Analysis of the Z^0 Resonant Amplitude in General R_ξ Gauges

Massimo Passera[1,*] and Alberto Sirlin[2,†]
[1]*Physics Department, Brookhaven National Laboratory, New York, New York 11973-5000*
[2]*Max-Planck-Institut für Physik, Föhringer Ring 6, 80805, München, Germany*
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The Z^0 resonant amplitude is discussed in general R_ξ gauges. When the original on-shell definition of the Z^0 mass M is employed, a gauge dependence of M emerges in the next-to-leading approximation which, although small, is of the same magnitude as the current experimental error. In the following order of expansion, these unphysical effects are unbounded. The gauge dependence of M disappears when modified, previously proposed definitions of mass or self-energies are used. The relevance of these considerations to the concept of the mass of unstable particles is pointed out. [S0031-9007(96)01543-8]

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Ever since LEP (Large Electron Positron Collider) commenced operations, major efforts have been devoted by experimentalists and theorists to study the Z^0 line shape. Indeed, this important observable leads to the determination of some of the most fundamental parameters in electroweak physics, namely the Z^0 mass, its width, and the cross sections at the peak. Of primary interest in the theoretical side of these studies is the structure of the transverse part of the dressed Z^0 propagator

$$D(s) = [s - M_0^2 - \mathrm{Re}A(s) - i\mathrm{Im}A(s)]^{-1}, \qquad (1)$$

where $s = q^2$ is the squared four-momentum transfer, M_0 is the bare mass, and $A(s)$ is the conventional Z^0 self-energy, which we have split into its real and imaginary components.

In the original formulation of the on-shell method of renormalization [1], the physical mass M was related to the bare mass by means of the expression

$$M^2 = M_0^2 + \mathrm{Re}A(M^2), \qquad (2)$$

so that the mass counterterm $\delta M^2 = M^2 - M_0^2$ was identified with $\mathrm{Re}A(M^2)$. Recalling that counterterms in field theory are real, Eq. (2) is the simplest generalization to unstable particles of the well known expression $m_e = m_e^0 + \Sigma(\not{p} = m_e)$ for the mass renormalization of the electron in QED. In particular, Ref. [1] contains a rather detailed discussion of the gauge invariance of the resulting one-loop corrections investigated in that work. Since 1980 Eq. (2) has been adopted by many theoretical physicists and, in fact, several standard analyses of the Z^0 line shape are based on such a definition (see, for example, Refs. [2,3]). However, it was later pointed out that, in spite of its simplicity and usefulness in the evaluation of the one-loop corrections, Eq. (2) contains theoretical limitations in higher orders of perturbation theory. Using special arguments, it was concluded in Refs. [4,5] that the use of Eq. (2) would generate gauge-dependent electroweak corrections in $O(g^6)$ and, in a restricted class of R_ξ gauges, even in $O(g^4)$. Since the pioneering work of Wetzel [6], effects of $O(g^4)$ are routinely incorporated in the analysis of $D^{-1}(s)$. In fact, in the resonance region $|s - M^2| \lesssim M\Gamma$, and therefore the leading contributions in $D^{-1}(s)$ are of $O(g^2)$. Thus, in the next-to-leading order (NLO) approximation employed by theorists to study the resonant amplitude, one must retain all contributions of $O(g^4)$ in $D^{-1}(s)$. Contributions of $O(g^6)$ in $D^{-1}(s)$ should be taken into account when one expands $D(s)$ around the resonance in order to obtain the corresponding contribution to the nonresonant amplitude. So far the gauge dependence induced by Eq. (2) has not been explicitly demonstrated in the analysis of the line shape. Fortunately, it was also anticipated in Refs. [4,5] that the $O(g^4)$ gauge dependence is numerically bounded, so that the effect in this order is expected to be small.

In this Letter we reexamine the electroweak corrections to Eq. (1) in the resonant region. Unlike previous studies, we work in the framework of general R_ξ gauges. Our aim is threefold: (i) to explicitly show that, as anticipated in Refs. [4,5], a gauge dependence emerges when Eq. (2) is employed, (ii) to evaluate its magnitude in $O(g^4)$, and (iii) to explicitly show that the gauge dependence disappears when previously proposed, modified definitions of M or $A(s)$ are employed.

