Collective Excitations of a Bose-Einstein Condensate in a Dilute Gas

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We observe phononlike excitations of a Bose-Einstein condensate (BEC) in a dilute atomic gas. ^{87}Rb atoms are optically trapped and precooled, loaded into a magnetic trap, and then evaporatively cooled through the BEC phase transition to form a condensate. We excite the condensate by applying an inhomogeneous oscillatory force with adjustable frequency and symmetry. We have observed modes with different angular momenta and different energies and have studied how their characteristics depend on interaction energy. We find that the condensate excitations persist longer than their counterparts in uncondensed clouds. [S0031-9007(96)00722-3]

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An important consequence of quantum statistics is that, below some critical temperature, bosons are predicted to pile up in the lowest energy state of a system [1]. This macroscopic quantum phenomenon, termed Bose-Einstein condensation (BEC), has recently been observed in a dilute atomic vapor [2,3]. This condensation provides the basis for theoretical understanding of unusual quantum phenomena such as superfluidity in liquid helium and superconductivity in metals. In these systems, however, strong interactions among the constituent particles fundamentally alter the features of the BEC and considerably complicate any theoretical analysis. In dilute atomic vapors the interactions are primarily binary and can be treated theoretically within an s-wave scattering approximation. The exact shape of the interparticle potential is ignored and a single parameter, a, the s-wave scattering length, characterizes the interactions. Thus, even with interactions, the system is amenable to theoretical understanding. We report herein on studies of low energy, collective excitations of a dilute condensate of ^{87}Rb. Characterizing these low-lying excitations is a first step towards understanding the dynamics of this novel quantum fluid.

The apparatus and procedures we use for creating BEC are described elsewhere [2,4]. In summary, we optically precool and trap ^{87}Rb atoms, then load them into a purely magnetic trap. We use a time-averaged orbiting potential (TOP) trap consisting of a static quadrupole field plus a small rotating transverse bias field [4]. The effective average potential is axially symmetric and harmonic, with a ratio of axial to radial (or equivalently, "transverse") trapping frequencies of $\sqrt{8}$. We further cool the atoms in the TOP trap with forced evaporation by applying a radio frequency (rf) magnetic field to selectively induce Zeeman transitions to untrapped spin states [5].

We observe the atom cloud with absorption imaging. A 26 μs pulse of light resonant with the $5S_{1/2}, F = 2$ to $5P_{3/2}, F = 3$ transition illuminates the atoms, while a lens system with a resolution of 6 μm FWHM images the shadow of the cloud onto a charge-coupled device array. The image is then digitized and stored for analysis. The probe geometry is such that we get both radial and axial sizes from a single image. By allowing the cloud to expand ballistically before taking the picture, we improve the imaging resolution [2]. Improvements in the procedure used in Ref. [2] now allow us to turn off the trap potential rapidly enough to allow free expansions from traps with any radial frequency between 9 and 308 Hz [6]. Our imaging procedure is destructive, but by repeating the cycle of loading, cooling, and imaging, we study time evolution of the cloud.

The final evaporation takes place in a trap with a radial frequency of 132 Hz (373 Hz axial). To approach the zero-temperature limit, we evaporate well below the BEC phase transition [7] so that the expanded clouds show no sign of having a thermal component. At higher temperatures, this component appears as a broad, symmetric Gaussian background [2]. Typically we have 4500 ± 300 atoms in the condensate.

The standard theory for BEC in a dilute atomic vapor uses a nonlinear Schrödinger equation, the Gross-Pitaevskii equation, to describe the condensate wave function in the limit of zero temperature [8]. This equation comes from a second-quantized mean-field theory with the interatomic interactions modeled by an s-wave scattering length. The elementary excitations of BEC in a finite, harmonically confined dilute gas have been studied theoretically using this model [9–12]. The lowest energy normal modes of the condensate, corresponding to rigid-body center-of-mass motion ("sloshing"), are predicted to occur at the trap frequencies. The frequencies of the next lowest condensate excitation modes, however, are expected to deviate from the spectrum of a cloud of ideal gas, for which the excitation frequencies are simply multiples of trap frequencies. Not surprisingly, the amount of deviation depends on the strength of the interatomic interactions.

