more loss per round-trip than do circular modes, increasing the power advantage after direction reversal; after calculating the nonorthogonality integrals this leads to a larger K factor in square geometry, $K_\square$, as compared to circular geometry, $K_\circ$. From the diffraction calculations of the transverse mode profiles for circular and square geometry, with $N_{\rm eq} = 1.44$ and $M = 2.03$ (these values correspond to the experiment described below) we find the K factors $K_\square = 21$ for the square and $K_\circ = 7.6$ for the circular case.

In the experiment we use a $1.68 \times 1.68\ \mathrm{mm}^2$ square aperture or a 1.68 mm diameter circular aperture inside the laser, positioned right in front of the 93% mirror, so that in both cases $N_{\rm eq} = 1.44$. The measured linewidths as a function of the inverse laser output power are plotted in Fig. 1. The measurements were performed under the same conditions of the gas discharge so that a direct comparison is possible; they show a clear difference between square- and circular-aperture laser linewidths. When using the measured cavity loss rates one finds as experimental values $K_\square = 22 \pm 7$ and $K_\circ = 6 \pm 1$. This is in good agreement with the values predicted above. These are, in fact, the first measurements to confirm the recent prediction that the K factor in square geometry is much larger than in circular geometry [6].

We will now proceed with the resonant behavior of the K factor with $N_{\rm eq}$. In this experiment the laser cavity length was 4.86 cm with a 36 cm radius of curvature convex gold mirror and a 93% reflectivity concave mirror with a radius of 3 m, resulting in a round-trip magnification

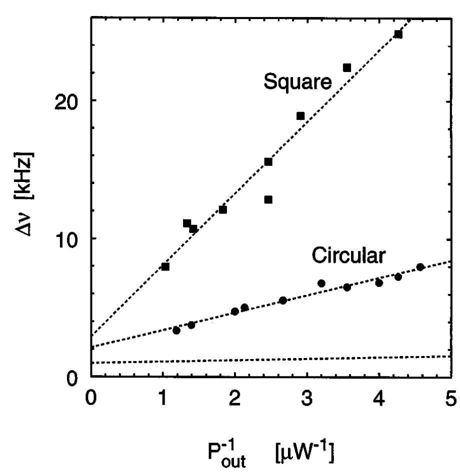

FIG. 1. Plots of the laser linewidth as a function of the inverse output power of an unstable cavity laser, for a circular and square intracavity aperture. The lowest dashed line has been calculated with $K = 1$ and corresponds to a stable cavity laser.

M of 1.95. The square aperture is inserted right in front of the gold mirror and can be varied in size from 0.80×0.80 to $1.45 \times 1.45\ \mathrm{mm}^2$, which changes N_{eq} from 0.34 to 1.1. The difference in horizontal and vertical size of the aperture is kept less than 50 μm. The average of the horizontal- and vertical-aperture size is used for the calculation of N_{eq}. After every change of the aperture size, the aperture was centered again on the laser axis with an additional HeNe 633 nm alignment beam, so that the mirror realignment could be limited to very small changes.

Virtual-source theory predicts mode crossing behavior near integer values of N_{eq}. For a square aperture the crossings occur for N_{eq} slightly below integer values since the effective Fresnel number is different along different radial directions [13]. The crossing that is relevant for our experiment is marked by the dashed circle in Fig. 2, where the absolute values of the eigenvalues, $|\alpha|$, of a number of transverse modes are plotted. The mode with the highest eigenvalue is the lowest-loss mode. Near the crossing point two modes have nearly identical values of $|\alpha|$, so that their round-trip losses are nearly equal (it has been shown that at the mode crossing the mode profile of the "growing" mode becomes identical to the mode profile of the "disappearing" mode [13]). This is also observed experimentally; when the aperture size is enlarged towards an integer value of N_{eq}, a second transverse mode appears, which has a slightly higher loss (this is reflected in both the smaller output power of this mode and in the higher mode pulling beat frequency). Upon further increasing the aperture size the losses of the two transverse modes approach each other until finally they interchange and their loss difference increases again. Note that when the modes have equal values of $|\alpha|$ they do not become frequency degenerate; they do not have the same complex eigenvalue α [13]. Since this transverse mode splitting is much larger than the gain linewidth they can be easily selected by tuning the cavity length.

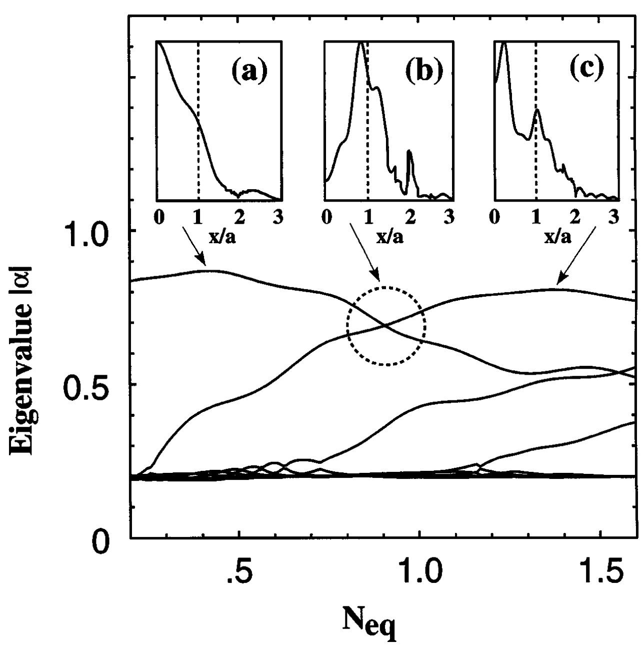

FIG. 2. Plot of the absolute value of the eigenvalues, $|\alpha|$, of a number of transverse modes as a function of equivalent Fresnel number N_{eq} for the square resonator with $M = 1.95$. A mode crossing occurs at $N_{\mathrm{eq}} = 0.90$, indicated by the circle. The insets show the numerically calculated transverse mode profiles that were used to calculate the K factor for N_{eq} equal to (a) 0.42, (b) 0.90 (crossing), and (c) 1.38.