We first discuss Eq. (1) in the NLO approximation. Inserting Eq. (2) into Eq. (1) and recalling that in the resonance region $s - M^2$ is $O(g^2)$, we have

$$D^{-1}(s) = (s - M^2)[1 - \mathrm{Re}A'(M^2)] - i\mathrm{Im}A(M^2) - i\mathrm{Im}A'(M^2)(s - M^2) + O(g^6). \qquad (3)$$

For $\mathrm{Im}A(M^2)$ we can employ the unitarity relation

$$-i\mathrm{Im}A(M^2) = iM\Gamma[1 - \mathrm{Re}A'(M^2)], \qquad (4)$$

where Γ is the radiatively corrected width. As $s - M^2 = O(g^2)$ it is sufficient to evaluate $\mathrm{Im}A'(M^2)$ in the last term to one-loop order. Calling A_f and A_b the fermionic and bosonic contributions, we have

$$\mathrm{Im}A_f'(M^2) = -\Gamma/M - (3/4\pi\sqrt{2})G_\mu m_b^2. \qquad (5)$$

The last term, which is very small, arises from the leading violation to the scaling behavior $\mathrm{Im}A_f(s) \sim s$, due to the finiteness of the b quark mass m_b. Its magnitude can be gleaned, for example, from Eq. (25) of Ref. [7] or from Ref. [3]. To evaluate $\mathrm{Im}A_b'(M^2)$ in the R_ξ gauges, we need the expression of the one-loop self-energies in such a general framework. They are given in compact form in Ref. [8]. In the analysis we restrict ourselves to $\xi_w \geq 0$, where ξ_w is the W-gauge parameter. Negative values of ξ_w are not tenable for at least two reasons: (i) as is well known, the Euclidean path integral representation of the generating functional does not exist in this case (i.e., it does not converge) and (ii) for $\xi_w < 0$ the unphysical scalar mass $m_{\phi^\pm}$ becomes imaginary and the $W^\pm$ and $\phi^\pm$ propagators develop pathological singularities at spacelike values of k^2. Taking this restriction into account and evaluating the corresponding imaginary parts through $O(g^2)$, we find

$$\begin{aligned}\mathrm{Im}A_b'(M^2) = -\theta[(4c_w^2)^{-1} - \xi_w](\alpha/24c_w^2 s_w^2) \\ \times (1 - 4c_w^2\xi_w)^{3/2} + \theta[(c_w^{-1} - 1)^2 - \xi_w] \\ \times (\alpha/12c_w^2)(a^2 - 4c_w^2)^{1/2}(a^2 + 8c_w^2)\,,\end{aligned} \quad (6)$$

where $c_w \equiv M_w/M_z$ and $a \equiv 1 + c_w^2(1 - \xi_w)$. To understand the origin of these gauge-dependent contributions, we recall that the relevant unphysical fields have masses $M_w\xi_w^{1/2}$. When ξ_w is sufficiently small, $A_b(s)$ develops imaginary parts in the neighborhood of $s = M^2$, which contribute gauge-dependent contributions to $\mathrm{Im}A_b'(M^2)$. We expect such contributions when $M_z \geq 2M_w\xi_w^{1/2}$, i.e., $\xi_w \leq (4c_w^2)^{-1}$ and when $M_z \geq M_w(1 + \xi_w^{1/2})$, i.e., $\xi_w \leq (c_w^{-1} - 1)^2$. They correspond to the two terms on the right-hand side of Eq. (6). Inserting Eqs. (4)–(6) in Eq. (3), we rewrite this expression in the form

$$\begin{aligned}D^{-1}(s) = [1 - \mathrm{Re}A'(M^2) - i\Phi](s - \tilde{M}^2 + is\tilde{\Gamma}/\tilde{M}) \\ + O(g^6)\,,\end{aligned} \quad (7)$$