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We excite these collective modes of the condensate by applying a small time-dependent perturbation to the transverse trap potential. We generate the perturbations by applying a sinusoidal current to the coils responsible for the rotating field of our TOP trap (in addition to the normal TOP currents). The response of the condensate depends on the symmetry of the driving force as well as the driving frequency ν_d. By appropriately setting the phases of the currents through the coils, we can generate perturbations with either of two different symmetries. We label these two driving symmetries $m = 0$ and $m = 2$, where m, the angular momentum projection onto $\hat{z}$, is a good quantum number because of the axial symmetry of our unperturbed magnetic trap. Equipotential contours for the two trap perturbations are shown in Fig. 1. The $m = 0$ drive preserves axial symmetry and corresponds to an oscillation in radial size. For the $m = 2$ drive symmetry, the trap spring constants along $\hat{x}$ and $\hat{y}$ are modulated 90° out of phase. This corresponds to a normal mode resembling a transverse ellipse whose major axis rotates in the x-y plane.

The basic spectroscopic approach is as follows: We distort the cloud by applying the perturbative drive for a short time, then allow the cloud to evolve freely in the unperturbed trap for a variable length of time. Finally, we turn off the confining potential suddenly and image the resulting cloud shape after 7 ms of free expansion.

Initial studies were made in a 132 Hz trap. The perturbative drive pulse duration was 50 ms, the center frequency was set to match the frequency of the excitation being studied, and the amplitude was 1.5% of the radial spring constant of our trap. We observe two different collective excitations of the condensate. The observables in both cases are the widths of the expanded clouds as a function of the free evolution time. In one case we observe a sinusoidal oscillation of the radial width at a frequency of $(1.84 \pm 0.01)\nu_r$, where ν_r is the radial trap frequency and the error quoted reflects only statistical uncertainties. This mode is driven by the axially symmetric $m = 0$ trap perturbation and is not observed for the drive with $m = 2$ symmetry (with the same drive amplitude and frequency). The cloud widths oscillate in both axial and radial directions, with approximately opposite phase. This response is shown in Fig. 2. The observed phase difference between the oscillations of axial and radial widths is not exactly π; however, the free expansion of the condensate prior to imaging complicates analysis of this phase shift [13]. The second excitation oscillates freely at $(1.43 \pm 0.01)\nu_r$, and appears in response to an $m = 2$ drive, and not to an $m = 0$ drive. In this case, the radial width oscillates, with no observable response in the axial width. The two-dimensional projection of an elliptical cloud whose major axis rotates in the transverse plane would exhibit this behavior.

We calibrate the observed excitation frequencies in units of the trap frequencies by making similar measurements on noncondensate clouds. The temperature of these clouds, in units of the BEC transition temperature, is $T/T_c \approx 1.3$; consequently, the density and correspondingly the interactions are very small. Here, we see a response that oscillates at 264 Hz and can be driven with either symmetry. A harmonically confined, noninteracting gas pulses at twice the radial trap frequency, so this gives $\nu_r = 132 \pm 1$ Hz. We have also checked that the thermal cloud does not respond when driven at $1.43\nu_r$.

In the s-wave scattering approximation, the interactions for ground state ^{87}Rb atoms in our trap are repulsive (positive scattering length) [14], providing an effective potential energy which favors a lower central density of

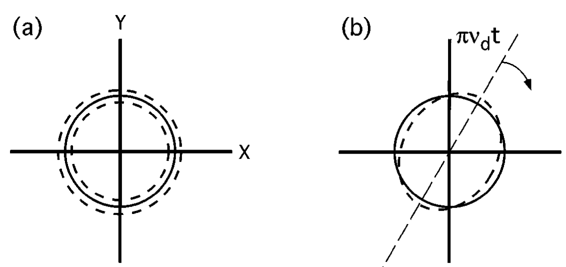

FIG. 1. In the unperturbed trap, contours of equipotential in the transverse plane are symmetric (solid line). To drive the $m = 0$ excitation (a) we apply a weak harmonic modulation with frequency ν_d to the trap radial spring constant. The $m = 2$ drive (b) breaks axial symmetry with elliptical contours which rotate at $\nu_d/2$. The amplitude of perturbation is shown exaggerated for clarity.

