Dynamic Matching of Vortex Lattice in Superconducting Multilayers

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We observed oscillations of a nonlinear flux flow resistivity $R(j,H)$ as a function of a parallel magnetic field $6 < H < 9$ T in Nb-Ti/Cu multilayers. We show that the oscillations in $R(H)$, which have the field period $\Delta H \approx 0.1$ T independent of temperature and current, indicate a long-range order in the rapidly moving vortex structure. The critical current $I_c(H)$ exhibits no oscillations characteristic of $R(H)$. We propose an explanation of the effect in terms of dynamic matching of the moving vortex lattice with periodic microstructure and show that both ΔH and the amplitude of the oscillations of $R(H)$ are inversely proportional to the sample thickness. [S0031-9007(96)01582-7]

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Dynamics and pinning of the flux line lattice (FLL) in a periodic potential $U(x)$ characteristic of artificial multilayers or layered high-temperature superconductors in a parallel magnetic field have attracted much attention [1,2]. For instance, a static interaction of the FLL with a periodic pinning structure can give rise to peaks in the critical current density $j_c(H)$ at matching magnetic fields $H_{mn_1n_2} = \sqrt{3}\,\phi_0 m^2/2(n_1^2 + n_1 n_2 + n_2^2)l^2$ for which the spacing $a(H)$ between neighboring vortex rows is commensurate with the period l of $U(x)$, where m, n_1, and n_2 are integers, and ϕ_0 is the flux quantum. This has been observed in Pb-Bi alloys with periodic concentration of Bi [3], films with modulated thickness [4], or periodic structure of holes [5], superconducting multilayers [6], Nb-Ti wires with artificial pinning centers [7], etc. The motion of the FLL can cause steps in the dynamic flux flow resistance R when the FLL is driven by superimposed dc and ac currents [8–11].

A mathematical description of the FLL in a periodic potential is similar to that of a Josephson contact, so the above phenomena have analogs in the physics of Josephson systems [12] and charge and spin density waves [13]. For instance, the peaks in $j_c(H)$ are analogous to those for the Josephson contact with a spatially modulated j_c [14], while the steps in R correspond to Shapiro steps [8–11]. In this Letter we report on oscillations in the nonlinear flux flow resistance $R(H)$ of NbTi/Cu multilayers. This new dynamic matching effect is described below in terms of the generation of standing electromagnetic waves in the moving FLL. This gives rise to an oscillating $R(H)$ which depends on the sample dimensions, similar to Fiske resonances in Josephson contacts [12].

The multilayers were fabricated by two-gun programmable dc magnetron sputtering of Nb $-$ 49 wt % Ti and Cu onto (1102) 5 mm $\times$ 10 mm sapphire substrates in an argon plasma at 2 mTorr and 300 K [15]. The thickness of Cu layers (d_n) was 10 nm for all samples, while the thickness of Nb-Ti layers (d_s) was varied from 10 to 100 nm. The number of layers varied from 50 to 11 in order to maintain the total sample thickness d at 1 μm. Low angle diffractometer scans showed distinct satellite peaks indicating good multilayer periodicity which was also confirmed by transmission electron microscopy. The samples were patterned photolithographically to produce 50 μm wide by 3 mm long bridges. The magnetic field $0 < H < 12$ T was applied parallel to the layers with an accuracy of 0.015° and was always perpendicular to the transport current density j flowing along the layers.

The resistance $R(H,T,j)$ for different T, j, and d_s was measured by a four probe method as a function of field H, which was continuously increased with a constant ramp rate $\dot{H}$ [$R(H)$ was independent of $\dot{H}$ for 1 mT/s $< \dot{H} <$ 28 mT/s]. The results were similar for multilayers with different d_s, so we show here representative data for $d_s = 25$ nm, $d_n = 10$ nm, and $T_c = 6.85$ K. Figure 1 shows typical quasiperiodic oscillations in $R(H)$ of order 1 $\mu\Omega$; the effect is absent for perpendicular field. The Fourier transform of $R(H)$ shows a sharp peak for the main harmonic of $R(H)$ at a period $\Delta H = 0.095$ T independent of T, $\dot{H}$, and j, except for j close to j_c (Fig. 2). The value of ΔH increases as d_s is decreased; for instance, we observed similar oscillating $R(H)$ with $\Delta H = 0.13$ T for $d_n = d_s = 10$ nm. The amplitude of the oscillations decreases with increasing T and j, dropping below the noise level at $T > 6.1$ K and $j > 5 \times 10^4$ A/cm^2.