$$\Phi \equiv \mathrm{Im}A_b'(M^2) - (3/4\pi\sqrt{2})G_\mu m_b^2\,, \quad (8)$$

$$\tilde{M}^2 \equiv M^2(1 + \Phi\Gamma/M)\,, \quad (9)$$

$$\tilde{\Gamma}/\tilde{M} \equiv \Gamma/M\,. \quad (10)$$

The gauge dependence of $\mathrm{Re}A'(M^2) + i\Phi$ in Eq. (7) cancels against corresponding contributions in the vertex functions. The very small scaling violation in $i\Phi$ is shifted to higher orders when evaluating $|D(s)|^2$ [3]. The amplitude $(s - \tilde{M}^2 + is\tilde{\Gamma}/\tilde{M})$ has the characteristic s-dependent Breit-Wigner form employed in the LEP analysis. It is clear, however, that what LEP measures is $\tilde{M}$ rather than M. As Φ in Eq. (8) contains the gauge-dependent contribution $\mathrm{Im}A_b'(M^2)$, it follows from Eq. (9) that M is gauge dependent. To evaluate the magnitude of the gauge dependence we note that the maximum value of $|\mathrm{Im}A_b'(M^2)|$ in Eq. (6) occurs at $\xi_w = (c_w^{-1} - 1)^2$ (the threshold of the second contribution), in which case $\mathrm{Im}A_b'(M^2) = -1.6 \times 10^{-3}$. On the other hand, $\mathrm{Im}A_b'(M^2) = 0$ for $\xi_w \geq (4c_w^2)^{-1}$. Thus the maximum shift in M due to the gauge dependence is $|\delta M| = 1.6 \times 10^{-3}\Gamma/2 = 2$ MeV. Although this is a small effect, it is of the same magnitude as the current experimental error $\Delta M = \pm 2.2$ MeV [9].

In order to circumvent the gauge dependence generated by Eq. (2) in higher orders, an alternative definition for the Z^0 mass was proposed in Refs. [4,5] which in $O(g^4)$ differs from Eq. (2) in very small but gauge-dependent terms. Calling $\bar{s} = m_2^2 - im_2\Gamma_2$ the complex-valued position of the pole in $D(s)$, so that $\bar{s} - M_0^2 - A(\bar{s}) = 0$, the physical mass and width were identified with

$$m_1^2 = m_2^2 + \Gamma_2^2, \qquad \Gamma_1/m_1 = \Gamma_2/m_2\,, \quad (11)$$

Noting that $m_2^2 = M_0^2 + \mathrm{Re}A(\bar{s})$, Eq. (11) corresponds to a mass counterterm

$$\delta m_1^2 = m_1^2 - M_0^2 = \mathrm{Re}A(\bar{s}) + \Gamma_2^2\,. \quad (12)$$

Through $O(g^4)$ this becomes $\delta m_1^2 = \mathrm{Re}A(m_1^2) + \Phi m_1\Gamma_1 + O(g^6)$. The additional $O(g^4)$ term $\Phi m_1\Gamma_1$ has important consequences. Repeating the previous analysis one readily finds

$$\begin{aligned}D^{-1}(s) = [1 - \mathrm{Re}A'(m_1^2) - i\Phi](s - m_1^2 + is\Gamma_1/m_1) \\ + O(g^6)\,.\end{aligned} \quad (13)$$

Again, the gauge dependence in $\mathrm{Re}A'(m_1^2) + i\Phi$ cancels against corresponding contributions in the vertex parts, and the scale violation in $i\Phi$ is promoted to higher orders in $|D(s)|^2$. It is important to note that, in contrast with Eq. (7), m_1 in the resonant factor is not modified. As a consequence, m_1 can be directly identified with the mass measured at LEP. In fact, there is an alternative and more elegant way for deriving Eq. (13), already outlined in Ref. [4]. Recalling that $\bar{s}$ is the position of the pole we can write $D(s) = [s - \bar{s} - A(s) + A(\bar{s})]^{-1}$, the resonant part of which is $D^{\mathrm{res}}(s) = (s - \bar{s})^{-1}[1 - A'(\bar{s})]^{-1}$. Multiplying and dividing by $1 + i\Gamma_2/m_2$ and neglecting terms of higher order in the cofactor $(1 + i\Gamma_2/m_2)/[1 - A'(\bar{s})]$ one obtains once more Eq. (13). This shows that the s-dependent Breit-Wigner form automatically emerges when the large imaginary part $-i\mathrm{Im}A'(m_2^2)$ in the pole's residue is absorbed into the resonant factor.

