Statistical Entropy of Four-Dimensional Extremal Black Holes

Juan M. Maldacena,[1] and Andrew Strominger[2]
[1]*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544*
[2]*Department of Physics, University of California, Santa Barbara, California 93106*
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String theory is used to count microstates of four-dimensional extremal black holes in compactifications with $N = 4$ and $N = 8$ supersymmetry. The result agrees for large charges with the Bekenstein-Hawking entropy. [S0031-9007(96)00674-6]

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Recently it has been shown [1–5] that string theory can, in some special cases, provide a statistical derivation of the Bekenstein-Hawking entropy [6,7] by representing the black holes as bound states of D-branes and strings. The statistical entropy is the logarithm of the bound state degeneracy, which was counted using D-technology introduced in [8–10]. Curiously the results so far have been limited to five dimensions. The reason for this is that four-dimensional black holes with nonzero horizon area cannot be constructed from D-branes alone as in [1]. Typically another type of object such as a symmetric fivebrane or Kaluza-Klein monopole is required, and further technology is needed. In this paper we will find the missing piece of technology in Refs. [11,12] and use it to compute the statistical entropy of certain four-dimensional extremal black holes in $N = 4$ and $N = 8$ supergravity theories. The result agrees with the Bekenstein-Hawking entropy, which was computed in a special $N = 4$ case in [13], more generally for $N = 4$ in [14,15] and for $N = 8$ in [16].

The statistical entropy of four-dimensional black holes has been recently analyzed in [17] with methods seemingly quite different from those used herein. It would be very interesting to understand the relation between the two approaches.

The required modification of [1] is rather simple and this presentation will be accordingly brief. Let us begin by rederiving the result of [1] in a T-dualized picture with one extra $\hat{S}^1$-compactified dimension. Consider type IIA string theory on $X = Y \times S^1 \times \hat{S}^1$, where Y is T^4 for the $N = 8$ case and $K3$ for the $N = 4$ case. A dual description of the D-brane configuration in [1] (obtained by T dualizing along $\hat{S}^1$) consists of Q_6 sixbranes wrapping X, Q_2 twobranes wrapping $S^1 \times \hat{S}^1$, and right-moving momentum n along the S^1. We take $n, Q_2 \gg 1$. The twobranes are marginally bound to the sixbranes [18–20]. For $Q_6 = 1$ the momentum is carried by massless, right-moving modes of $(2,2)$ open strings that end on the twobranes. It is sufficient to consider the case $Q_6 = 1$ because duality implies the results can depend only on the product Q_2Q_6. (This has been explicitly verified in some cases [18–22].) BPS excitations of these $(2,2)$ strings correspond to transverse motion of the Q_2 twobranes within Y (and the sixbrane). [Since the two branes are separated in Y the $(2,2)$ open strings going between different twobranes are massive and do not contribute to the extremal entropy as in [1]. $(2,6)$ strings also do not contribute in this case ($Q_6 = 1$) because of charge confinement.] Because Y is four dimensional this means there are $4Q_2Q_6$ bosons and their $4Q_2Q_6$ fermionic superpartners available to carry the momentum. (We suppress here the anomalous shift of Q_2 for $K3$ [20,21] which is subleading for large Q_2). The number of BPS-saturated states of this system as a function of Q_2, Q_6, and n follows from the standard $(1 + 1)$-dimensional entropy formula

$$S = \sqrt{\frac{\pi(2N_B + N_F)EL}{6}}, \tag{1}$$

where N_B (N_F) is the number of species of right-moving bosons (fermions), E is the total energy, and L is the size of the box. Using $N_B = N_F = 4Q_2Q_6$ and $E = 2\pi n/L$, we find the L-independent result for the large n thermodynamic limit [1]

$$S_{\mathrm{stat}} = 2\pi\sqrt{Q_2Q_6n}. \tag{2}$$

The Bekenstein-Hawking entropy was computed from the corresponding four-dimensional extremal black hole solutions in [13–16]. The result in our notation for either $N = 4$ or $N = 8$ is