The above oscillations in $R(H)$ have a *dynamic* origin, since they can be observed only in the nonlinear flux flow regime for $j \approx (2-10)j_c$. As j approaches $j_c(H)$, the period ΔH sharply increases, becoming of order H_m at $j \approx j_c$ (see Figs. 1 and 2). At the same time, our direct j_c measurements [15] have shown that the static $j_c(H)$ does not exhibit any oscillations characteristic of $R(H)$ and has just a single broad maximum around the first matching field, $H \approx 1$ H. This indicates that the static matching of the FLL with the multilayer structure cannot give rise to the oscillations in $R(H)$ with the period $\Delta H \approx 0.1$ T at $H \approx 7-8$ T.

It turns out that the observed ΔH is close to the field increment needed to increase the number of vortex rows N_v in the sample by one,

$$\Delta H = (2\sqrt{3}\,\epsilon H \phi_0)^{1/2}/d\,. \quad (1)$$

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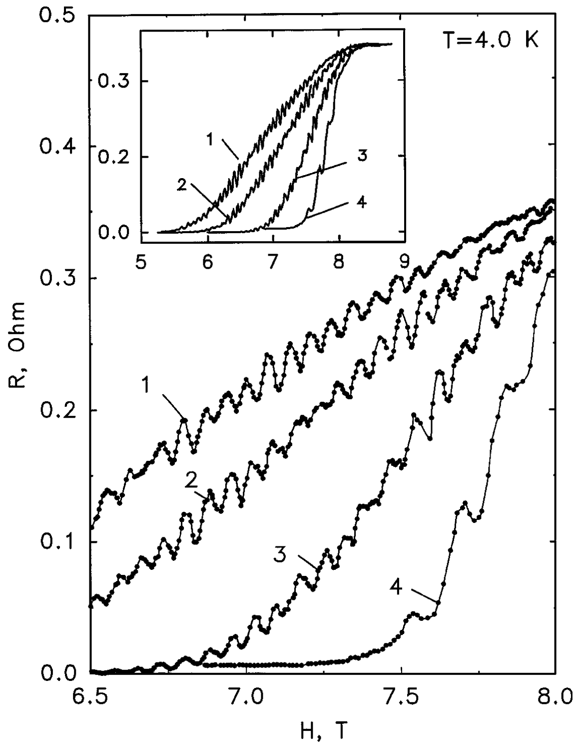

FIG. 1. $R(H)$ curves for the NbTi/Cu multilayer with $d_n = 10$ nm and $d_s = 25$ nm for different $j[\mathrm{A/cm^2}]$: 10^4 (1); 6×10^3 (2); 2×10^3 (3); 2×10^2 (4). Inset shows the same $R(H)$ curves on a larger field scale.

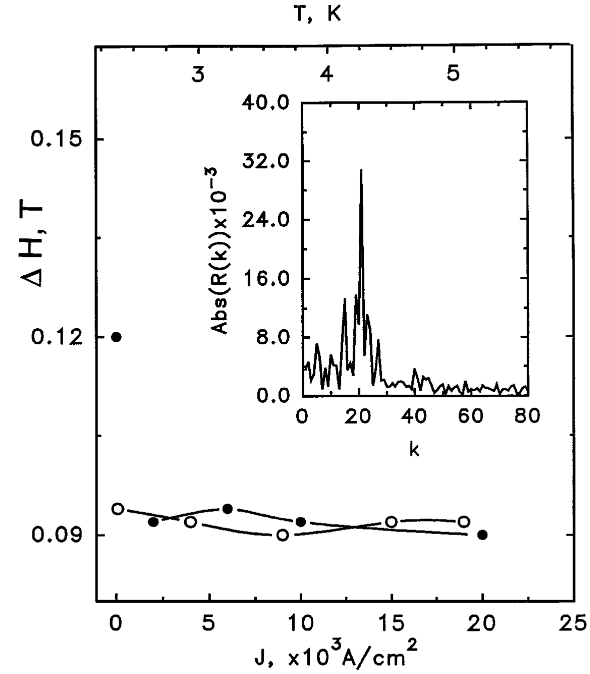

FIG. 2. ΔH as a function of current density j (filled symbols) and temperature T (open symbols). Inset shows the Fourier transform of $R(H)$ in Fig. 1, where $k = 2.05/\Delta H$ [T]. The main peak in $R(k)$ corresponds to $\Delta H = 0.095$ T.