In order to obtain the leading nonresonant contribution, one expands $D^{-1}(s)$ to the next order in $s - M^2$. If Eq. (2) is employed we have

$$\begin{aligned}D^{-1}(s) = s - M^2 - i\mathrm{Im}A(M^2) - (s - M^2)A'(M^2) \\ - (s - M^2)^2A''(M^2)/2 + O(g^8)\,.\end{aligned} \quad (14)$$

To simplify the analysis we now consider the class of gauges $\xi_w \geq (4c_w^2)^{-1}$, within which the previous calculation was gauge independent, and further neglect the small scaling violation in the one-loop amplitude $\mathrm{Im}A_f(s)$. In this case we can set $\mathrm{Im}A''(M^2) = 0$ in Eq. (14). For

Im$A(M^2)$ we again employ the unitarity relation [Eq. (4)]. After some elementary algebra, Eq. (14) becomes

$$D^{-1}(s) = \left[\frac{1 - A'(M^2 - iM\Gamma)}{1 + i\Gamma/M}\right]\left(s - \hat{M}^2 + is\frac{\hat{\Gamma}}{\hat{M}}\right) \times [1 - \mathrm{Re}A''(M^2)(s - \hat{M}^2 + is\hat{\Gamma}/\hat{M})/2] + O(g^8), \quad (15)$$

$$\hat{M}^2 = M^2 - \delta, \qquad \hat{\Gamma}/\hat{M} = \Gamma/M, \quad (16)$$

$$\delta = -\mathrm{Im}A'(M^2)M\Gamma[1 + \mathrm{Re}A'(M^2)] - \Gamma^2 + \mathrm{Re}A''(M^2)M^2\Gamma^2/2. \quad (17)$$

In the first term of Eq. (17), Im$A'(M^2)$ is evaluated through $O(g^4)$. In $O(g^2)$, $-$Im$A'(M^2)$ equals Γ/M [cf. Eq. (5)]; thus the $O(g^4)$ terms cancel in Eq. (17) and δ is $O(g^6)$. This quantity δ already occurred in the arguments of Refs. [4,5], where it was pointed out that it is afflicted by an unbounded gauge dependence (i.e., it diverges in the unitary gauge). As the LEP measurement can be identified with $\hat{M}^2$, it follows from Eq. (16) that M, defined in Eq. (2), is also afflicted by an unbounded gauge dependence in $O(g^6)$. Once more the gauge dependence disappears if the physical mass is identified with Eq. (11). In fact, expanding Eq. (12) in the range $\xi_w \geq (4c_w^2)^{-1}$, one has $\delta m_1^2 = \mathrm{Re}A(m_1^2) - \delta + O(g^8)$. The additional term cancels the gauge dependence in the resonant factor of Eq. (15) and m_1 can be identified with the observed mass $\hat{M}$.

Aside from Refs. [4,5], a number of authors have advocated the idea of defining the Z^0 mass and width in terms of m_2 and Γ_2 [10]. All such proposals should also lead to correct, gauge invariant answers. One significant phenomenological difference is that the definitions in Eq. (11) lead to the s-dependent Breit-Wigner resonance employed in the LEP analysis, so that m_1 and Γ_1 can be identified with the LEP measurements. The other proposals, instead, differ numerically from such determinations by amounts much larger than the experimental error.

