Coherence and Localization in 2D Luttinger Liquids

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Recent measurements on the resistivity of $(La\text{-}Sr)_2CuO_4$ are shown to fit within the general framework of Luttinger liquid transport theory. They exhibit a crossover from the spin-charge separated "holon nondrag regime" usually observed, with $\rho_{ab} \sim T$, to a "localizing" regime dominated by impurity scattering at low temperature. The proportionality of ρ_c and ρ_{ab} and the giant anisotropy follow directly from the theory. [S0031-9007(96)01626-2]

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Recently Ando *et al.* [1] have measured the resistivity of two $(La\text{-}Sr)_2CuO_4$ samples, one of them in the fully metallic optimally doped range, in magnetic fields up to 60 T, which completely destroys superconductivity. The result confirms other measurements which showed that at sufficiently low temperature the resistivity of the cuprate ab plane in the normal state crosses over to a negative temperature derivative with an approximately logarithmic T dependence, as opposed to the conventional linear T behavior (at least where the spin gap does not intervene as it does in underdoped bilayer materials such as BISCO-2212 and $YBCO_{6.6-6.8}$). What is new about the Ando-Boebinger experiments is that the c-axis and ab-plane conductivities become proportional to each other at low temperature although with an extraordinarily large anisotropy of the order 1000.

The c-axis conductivity in these measurements follows a smooth extrapolation from the normal state values above T_c. It is reasonable to suppose that the c-axis conductivity is therefore not crossing over to a new behavior; we must then suppose that the ab plane does so.

We [2] have derived the c-axis conductivity as a function of frequency and at $T = 0$, for Luttinger liquid planes or chains weakly coupled by a tunneling matrix element $t_\perp$, presumed diagonal in the momentum parallel to the chain or plane. We get

$$\sigma = \alpha \frac{e^2}{\hbar} \frac{t_\perp^2}{t_\parallel^2} \left(\frac{\omega}{\Lambda} \right)^{2\alpha}, \tag{1}$$

where this is measured per electron and between one pair of planes: i.e., it is a conductance in $(\Omega)^{-1}$. α is the Fermi surface exponent (considerably less than 1) giving the power law for $n(k)$ at k_F, and Λ is a cutoff of order $t_\parallel$. This ω dependence will be reflected in a T dependence since the derivation of (1) depends only on the scaling properties of the one-particle Green's function, which at finite temperatures $kT > \hbar\omega$ will have an infrared cutoff $\sim kT$ rather than $\hbar\omega$ [no extraneous cutoff such as $(\hbar/\tau)$ enters the expressions, since they converge at the low-frequency end]. Thus our interpretation of the c-axis conductivity is that it follows, over most of the range, a power law

$$\sigma_c \propto \omega^{2\alpha},$$

and in Figs. 1 and 2 we plot the c-axis results of Ando *et al.* [1] as $\ln\sigma$ vs $\ln T$ to show an approximate fit, with $2\alpha = 0.35$ ($\delta = 0.08$) and 0.22 ($\delta = 0.13$). At higher temperatures $T > t_\perp$, σ_c eventually crosses over to a positive temperature coefficient. $t_\perp$ is several hundred degrees K or greater according to band calculations [3].

There is some question in our minds whether the expression (1) is valid over the whole range of temperature. Its source is simply the golden rule: the joint density of states for hopping between two Luttinger liquids is $\propto\omega^{2\alpha}$, and the hopping rate is proportional to the density of states. But when the density of states is varying rapidly compared to the calculated decay rate, it is not obvious that the rate should continue to scale in the same way. Some experimental data support the present interpretation, some the possibility that the rate scales to a constant.

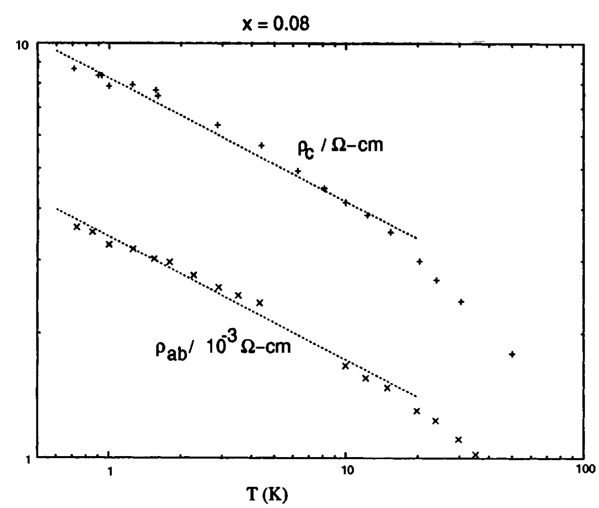

FIG. 1. Fit of $T^{-2\alpha}$ to resistivity of $La_{2-x}Sr_xCuO_4$ at 60 T: $x = 0.08$.

