A Causal Source Which Mimics Inflation

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How unique are the inflationary predictions for the cosmic microwave anisotropy pattern? In this paper, it is asked whether an arbitrary causal source for perturbations in the standard hot big bang could effectively mimic the predictions of the simplest inflationary models. A surprisingly simple example of a scaling causal source is found to closely reproduce the inflationary predictions. This Letter extends the work of a previous paper [N. Turok, Phys. Rev. D **54**, 3686 (1996)] to a full computation of the anisotropy pattern, including the Sachs-Wolfe integral. I speculate on the possible physics behind such a source. [S0031-9007(96)01649-3]

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The prospect of mapping the cosmic microwave background (CMB) anisotropies [1] to high resolution has raised the exciting possibility of confirming fundamental theories of the origin of structure in the Universe. The inflationary theory is the present front runner, and the latest CMB measurements do even hint at support for the simplest, spatially flat inflationary models. The angular power spectra these theories predict are distinct from those in cosmic defect or baryon isocurvature models, and it is an important question whether spectra of this form are really a unique prediction of inflation. Or could a noninflationary mechanism somehow replicate them?

The fundamental difference between inflationary and noninflationary mechanisms of structure formation is that inflation alters the causal structure of the early Universe, adding on a prior epoch during which correlations are established on scales much larger than the Hubble radius. This is, of course, how the standard horizon puzzle is solved. Similarly, the perturbations produced during inflation possess "super-horizon" correlations (quotes indicate a standard big bang definition). If these "super-horizon" correlations were shown to exist, it would strongly support the idea of inflationary structure formation, for no causal mechanism could have produced them within the standard big bang.

Since COBE observed perturbations on the CMB sky on scales larger than the "horizon" at last scattering, one might think the issue was settled. But these large angle anisotropies could have been produced causally within the standard big bang, by time dependent gravitational potentials along the line of sight. Cosmic defects as well as open Universe or Λ dominated models provide examples of theories where this happens.

The smaller angle anisotropies are a more promising probe because they are due to local effects which are strongly constrained by causality (Fig. 1). In particular, the Doppler peaks caused by phase coherent oscillations [2] in the photon-baryon fluid provide a possible signature of "super-horizon" curvature perturbations [3–7]. Reference [6] developed a formalism for causal sources, and the present Letter follows up that work.

Here I ask whether a causal source acting purely via gravity in a smooth Universe could generate CMB anisotropies similar to those in flat inflationary models. For simplicity I restrict consideration to scaling sources, which nevertheless turn out to provide a surprisingly simple inflationary mimic. I emphasize that the mimic is not a theory, but an ansatz constructed by hand to provide a consistent solution to the Einstein equations. But it *is* sufficiently simple that it might actually be realized in a future theory of structure formation—in that sense the counterexample may turn out to be constructive.

I deal with the linearized Einstein equations in the stiff approximation, where the perturbations are assumed to have negligible effect on the source [8]. The source stress energy tensor $\Theta_{\mu\nu}$ is then covariantly conserved with respect to the background metric:

$$\dot{\Theta}_{00} + \frac{\dot{a}}{a}(\Theta_{00} + \Theta) = \Pi; \qquad \dot{\Pi} + 2\frac{\dot{a}}{a}\Pi = \partial_i\partial_j\Theta_{ij}, \tag{1}$$

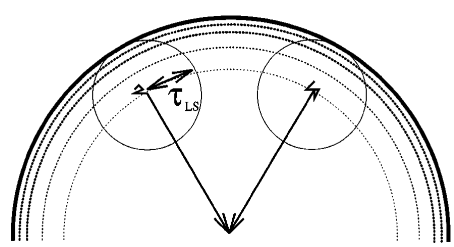

FIG. 1. The causality constraint on the microwave background anisotropy. The picture is in comoving coordinates: The outer circle represents our causal horizon in the standard big bang. The inner circle is the surface of last scattering, on which photons are set free from the hot plasma. Circles show the domains of influence on these photons—the radius is the light travel distance since the hot big bang τ_{LS}. This subtends an angle $\Theta_{LS} \sim 1.1°$ in a flat Universe with standard recombination. No causal physics operating within the standard big bang could have generated correlations between photons on the last scattering surface at points separated by more than $2\Theta_{LS}$ on the sky.