Here $\epsilon = \lambda_x/\lambda_y < 1$ is the anisotropy factor which accounts for a uniaxial compression of the hexagonal FLL [1,2], and λ_x and λ_y are effective magnetic penetration depths perpendicular and parallel to the normal (N) layers, respectively. For $H = 7$ T and $d = 1$ μm, Eq. (1) yields $\Delta H = 0.095$ T if $\epsilon = \lambda_x/\lambda_y \approx 0.2$. This comparatively large ϵ corresponds to a good proximity coupling and a weak H_{c2} suppression in this multilayer [15]. However, the oscillations in $R(H)$ cannot be explained by a static surface pinning characteristic of thin films with $d < \lambda$ when $a(H)$ is commensurate with the film thickness [16]. In our case the $R(H)$ curves display many pronounced oscillations in the field region, where $N_v \sim 10^2$, so the commensurability of the static FLL with the sample thickness would be suppressed by very strong bulk pinning typical for Nb-Ti. Nevertheless, to check the relevance of the surface pinning, we measured $R(H)$ and $j_c(H)$ on a 1 μm thick Nb-Ti film made by the same sputtering system as the multilayers. We observed no oscillations in $R(H)$ and $j_c(H)$, which indicates that the effect is related to the multilayer structure.

The fact that $R(H)$ exhibits the oscillations with the period (1) indicates the effect of commensurability of the vortex structure with the sample thickness, which requires a good periodicity of the rapidly moving FLL. This implies that for $j \gg j_c$, the FLL correlation length L_t [1] perpendicular to the layers exceeds the sample thickness. At the same time, the absence of the oscillations in $j_c(H)$ indicates that for $j \approx j_c(H)$, pinning destroys the long-range translational order in the FLL; thus, L_t becomes smaller than d, and the FLL exhibits the same behavior as in an infinite sample. This scenario implies that $L_t(j)$ considerably increases with j, which is indeed a specific feature of the moving FLL recently discovered both in neutron difraction experiments and theoretical calculations [17]. The increase of $L_t(j)$ is due to the recovering of the long-range order in the moving FLL, as compared to the disordered FLL at $j \approx j_c$. Thus, the oscillations in $R(H)$ at $j \gg j_c$ may indicate a dynamic matching of the moving FLL with the layered structure, unlike the static matching responsible for the peaks in $j_c(H)$ in thin films and multilayers [6,16].

To show that the FLL moving perpendicular to the layers does result in the oscillating $R(H)$, we calculate $R(j,H)$ in the second order in $U(x)$ which is a weak perturbation for $j \gg j_c$. We use equations of anisotropic elasticity theory for vortex displacements $\mathbf{u}(x,y,t)$ [1,2].

$$\eta_x \frac{\partial u_x}{\partial t} = K \frac{\partial}{\partial x} \operatorname{div} \mathbf{u} + \mu \tilde{\nabla}^2 u_x - \sum_{m=1}^{\infty} f_m \sin(k_m x + Qmu_x + \varphi) - f\,, \tag{2}$$

$$\eta_y \frac{\partial u_y}{\partial t} = K \frac{\partial}{\partial y} \operatorname{div} \mathbf{u} + \mu \tilde{\nabla}^2 u_y / \epsilon^2\,, \tag{3}$$

where the x axis is perpendicular to the layers, $\tilde{\nabla}^2 = \partial^2/\partial x^2 + \epsilon^2 \partial^2/\partial y^2$, η_x and η_y are vortex viscosities across and along the layers, respectively, $K = (C_{11} - \epsilon C_{66})\phi_0/H$, $\mu = \epsilon C_{66}\phi_0/H$, C_{11} and C_{66} are the bulk and shear elastic moduli, respectively, of the FLL in the superconducting matrix, $f = \phi_0 j/c$ is the Lorentz force, c is the speed of light, and f_m are Fourier coefficients of the pinning force $f_p(x)$. For simplicity, the dispersion of C_{11} is not included in Eqs. (2) and (3), since $R(j,H)$ will be determined by the nondispersive C_{66}.