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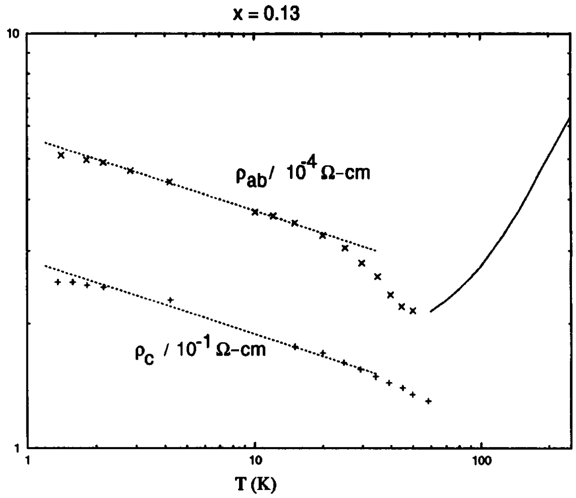

FIG. 2. Same as Fig. 1 except $x = 0.13$.

A second problem is that although the power law is acceptable according to any reasonable estimate of the errors of these quite complex experimental measurements, it is not by any means the best fit, which appears to be $\ln T$.

An additional experimental caveat is the gigantic magnetic field the experimentalists find necessary to suppress superconductivity. Experimentally, magnetoresistance in this regime seems quite small, and this is rather a problem for theory. The field, amounting to a $g\mu_B H$ of about 20–60 K, surely suppresses any tendency towards spin-charge separation below that temperature. However, we will argue that the intrinsic scattering rate is of the same order of magnitude and has already suppressed spin-charge separation below the crossover from $\rho \propto T$ to $\rho \propto T^{-\alpha}$. (See below.)

The remarkable observation of Ando *et al.* is that as $T \to 0$ $\sigma_{ab} \propto \sigma_c$. From the Luttinger liquid theory we predict a crossover phenomenon in σ_{ab}. The essence of Luttinger liquid transport theory is summarized in Ref. [4]. (See also [5].) There are 3 regimes (see Fig. 3) for in-plane transport, crossing over from one regime to another with increasing elastic scattering rate $\hbar/\tau_{\mathrm{el}}$ or decreasing temperature kT.

The regime characteristic of the usual normal state of the cuprates is the "holon nondrag" regime, the middle one of the three. In this regime the fastest dissipative process is the decay of the electron into spinon + holon,

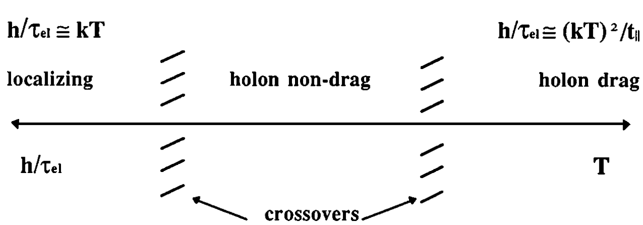

FIG. 3. Regimes of transport theory of composite electrons. T increases to the right, $\hbar/\tau_{\mathrm{el}}$ to the left.

at a rate $\propto (kT, \hbar\omega)$. The holons are then incoherently scattered at a rate $1/\tau_{\mathrm{el}}$ with

$$kT > \hbar/\tau_{\mathrm{el}} > (kT)^2/E_F\,, \tag{2}$$

the latter being the rate of the inverse process holon + spinon $\to$ electron (by detailed balance). Under these circumstances the holons lose their momentum before they are able to recombine, and the rate of current decay is simply kT. As shown in Refs. [5,6] σ has a small power-law correction, $\sigma \propto \omega^{-1+2\alpha}$; the expression is

$$\sigma = \frac{e^2}{\hbar}\,\frac{\alpha}{\pi}(\Lambda/\omega)^{1-2\alpha}. \tag{3}$$

Here as elsewhere we use $\hbar\omega$ and kT interchangeably in the power-law dependences. The ω dependence of (3) is evident in the infrared data quoted in Ref. [5]. Note that under the condition (2) this conductivity is *less* than it would have been under impurity scattering alone; i.e., spinon-holon decay is an extra dissipative mechanism. It is anomalous in that Matthiesson's rule fails, and the two mechanisms are not additive.