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where $\Pi = \partial_i \Theta_{0i}$. Dots denote derivatives with respect to conformal time τ, and $a(\tau)$ is the scale factor.

A formalism for dealing with such sources was proposed in [6]. Here I shall consider only coherent sources, representable in terms of a single set of master functions, which are solutions $\Theta_{\mu\nu}(\mathbf{x}, \tau)$ of (1). The correlator $\langle \Theta_{\mu\nu}(r, \tau) \Theta_{\rho\lambda}(0, \tau') \rangle$ equals the spatial convolution of the master functions $\Theta_{\mu\nu}$ and $\Theta_{\rho\lambda}$.

As argued in [6], causality implies that the master functions $\Theta_{\mu\nu}(r, \tau)$ vanish for $r > \tau$. For scalar perturbations, the master functions are spherically symmetric and can be written as $\Theta_{00}(r, \tau)$, $\Theta_{0i} = x_i J(r, \tau)$, $\Theta_{ij} = \frac{1}{3}\Theta(r, \tau)\delta_{ij} + \Theta^A(r, \tau)(x_i x_j - \frac{1}{3} r^2 \delta_{ij})$. The term $\frac{1}{3}\Theta$ is the pressure P, and Θ^A the anisotropic stress. In situations where matter is being actively moved, as it will be here, the anisotropic stresses are generally of the same order as the pressure.

Some general properties can now be seen. The Fourier transforms (assumed to exist) are analytic about $k_i = 0$, and can be Taylor expanded. By isotropy, the leading terms are $\Theta_{00} \sim k^0$, $\Theta_{0i} \sim k_i$, $\Pi \sim k^2$, and $\Theta_{ij} \sim \delta_{ij}$. In Fourier space we write $\Theta_{ij}(\mathbf{k}) \equiv \frac{1}{3}\delta_{ij}\Theta + (k_i k_j - \frac{1}{3}\delta_{ij})\Theta^S$. One sees that $\Theta^S(k) = k^{-1} d\Theta^A/dk - d^2\Theta^A/dk^2 \sim k^2$.

I now specialize to scaling sources, where the source (a) involves a number with dimensions of the inverse of Newton's constant G and (b) involves no other length scale apart from the horizon scale τ. With these conditions the source-perturbation equations are scale-invariant, apart from the violation of scaling caused by the radiation-matter transition. Scaling and dimensional analysis imply that (see, e.g., [9]) $\Theta_{00}(k, \tau) \sim \tau^{-\frac{1}{2}} f_1(k\tau)$, $\Theta(k, \tau) \sim \tau^{-\frac{1}{2}} f_2(k\tau)$, $\Pi(k, \tau) \sim \tau^{-\frac{3}{2}} f_3(k\tau)$, $\Theta^S(k, \tau) \sim \tau^{-\frac{1}{2}} f_4(k\tau)$, where the f_i have Taylor expansions in k^2 obeying the restrictions noted above. These four f_i's are related by the two energy momentum conservation equations (1). So, for example, the k^0 term in the first equation relates the leading terms in f_1 and f_2, and the k^2 term in the second equation relates the leading terms in f_2 and f_3. But even after applying these equations, we still have essentially two free functions remaining. We also have some freedom in how to incorporate the matter-radiation transition into the source.

I assume the background spacetime is flat, and has metric $ds^2 = a^2(\tau)\{-d\tau^2 + [\delta_{ij} + h_{ij}(x, \tau)]\, dx^i\, dx^j\}$, with τ conformal time and $a(\tau)$ the scale factor. I work in initially unperturbed synchronous gauge, in which the Einstein equations are manifestly causal.

We are interested in computing the CMB temperature distortion in a direction $\mathbf{n}$ on the sky: In the "instantaneous recombination" approximation this is

$$\frac{\delta T}{T}(\mathbf{n}) = \frac{1}{4}\,\delta_R(i) - \mathbf{n} \cdot \mathbf{v_R}(i) - \frac{1}{2}\int_i^f d\tau \dot{h}_{ij} \mathbf{n}^i \mathbf{n}^j\,, \tag{2}$$

where δ_R is the density contrast, and $\mathbf{v_R}$ the velocity, of the photon fluid on the surface of last scattering. The last term is the Sachs-Wolfe integral, representing the change in the proper path length along the line of sight.