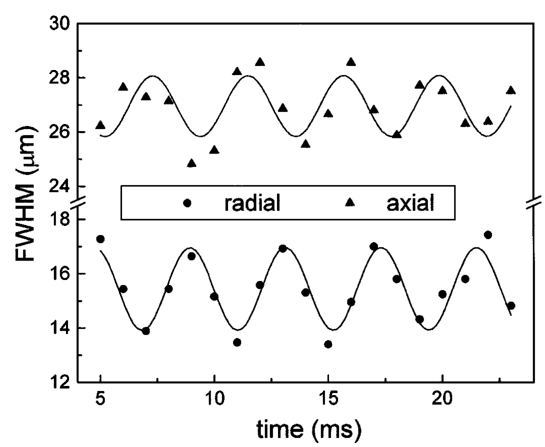

FIG. 2. We apply a weak $m = 0$ drive to an $N \approx 4500$ condensate in a 132 Hz (radial) trap. Afterward, the freely evolving response of the condensate shows radial oscillations. Also observed is a sympathetic response of the axial width, approximately 180° out of phase. The frequency of the excitation is determined from a sine wave fit to the freely oscillating cloud widths. Each data point represents a single destructive condensate measurement.

the condensate compared to the noninteracting case. This interaction energy determines the excitation spectrum of the condensate [15]. We can examine this effect because BEC in trapped neutral atoms offers the advantage of an adjustable interaction energy. In the standard mean-field picture, the strength of the nonlinear interaction term in the Gross-Pitaevskii equation, relative to the harmonic trap's energy-level spacing, scales with $Na\sqrt{\nu_r}$ [16]. Thus, by varying the trap frequency or the number of atoms N we can change the relative importance of interactions in the condensates.

We measure the excitation frequencies of the $m = 0$ and $m = 2$ modes of the condensate as a function of both N and ν_r. In the 132 Hz trap, we change the relative interaction strength by reducing the number of atoms. To change ν_r, we evaporate to BEC in the 132 Hz trap, then adiabatically ramp the trap fields until the condensate is held in a trap with an axial frequency of 43.2 Hz. In this lower frequency trap, we excite the condensate with a 100 ms pulse and a drive amplitude equal to 3% of the radial spring constant. The observed fractional amplitude of the oscillations in the cloud width (approximately 11% of the mean width) for this drive is the same as observed in the 132 Hz trap. We measure the free oscillation frequency of the $m = 0$ and $m = 2$ modes in this trap to be $(1.90 \pm 0.01)\nu_r$ and $(1.51 \pm 0.01)\nu_r$, respectively, for $N \approx 3000$.

The measured excitation frequencies as a function of interaction strength are shown in Fig. 3. By using the product $N\sqrt{\nu_r}$ for the dependent variable we combine our different number and trap frequency data into one graph. The solid lines in Fig. 3 show the mean-field theory calculation by Edwards *et al.* [9], using the current best value of the scattering length for ground state ^{87}Rb atoms, $a = 110a_0$ [14], where a_0 is the Bohr radius. An extension of this calculation by Esry and Greene is shown with dotted lines [10]. Finally, dashed lines indicate the prediction by Stringari for the "strongly interacting" limit [11], in which the kinetic energy of the ground state is ignored. Our data agree reasonably well with these mean-field theory results; the measured energies of the low-lying collective excitations of the condensate deviate from the simple harmonic trap spectrum as predicted, with larger deviation for larger interaction strength. Error bars in Fig. 3 indicate statistical error in the determination of the frequencies, but do not include possible systematic errors such as day-to-day variations in the trap magnetic fields, and therefore frequencies (estimated to be less than 0.5%). Also, the theoretical curves are strictly valid only in the limit of zero temperature and zero amplitude.