The other regime of interest here is the "localizing" regime, where $kT < \hbar/\tau_{\mathrm{el}}$. Here the electron has no time to decay before scattering, so charge-spin separation is no longer relevant: The transport properties are those of electrons. However, we still assume that the Green's function in the absence of scattering would exhibit anomalous scaling with x, t or conversely with k, ω,

$$\begin{aligned} G(r,t) &= t^{-1-\alpha} \qquad F\left(\frac{x}{t}\right), \\ G(k,\omega) &= \omega^{-1+\alpha} \qquad F'\left(\frac{k}{\omega}\right). \end{aligned} \tag{4}$$

The scattering processes are sufficiently frequent that they can be treated as matrix elements connecting whole electron states, i.e., simply as one-electron operators between different channels of electron propagation at different points on the Fermi surface.

We argue that in two dimensions, as opposed to one, there is no severe renormalization of the actual scattering potential such as was described by Kane and Fisher [7]. In one dimension, the singular $2k_F$ response enhances or screens the $2k_F$ scattering matrix elements and leads to an infinite renormalization, but this effect is not serious for scattering around a 2D Fermi surface, where at the general Q vector there is no singular response. The Kane-Fisher effect is linear in η, the phase shift caused by the interaction, hence is of opposite sign for attractive and repulsive potentials and overwhelms in 1D the effect we shall discuss.

In 2D the effect of singular interactions is to modify the propagation of electrons between scatterings. In a sense, the Fermi surface exponent α can be thought of as modifying the dimensionality of the scattering problem. For small η it is always positive and proportional to η^2.

Localization in a Luttinger liquid is a complex problem, and the following is only a first attempt at a discussion, expected to be valid only in the "classical" limit $g = \sigma\hbar/e^2 \gg 1$ of weak scattering (fortunately this condition is satisfied in our case).

In this limit we can draw a close analogy between the calculation by Clarke and Strong [2] of the c-axis conductivity, and the impurity scattering problem of ab-plane conductivity, using the generalized Landauer formula [8]

$$\sigma = \frac{e^2}{\hbar} \sum_{ij} |t_{ij}|^2, \tag{5}$$

where $\sum_{ij}$ is a sum over outgoing and incoming channels, respectively. In any system approaching macroscopic dimensions, any one matrix element t_{ij} is small, since every incoming channel connects randomly with every outgoing one: the electrons are certain to have been scattered. In the weak scattering case we can think of the channels as labeled by directions k_F on the Fermi surface.

The question then is: After scattering, what is the amplitude for the electron to arrive in the final channel as a coherent entity? The final, outgoing channel will normally be at a different k vector on the Fermi surface from the initial one, so the strong forward-scattering interaction which causes spin-charge separation and anomalous dimensionality does not act between initial and final states. The relevant diagram is as shown in Fig. 4, with no vertex connections to t_{ij}. But this is the identical diagram to that calculated by Clarke and Strong. In essence, we think of the impurity scattering as a barrier between ingoing and outgoing channels, in exact correspondence to the barrier between layers, with a weak transmission coefficient t_{ij} which can be treated in lowest-order perturbation theory. The result of that treatment is

$$\sigma \propto (\hbar\omega, kT)^{2\alpha},$$

as for the c-axis conductivity. This can be normalized to the value of conductivity at the crossover temperature to give us a quantitative formula,

$$\sigma_{ab} \simeq \frac{e^2}{\hbar}\,\frac{\alpha}{\pi}\left(\frac{\Lambda\tau_{\rm el}}{\hbar}\right)\left(\frac{kT}{\Lambda}\right)^{2\alpha}. \tag{6}$$

In Figs. 1 and 2 we give a log-log plot of the ab-plane conductivity observed by Ando *et al.*, showing the crossover (measurements at high T are inferred from other work) [9]. Figure 5 shows data, also agreeing roughly with (6), on the one-layer BISCO compounds, taken by Ong's group [10]. This also seems to fit Eq. (6) but with quite small values of (2α).