As known from a theory of commensurate structural transitions [18], the values k_m in Eq. (2) can differ from Qm due to matching effects. Indeed, if mQa is not too close to 2π, the mth harmonic $f_m \sin(mQx_n)$ of $f_p(x)$ oscillates with the period $\approx l/m$ as a function of the position of the nth vortex row, $x_n = na$. However, if $ma \approx l$, the pinning force $f_p(x)$ contains the slowly varying "resonance" term $f_m \sin k_m x$ with the period $2\pi/k_m \gg l/m$, since $\sin(mQna) = \sin mQ(a - a_m)n$. Here $ma_m = l$, and $k_m = mQ(a - a_m)/a$ equals

$$k_m = 2\pi m(\sqrt{H/H_m} - 1)/l\,, \qquad (4)$$

where $H_m = \sqrt{3}\,\epsilon\phi_0 m^2/2l^2 = 1.4\epsilon m^2$[T] for $l = 35$ nm. Since f_m decreases with m, usually only the first few harmonics dominate. However, the summation over m in Eq. (2) should also include resonance terms with $m > 1$ for which k_m depends on H, so the smallness of f_m can be compensated by large values of $Q/k_m(H)$ [18].

We consider solutions $u_x = vt + py/Q + w_x$, $u_y = -px/Q + w_y$, where $v = f/\eta_x$, and the parameter p determines the orientation of the FLL with respect to $\mathbf{v}$ ($p = 0$ corresponds to the FLL moving along its nearest neighbor direction). The pinning-induced displacement $\mathbf{w}(x,y,t)$ can be written in the form

$$\mathbf{w}(x,y,t) = \mathrm{Re}\sum_{n,m}\mathbf{A}(n,m)\exp(iqnx + i\Psi_m)\,, \qquad (5)$$

where $\Psi_m = m(py + \omega t) + \varphi$, $q = 2\pi/d$, $\omega = vQ$. Here we use the periodic boundary condition, $u_x(0) = u_x(d)$, which provides the conservation of total magnetic flux in the sample, since for $H \simeq H_{c2}$ the FLL is almost incompressible ($K \gg \mu$) [1,2]; thus, div $\dot{\mathbf{u}} \propto \dot{H} = 0$.

For $w \ll a$, we can replace the sine terms in Eq. (2) with $\sin(k_m x + \Psi_m)$. Then A_x can be found by multiplying both sides of Eqs. (2) and (3) by $\exp(-iqnx)$ and integrating from $x = 0$ to $x = d$. For $K \to \infty$, we obtain

$$A_x = \frac{ij_m\beta m^2(e^{ik_m d} - 1)Q^{-1}}{[imj(\beta m^2 + \gamma n^2) + j_q(\epsilon^2 m^2\beta + n^2)^2]d(k_m - qn)}\,, \qquad (6)$$

where $\gamma = \eta_y/\eta_x$, $j_m = cf_m/\phi_0$, $j_q = c\mu q^2/\epsilon^2\phi_0 Q = 2\pi clC_{66}/Hd^2\epsilon$, and $\beta = (p/q)^2$. Expanding Eq. (2) in the first order in w_x and integrating over x, we can calculate the mean dc electric field $E_y = -H\int\langle\dot{u}_x\rangle\,dx/cd$ and thus obtain $R(j)$ using the scheme developed for the moving FLL [1,11], or Fiske resonances [12]. This yields

$$\frac{R}{R_f} = 1 - \frac{1}{2\pi^2}\sum_{n,m}\frac{\sin^2(\pi\theta_m)G(n,m)}{(n - \theta_m)^2}\,, \qquad (7)$$

where R_f is the flux flow resistance, $\theta_m = k_m d/2\pi$, n runs from $-\infty$ to ∞, and

$$G = \frac{\beta m^3 j_m^2(\beta m^2 + \gamma n^2)}{j^2(\gamma n^2 + \beta m^2)^2 m^2 + j_q^2(n^2 + \epsilon^2\beta m^2)^4}\,. \qquad (8)$$

Here $R(j,H)$ depends on the parameter $\beta = (pd/2\pi)^2$ which fixes the orientation of the FLL with respect to the layers. For a static FLL, p is determined by the minimum of elastic energy [18]. The orientation of the moving FLL can be found from the minimum dissipation principle [11], which reduces to the minimum of $R(j,p)$; that is, $\partial R/\partial\beta = 0$ for fixed current.