The insets in Fig. 2 show examples of the calculated mode profiles that were used to calculate the K factor as a function of N_{eq}. The three mode profiles have been calculated for a square aperture and $M = 1.95$. In all three figures the vertical dashed line represents the edge of the aperture. Figure 2(b), which corresponds to the mode profile at the mode crossing, shows a clearly reduced intensity on the axis of the resonator, as compared to 2(a) and 2(c), and spreads out well beyond the edge. Therefore profile 2(b) has the highest round-trip diffraction loss; a relatively large fraction of the mode power is spilled over the aperture each round-trip. This mode profile will result in a large K factor; the maximum of the K factor occurs at the position of the mode crossing, i.e., at $N_{\mathrm{eq}} = 0.90$. The excess noise factors have been calculated from the transverse mode profiles belonging to the highest eigenvalue $|\alpha|$ using 2048 points along the transverse coordinate. The result of these calculations is shown in Fig. 3(a). An abrupt change in the K factor occurs at the crossing point $N_{\mathrm{eq}} = 0.90$, where the mode profiles interchange [6].

Figure 3(b) shows the experimental results for the lowest-loss mode at each value of N_{eq}. The resonant behavior of the K factor with N_{eq} is clearly visible. The maximum in the experimental K factor is at $N_{\mathrm{eq}} = 0.88$, in good agreement with the position of the theoretical maximum at 0.90. Each linewidth measurement was accompanied by a measurement of the cavity loss rate (using the technique discussed above). Also the latter measurements show the expected maximum in loss rate at the position of the peak, although this is a much less pronounced effect. Near the resonance the cavity loss hardly changes (a few %), whereas the K factor changes by an order of magnitude; this proves, as stated above, that there exists no obvious relation between cavity losses and excess noise. Further, it can be seen in Fig. 3 that the resonance of the K factor is superimposed on a "baseline" that is increasing with N_{eq}. This is seen in both the theoretical and experimental curves. Again, this agrees with theory: it can be shown in a simple geometrical model that for half-integer values of N_{eq} the K factor increases linearly with N_{eq} [22]. The experimental peak in Fig. 3 appears to be somewhat narrower and higher than the theoretical prediction, which may be the result of using an aperture at a finite distance (~0.5 mm) in front of the gold mirror. This situation is slightly different from using a small feedback mirror since the aperture is

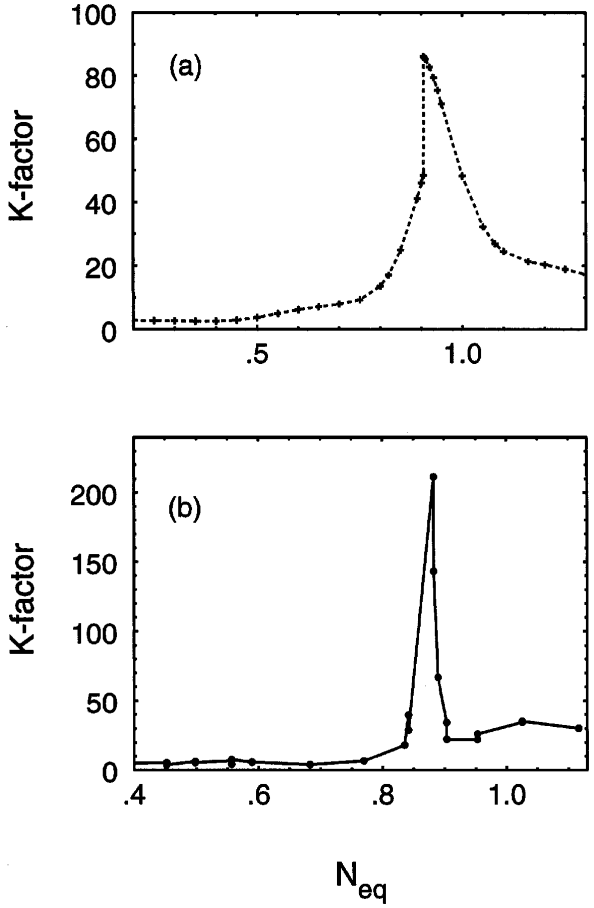

FIG. 3. Resonance behavior of the excess-noise factor as a function of N_{eq}; (a) theoretical (b) experimental. The curves in (a) and (b) have been added to guide the eye.

traversed twice per round-trip [23]. The difference in horizontal and vertical size of the square aperture cannot be held responsible for the narrowing. Since the K factor in two dimensions is the product of the horizontal and the vertical one-dimensional K factor [6] it can only lead to a broadening.

In conclusion, we have presented an experimental verification of a key aspect of quantum-noise theory for unstable resonators. Our experiments confirm the intriguing prediction of resonant behavior of the quantum noise with equivalent Fresnel number, which is a unique property of hard-edged unstable resonators. Furthermore, we have demonstrated that a square-geometry hard-edged unstable cavity laser has a considerably larger linewidth than the corresponding circular-geometry laser, confirming a recent theoretical prediction.

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