The first two terms are local effects, determined from

$$\ddot{\delta}_C + \frac{\dot{a}}{a}\,\dot{\delta}_C = 4\pi G \times \left[\sum_N (1 + 3c_N^2)\rho_N \delta_N + \Theta_{00} + \Theta\right], \tag{3}$$

$$\dot{\delta}_R = \frac{4}{3}\,\dot{\delta}_C - \frac{4}{3}\nabla \cdot \mathbf{v_R}; \qquad \dot{\mathbf{v}}_\mathbf{R} = -(1 - 3c_S^2)\frac{\dot{a}}{a}\,\mathbf{v_R} - \frac{3}{4}\,c_S^2 \nabla \delta_R\,, \tag{4}$$

where c_S is the speed of sound in the photon-baryon fluid. Note that only $\Theta_{00} + \Theta$ enters. Thus prior to last scattering, only one of the two free functions in $\Theta_{\mu\nu}$ contributes—the other is literally invisible in the CMB anisotropy.

In the simplest inflationary theory, the surface term $\frac{1}{4}\delta_R$ dominates in determining the Doppler peaks—the $\mathbf{v}_R$ term and the Sachs-Wolfe integral are subdominant. In Ref. [6], I found a causal source $\Theta_{00} + \Theta$ for which the δ_R surface term matched that from inflation. The idea was simply to choose

$$\Theta_{00} + \Theta \propto f_1(r) + f_2(r) \propto \delta(r - A\tau), \qquad 0 < A \le 1\,, \tag{5}$$

representing a spherical shell expanding at some fraction of the speed of light. For A close to unity, the match was excellent [6]. Such a shell of matter is similar in form to a supernova explosion—for a spherical shell of neutrinos, one has $\Theta_{ij} \sim \Sigma p^i p^j \propto x^i x^j / r^2$.

Here I extend the computation to the entire expression (2). The Sachs-Wolfe integral has some dependence on the anisotropic part of the metric perturbation, and to compute this it is necessary to further specify $\Theta_{\mu\nu}$. The simplest choice leaving $f_1 + f_2$ fixed is to specify f_3. Then Eqs. (1) are used as follows: The energy equation is integrated to determine Θ_{00}, and the momentum equation is differentiated to determine Θ^S. Of course this must be done consistently with the matching of the leading terms as discussed above.

In Fourier space, the choices I make for the source are

$$\Theta_{00} + \Theta = \frac{a}{\dot{a}}\,\frac{\sin Ak\tau}{Ak\tau^{5/2}} \tag{6}$$

as in (5), with the prefactor incorporating the radiation-matter transition in a simple way. For Π, we must satisfy $\Pi(k) \sim k^2$ at small k. Equivalently, the integral $\int_0^\tau r^2\, dr\, \Pi(r, \tau) = 0$. This is most easily satisfied by taking $\Pi(r, \tau)$ to be the sum of two delta functions of equal weight but opposite sign. Their Fourier transform produces

$$\Pi = -\frac{E(\tau)}{k\tau^{\frac{5}{2}}}\,\frac{6}{B^2 - C^2}\left(\frac{\sin Bk\tau}{B} - \frac{\sin Ck\tau}{C}\right), \tag{7}$$

where $E(\tau)$ is a messy function obtained by analytically solving for the coefficient of k^2 term in the momentum equation (1). It equals $\frac{2}{15}$ in the radiation era and $\frac{2}{18}$ in the matter era. A set of values which leaves the Sachs-Wolfe integral sub-dominant is $B = 1.0$ and $C = 0.5$.

The initial conditions for the Θ_{00} and the perturbations are set up deep in the radiation era $\tau \ll \tau_{EQ}$, well outside the horizon $k\tau \ll 1$. From the k^0 terms in the energy conservation and perturbation equations, one has

$$\Theta_{00} = 2\tau^{-\frac{1}{2}}, \quad \delta_R = \delta_\nu = \frac{4}{3}\,\delta_C = D\tau^{\frac{3}{2}}, \quad \mathbf{v}_R = 0\,, \tag{8}$$

with the constant D determined by setting the total pseudoenergy $\tau_{00} = k^2(h - h^S)/(24\pi G) = \Theta_{00} + \sum_N \rho_N a^2 \delta_N + (\dot{a}/a)\dot{\delta}_C/(4\pi G)$ to zero. With these choices there are no superhorizon perturbations in the photon-to-CDM -baryon or -neutrino ratios. Setting the pseudoenergy zero means there are no curvature perturbations either. In the full calculation, the anisotropic metric perturbation is given by $\dot{h}^S - \dot{h} = -24\pi G[\Pi + \sum_N(P_N + \rho_N)a^2 i\mathbf{k}\cdot\mathbf{v_N}]/k^2$. The free streaming of photons and neutrinos after last scattering is modeled following Ref. [3]. The complete C_l spectrum of the causal model defined in Eqs. (6)–(8) is shown in Fig. 2.