In the limit of low energy, the spectrum of low-lying collective excitations corresponds exactly to the Bogoliubov quasiparticle spectrum [9]. The collective condensate response to our trap perturbation, in the limit of low amplitude, is simply a coherent state of these elementary excitations. To explore the question of whether or not our experiments are performed in this limit, we measure the condensate response for different driving force amplitudes. In this test, we drive the $m = 2$ mode in the 132 Hz trap, with $N \approx 4500$. The results are shown in Fig. 4 where we plot the frequency of the oscillating radial width as a function of the amplitude of that response. The solid line shows a fit with a parabola, a form which describes an oscillator with anharmonic terms. As our measurements of excitation frequency were performed for a response amplitude between 9% and 14%,

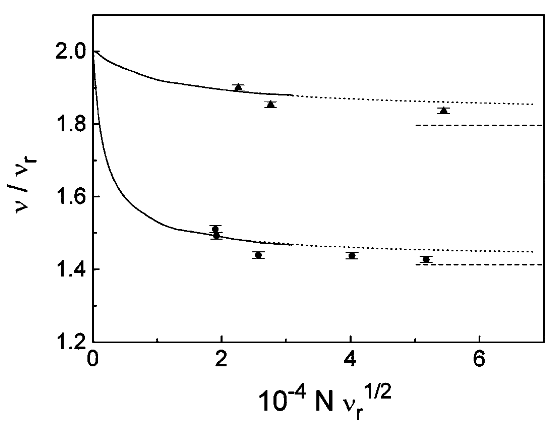

FIG. 3. We measure the frequency of the $m = 0$ (triangles) and $m = 2$ (circles) condensate modes as a function of interaction strength. The relative interaction strength in the condensate varies as the product of number of atoms, N, and the square root of the radial trap frequency, ν_r. Solid lines show the mean-field calculation by Edwards and co-workers [9], dotted lines show the results of similar calculations by Esry and Greene [10], and dashed lines show the prediction by Stringari for the strongly interacting limit [11].

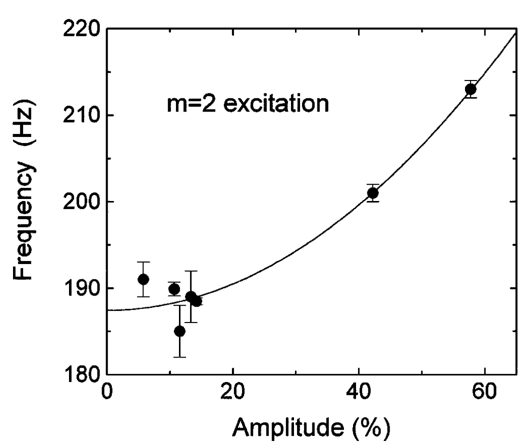

FIG. 4. The freely oscillating frequency of the condensate is shown as a function of response amplitude. The condensates, consisting of 4500 atoms, are held in a 132 Hz radial frequency trap and driven with $m = 2$ symmetry. The solid line shows a parabolic fit to the data.

which causes a shift of only 1% in the frequency, these data suggest that we are in the regime where the measured spectrum corresponds quite closely to the elementary excitations of BEC in a dilute gas.

Finally, we examine the damping of a condensate excitation. For comparison purposes we first study the damping in a noncondensed thermal cloud ($T/T_c \approx 1.3$) in the 132 Hz trap. We excite the 264 Hz $m = 0$ mode, because damping in this mode is not influenced by angular momentum conservation. We fit a sine wave with an exponentially decaying amplitude to the observed oscillations in the radial cloud width. This gives an excitation lifetime of 49 ± 13 ms. Since the mean free path in these clouds is long compared to the excitation wavelength and the effect of the trap anharmonicity is small, the excitation lifetime should scale inversely as the atom-atom collision rate. This rate in turn scales with the product of the density times the velocity of the atoms. For a given harmonic oscillator confining potential, this collision rate is proportional to the optical depth of the cloud. Using this scaling principle we predict that the damping lifetime in a classical cloud with the same optical depth as the condensate would be 28 ± 8 ms.

But when we perform the same experiment on the 4500 atom condensate we obtain an excitation lifetime of 110 ± 25 ms. Thus, the condensate excitation persists nearly four times longer than can be explained in a classical picture.

In summary, we have observed low-lying collective excitations of BEC in a dilute atomic vapor. Both $m = 0$ and $m = 2$ modes were identified, and their frequencies measured as a function of relative interaction strength. The data were taken in a linear regime where the collective modes should correspond to the elementary excitations of BEC in this system, and reasonable agreement was found between the experiment and mean-field theory results. The damping lifetime of the $m = 0$ excitation was measured and found to be significantly longer than the prediction of a classical model. We believe further study of these elementary excitations, particularly at different temperatures, will help deepen the understanding of the quantum phenomena of Bose-Einstein condensation of a gas.

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