Comparing (6) and (1), we obtain an expression for the anisotropy (factors of order 1 are possibly missing, since we have no reason to assume Λ is the same in the two formulas),

$$\frac{\sigma_{ab}}{\sigma_c} = \frac{1}{\pi}\left(\frac{\Lambda\tau_{\rm el}}{\hbar}\right)\frac{t_\parallel^2}{t_\perp^2}. \tag{7}$$

The extra factor $\Lambda\tau_{\rm el}/\hbar = \Lambda/(kT)_{\rm crossover} \simeq 100$ accounts for the enormous numerical value of the anisotropy, which is far too great to be a ratio of masses or Fermi surface areas.

A final note on the crossover to localization. At this crossover the mechanism we have proposed for the temperature-dependent Hall effect, which, when analyzed, requires that the spinon and holon currents be noncolinear, ceases to hold, and in the localizing regime we expect

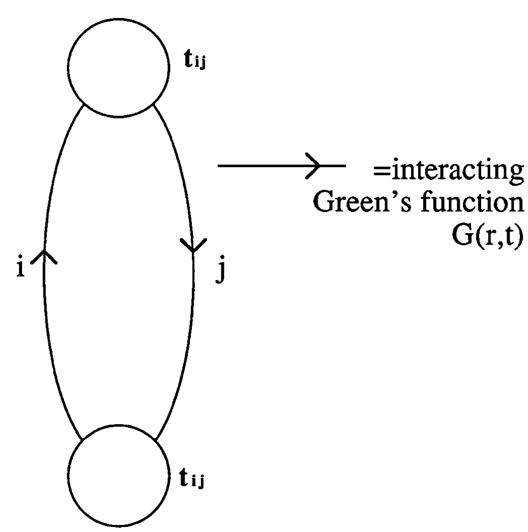

FIG. 4. Diagram for conductivity in the "holon nondrag" regime.

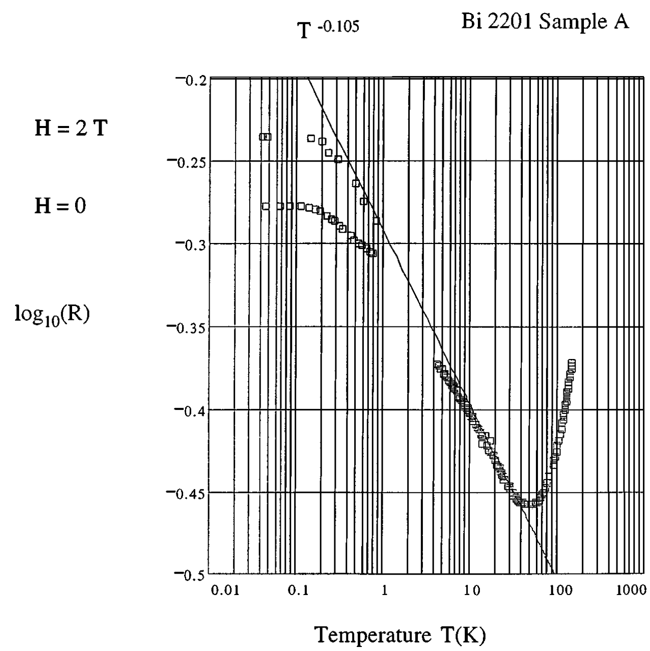

FIG. 5. Another example of fit of $T^{-2\alpha}$ to resistivity. Data of Ong *et al.* on BISCO 2201.

the Hall resistivity to become temperature independent following conventional theory. The Hall angle $\omega_c \tau$, then, ceases to follow the T^{-2} behavior characteristic of the holon nondrag regime, and instead crosses over to

$$\Theta_H = R_{xy}/p_{xx} \propto T^{2\alpha}. \tag{8}$$

This would be a spectacular effect and should be investigated. Some evidence that $R_{xy} \longrightarrow$ const does exist, in agreement with (8) [11].

The above contains some aspects which are admittedly conjectured, and some which appear to be solid. As we have said, it is conjectured that the interplanar or interchannel conductivity scales as $\omega^{2\alpha}$ even below $\omega \sim \Gamma$, the decay rate. The experiments may be the best evidence on this matter. It is also problematic what effect a spin-dependent energy might have on α, and this could be the source of the poor fit to $T^{2\alpha}$. It does not seem as conjectural that the crossover from holon nondrag to localizing regime is responsible for the striking change in behavior, or, controversial as it may seem, that the essence of the physics in the localizing regime is channel-channel scattering, which fact is responsible for the parallel T dependence.

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[11] Since this Letter was submitted (and rejected) data in strong support of this prediction have accumulated. Y. Ando, private communication.