In the construction above, where I integrate the energy equation to determine Θ_{00}, there is no reason it should go to zero inside the horizon. However, because I explicitly turn off $\Theta_{00} + \Theta$ inside the horizon, the source ceases to have any effect on the fluid perturbations and the trace part of the metric $h = -2\delta_C$. The effect on the anisotropic part h^S is similarly turned off because Π goes to zero. In effect, I have turned off all the "gravitationally active" components of the source, but there is no reason for the energy Θ_{00}, the pressure Θ, or the anisotropic stresses Θ^S to vanish separately—they need only satisfy the relations $\Theta_{00} + \Theta = 0$ and $\Theta + 2\Theta^S = 0$. This is reminiscent of the behavior of a straight cosmic string—it carries energy but generates no gravitational field. In any case, the sub-horizon source is removed if one adds a term $c_1 k^2\tau(a/\dot{a})\Theta_{00}$ to $\Theta_{00} + \Theta$, or a term $-c_2 k^2\tau\Theta_{00}$ to Π. Either of these makes Θ_{00}, Θ, and Θ^S go to zero inside the horizon as $\exp[-\mathrm{const}\; k^2\tau^2]$. Figure 3 shows the evolution of the components of $\Theta_{\mu\nu}$ with and without these modifications, and Fig. 4 shows the corresponding C_l spectra. The moral is that there is a lot of freedom inside the horizon to make large alterations in the source without introducing a significant integrated Sachs-Wolfe effect.

How sensitive to the particular choice of ansatz is the result? Figure 4 compares the $A = 1$ model with the cases $A = 0.7$ and $A = 0.1$. As A decreases, the source changes sign at larger $k\tau$. The radiation perturbation starts off with the opposite sign to the source: If the latter does not change sign, the radiation perturbation eventually must change sign, as it is driven by the source. This causes the leftward shift of the peak. It is also easy to arrange for a shift to the right by taking linear combinations of sources like (6) [6]. So there is a substantial region of parameter space around the

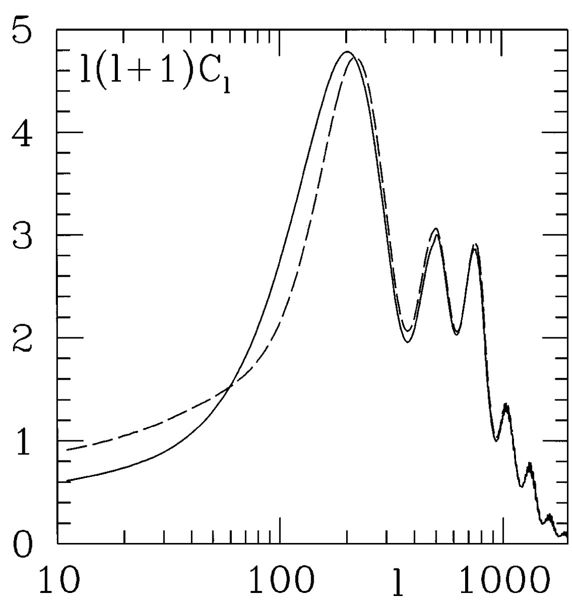

FIG. 2. Comparison of the simplest inflationary theory (dashed line) with its "mimic" causal source model (solid line) discussed here. The vertical axis is $l(l+1)C_l$, with C_l the angular power spectrum and l the Legendre index. Both curves were calculated in a flat Universe with canonical parameters $\Omega_B = 0.05$, $\Omega_{\mathrm{CDM}} = 0.95$. The vertical scale is arbitrary.