$$S_{\mathrm{BH}} = 2\pi\sqrt{Q_2Q_6nm}. \tag{3}$$

The integer m here is the axion charge carried by a symmetric fivebrane which wraps $Y \times S^1$. [To facilitate comparison with [14,15], we note that under type II heterotic duality an m-wound symmetric fivebrane together with momentum n becomes a fundamental heterotic string with (winding, momentum) $= (m, n)$ around S^1. The twobranes and sixbranes become the magnetic heterotic S duals of a fundamental heterotic string with (winding, momentum) $= (Q_2, Q_6)$ associated to the $(20,4)$ part of the Narain lattice.] Since that charge is absent in this $\hat{S}^1$ compactification of the configuration of [1], $S_{\mathrm{BH}} = 0$. This is not a contradiction because in four dimensions S_{BH} as computed from the leading low energy effective action always scales like (charge)^2, in contrast to five dimensions where it scales like (charge)$^{3/2}$.

Since (2) scales like $(\text{charge})^{3/2}$, it appears at leading order in five dimensions but is an invisible subleading correction in four. (In fact the four-dimensional solution with $m = 0$ contains scalar fields that blow up at the horizon, rendering the classical geometry at the horizon singular.) These fivebranes do not break any additional supersymmetry so that the final configuration still preserves one supersymmetry.

In order to get a nonzero area in four dimensions, we must add m NS fivebranes wrapping $Y \times S^1$. Hence we need to understand the effect of fivebranes on the state counting. Since we are counting BPS states, the result is independent of the moduli of $Y \times S^1 \times \hat{S}^1$ and we are free to work at any point in the moduli space. It is convenient to take all of the compactification radii to be very large and the string coupling to be very small. In this limit the transverse size of a fivebrane is much smaller than the radius of $\hat{S}^1$, and the locations of the m fivebranes are described by m points along $\hat{S}^1$. Since the twobranes wrap $S^1 \times \hat{S}^1$, topologically each of the Q_2 twobranes must intersect each of the m fivebranes. The mQ_2 intersections are circles (for fixed time) which wrap S^1. As explained in [11,12] a twobrane which intersects a fivebrane can break along the intersection line, just as a string can break along its intersection point with a D-brane. The two ends of the twobrane are then free to separate in Y. When the volume of Y is very large the ends of the twobranes will generically be well separated: It cost energy to localize the wave functions for the twobrane ends near one another. Hence the original Q_2 twobranes wrapped on $S^1 \times \hat{S}^1$ become mQ_2 open, toroidal twobranes which end on neighboring fivebranes. The momentum is oriented around the S^1 parallel to the twobrane boundaries. The momentum-carrying open strings now have an extra label describing which pair of fivebranes they lie in between. Hence the number of species becomes $N_B = N_F = 4mQ_2 = 4mQ_2Q_6$. Inserting this into (1) together with $E = 2\pi n/L$ we obtain

$$S_{\text{stat}} = 2\pi\sqrt{Q_2 Q_6 n m}\,, \tag{4}$$

in agreement with the semiclassical result (3) for S_{BH}.

For the $N = 4$ case there are, in general, 28 electric charges $\vec{Q}$ and 28 magnetic charges $\vec{P}$ which lie in the (22, 6) Narain lattice. In our notation $2Q_2Q_6 = \vec{P}^2$ and $2nm = \vec{Q}^2$. Duality implies that the entropy depends only on $\vec{P}^2$, $\vec{Q}^2$, and $\vec{Q} \cdot \vec{P}$. The general formula for the Bekenstein-Hawking entropy is [14,15]

$$S_{\text{BH}} = \pi\sqrt{\vec{P}^2\vec{Q}^2 - (\vec{Q} \cdot \vec{P})^2}\,. \tag{5}$$

For our example the last term vanishes. It would be interesting to construct a more general example for which this last term does not vanish, and so verify the general formula.

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