An alternative procedure to define a gauge invariant mass is to employ a gauge invariant self-energy in Eqs. (1) and (2), instead of the conventional amplitude. The possibility of using the pinch technique (PT) self-energy was suggested in Ref. [11]. Recently, there has been significant progress in the construction of the PT self-energies in higher orders [12–14]. In the expansion analogous to Eq. (3) the conventional self-energy A is replaced by its PT counterpart $\hat{A}$ which is ξ independent and, moreover, Im$\hat{A}_b'(M^2) = 0$. Thus, the gauge dependence does not arise and one obtains [14] an expression analogous to Eq. (7), with Re$A'(M^2) \to$ Re$\hat{A}'(M^2)$ and $\Phi \to \hat{\Phi} = -(3/4\pi\sqrt{2})G_\mu m_b^2$, a very small and gauge independent scale-breaking term. We have explicitly shown that the two methods described above eliminate the gauge dependence in the NLO approximation. However, the approach based on Eq. (11) also cancels the scale-breaking correction to the mass in Eqs. (8) and (9). In the Z^0 case this effect is extremely small, a shift $\approx$0.05 MeV in m_1. However, it may be more significant in other cases. Furthermore, in the PT approach the gauge independence of $D^{-1}(s)$ in the resonance region has so far been demonstrated only through $O(g^4)$ [14]. On the other hand, the PT analysis can be easily recast in a form very similar to Eq. (13), which conforms with the mass definition in Eq. (11) rather than Eq. (2). It suffices to note that the PT result in any gauge can be expressed as [14]

$$D^{-1}(s) = (s - \bar{s})[1 - \hat{A}'(\bar{s})] + O(g^6), \quad (18)$$

which is explicitly ξ independent and shows that the pole position is not displaced. Following the steps outlined after Eq. (13), one obtains an expression analogous to that equation with Re$A'(m_1^2) \to$ Re$\hat{A}'(m_1^2)$ and $\Phi \to \hat{\Phi}$. The difference with Eq. (13) is that the first factor (as well as the vertex parts) are now separately ξ independent. In other words, as far as it is presently known, in the PT approach one can define the mass either from the expression analogous to Eq. (2) (with $A \to \hat{A}$) or from Eq. (11).

Previous detailed studies of the Z^0 resonant amplitude have not uncovered the gauge dependence found in the present analysis when Eq. (2) is employed. The reason is that the gauge-dependent contribution involving Im$A_b'(M^2)$ is routinely disregarded, a procedure that is correct in the subclass of gauges $\xi_w \geq (4c_w^2)^{-1}$ (this includes the frequently employed 't Hooft-Feynman and unitary gauges). However, in the range $\xi_w \leq (4c_w^2)^{-1}$ (this includes the Landau gauge) one must consider such terms. Similarly, it appears that the contribution δ, which afflicts all gauges and is moreover unbounded, has also not been considered. Reference [5] did detect these effects by sitting exactly at the resonance and using indirect arguments. However, to make contact with experiment one must consider the full resonance region and demonstrate, as shown in the present Letter, how the gauge dependence arises in the analysis of the line shape. It is likely that the $O(g^4)$ gauge-dependent effects discussed above are larger in the case of other unstable particles, such as W and H. In particular, in the W case Im$A_b'^{w}(M_w^2) \neq 0$ for $\xi_w < 1$, so that the gauge dependence induced by the definition analogous to Eq. (2) arises just below the 't Hooft-Feynman gauge. These observations are relevant to elucidate the concept of mass for unstable particles, at least in the context of gauge theories. Extrapolating the lessons learned in the Z^0 case to other particles, one is lead to the conclusion that such a concept should be based on the parameters that define the complex-valued position of the pole, as, for example, in Eq. (11), or on gauge-independent self-energies, as in the discussions in the PT framework. In the first case, Eq. (12) gives the relevant mass counterterm in compact form. Meanwhile, in cases in which Γ is perturbatively small ($\Gamma \ll M$), Eq. (2) remains a very useful approximation that can be applied at the one-loop level and, in an important but restricted class of R_ξ gauges, in $O(g^4)$.

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*Electronic address: passera@mafalda.physics.nyu.edu

†Permanent address: Dept. of Physics, New York University, 4 Washington Place, New York, NY 10003. Electronic address: sirlin@mafalda.physics.nyu.edu

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