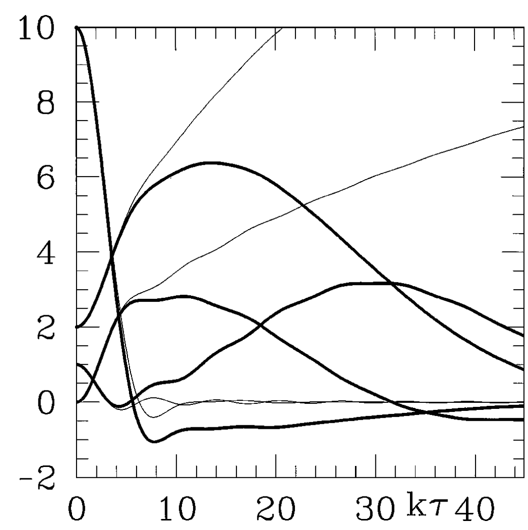

FIG. 3. Behavior of the stress tensor inside the horizon. The thin lines show the original source defined in Eqs. (6) and (7) as a function of $k\tau$ in the radiation era (in order as the curves intersect the y axis moving upwards) $\tau^{1/2}\Theta_S$, $\tau^{1/2}(\Theta + \Theta_{00})$, $\tau^{1/2}\Theta_{00}$, and $75k^{-2}\tau^{-1/2}\Pi$. The bold lines show the same curves for the model specified by $c_1 = 0.001$ and $c_2 = 0.003$ (see text), where all components are turned off inside the horizon. The C_l spectrum for the latter model is shown in Fig. 4.

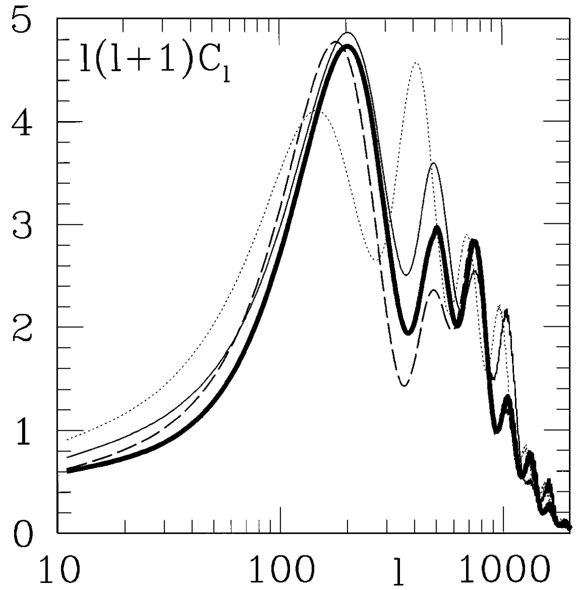

FIG. 4. The C_l spectrum of the $A = 1$ model (bold line) is compared with that for $A = 0.7$ (dashed line) and $A = 0.1$ (dotted line). The model illustrated in Figure 3, for which the stress tensor vanishes inside the horizon, is also shown as the thin line (altering c_2 to 0.001 makes the C_l spectrum virtually indistinguishable from the original model, except for the fourth peak, which is still a little high).

simplest $A = 1$ model which has similar C_l spectra. It would be very difficult to observationally separate these models from inflation, especially in the realistic case where we are unlikely to know all the relevant cosmological parameters (Ω, Ω_B, h, Ω_Λ, and so on) in advance. I have so far explored only a very tiny part of model space, and preliminary investigations of more general ansatzes indicate that a very wide range of C_l spectra is possible, especially when the Sachs-Wolfe integral becomes a dominant effect.

I have given myself a great deal of freedom in constructing this source, and it is far from clear that realistic physics could produce it. Nevertheless, the reader may tolerate some speculations. As mentioned, the form of $\rho + 3P$ required is similar to that resulting from a supernova explosion, but whereas in the latter case the energy redshifts away as $a(t)^{-1}$, here scaling evolution requires that the energy in the shell increases—by dimensions, $E \sim G^{-1}t$. This requires some positive feedback, which one could conceivably arrange with unstable dark matter, decaying via stimulated emission of Goldstone bosons or even gravity waves. Another issue is the Gaussianity or otherwise of the perturbations. If the source is made up of a very large number of "explosions" which are allowed to superpose, it can be made very Gaussian. But if the shells interact, there is a limit to their number density, and the perturbations would be non-Gaussian.

To conclude, causality alone is insufficient to distinguish the inflationary C_l predictions from those of noninflationary models. Of course the observational confirmation of one of these spectra would be a tremendous success for inflation, but the door would still be left open to other explanations of cosmic structure formation.

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