We first consider an infinite sample $d \to \infty$ for which $G(n,m)$ in Eq. (7) can be replaced by $G(\theta_m, m)$ [19]. Then the summation over n gives $\pi^2/\sin^2\pi\theta_m$; thus

$$\frac{R_0}{R_f} = 1 - \frac{1}{2}\sum_m\frac{mj_m^2 g_m(\gamma + g_m)}{m^2(\gamma + g_m)^2 j^2 + j_k^2(1 + \epsilon^2 g_m)^4}\,, \qquad (9)$$

where $j_k(m) = ck_m^2 C_{66}/\epsilon QH$, and $g_m = (mp/k_m)^2$. For all nonresonance terms with $k_m = mQ$, the value $j_k(m) = c\phi_0 m^2(1-b)^2/32\pi\lambda^2 l\epsilon$ is of order the depairing current density, $j_d = c\phi_0/16\pi^2\lambda^2\xi$, if $l\epsilon$ is of order the coherence length, ξ. Here $b = H/H_{c2}$, and $l\epsilon \approx 1.4\xi$ for our sample. Since $j_m \ll j_d$, the main contribution to the sum in Eq. (9) thus comes from few resonance harmonics for which $k_m(H) < 1/l$ and $j_k(m) \ll j_p$. This enables us to obtain p, at $H \approx H_m$, taking into account only one resonance term in Eq. (9). Furthermore, we set $\gamma = \eta_y/\eta_x \to 0$, due to the large ratio of normal conductivities $\sigma_{\mathrm{Cu}}/\sigma_{\mathrm{NbTi}} \sim 10^4$ in our multilayers. Then the condition $\partial R/\partial g = 0$ yields $\epsilon^2 g_m = 1$, whence

$$p = \pm k_m/\epsilon m\,. \qquad (10)$$

Now we return to Eq.(7) and consider the oscillating component $\delta R(H)$ due to a finite sample thickness. Performing the summation over n in Eq. (7) for $\exp(pd) \gg 1$, we obtain $R = R_0 + \delta R$, where [19]

$$\delta R/R_f = \sum_m[M_m\sin(dk_m) - N_m\sin^2(dk_m/2)]\,. \qquad (11)$$

Here $M_m = G'(\theta_m, m)/4\pi$, G is given by Eq. (8), the prime denotes the differentiation over n, and

$$N_m = \frac{1}{2\pi^2}\int_{-\infty}^{\infty}dn\frac{G'(n,m) - G'(\theta_m,m)}{n - \theta_m}\,. \qquad (12)$$

From Eqs. (8) and (11) it follows that $G'(\theta,m) \propto 1/d$, so the value $\delta R \propto 1/d$ vanishes for an infinite sample and oscillates with the period ΔH determined by $k_m(H)d = 2\pi n$. This condition, along with Eq. (4), yields the position of the nth peak in $\delta R(H)$ in the form $H_n = (nl/md + 1)^2 H_m$; whence $\Delta H = H_{n+1} - H_n = 2lH_m(nl/md + 1)/dm$ for $l \ll d$. Expressing n via H, we arrive at Eq. (1), where H corresponds to the middle of the resistive transition in Fig. 1 ($H \gg \Delta H$). Here the oscillations of $R(H)$ result from a coherent interaction of the moving FLL with the multilayer structure, since $\delta R(H) \propto f_m^2$. Although the effect is dynamic, the period ΔH basically depends only on H and d, similar to that of Fiske resonances [12].

As follows from Eqs. (1) and (11), the period ΔH is the same for all m, so the model accounts for the smallness of ΔH as compared with H_m, and the independence of ΔH from m, T, j, and the form of $f_p(x)$ which affects only the amplitude of $R(j,H)$. As an illustration, we calculate $R(H)$ for one resonance term $m = 5$ in Eq. (7) for which $H_m = 1.4\epsilon m^2[\mathrm{T}] = 7$ T corresponds to the middle of the resistive transition in Fig. 1. We take $p(H)$ given by Eq. (10), $C_{66} = H\phi_0/(8\pi\lambda_x)^2(1-b)^2$, and the "sawtooth" $f_p(x)$ which changes linearly between the N layers on which $f_p(x) = \pm f_0$. Then we have $f_m = f_0/m$ if $d_n \ll d_s$, and $j_m = j_1(1-b)/m$. In this case the calculated $R(H)$ shown in Fig. 3 displays the oscillating component $\delta R(H)$ due to a finite d. This $R(H,j)$ exhibits the qualitative features of the observed $R(H,j)$, such as the independence of ΔH of j and the decrease of the amplitude of $\delta R(H)$ as j is increased. A similar oscillating $R(H)$ was reported for the FLL driven by transverse current along narrow channels [20].

In conclusion, we have observed pronounced oscillations in nonlinear dynamic resistance $R(H)$ of NbTi/Cu multilayers. We account for this effect in terms of dynamic matching of the moving FLL with the periodic multilayer structure. Similar effects may occur in high-T_c superconductors in a parallel field for which the measurements of the oscillations in $R(H,j)$ could probe the intrinsic pinning by the ab planes.

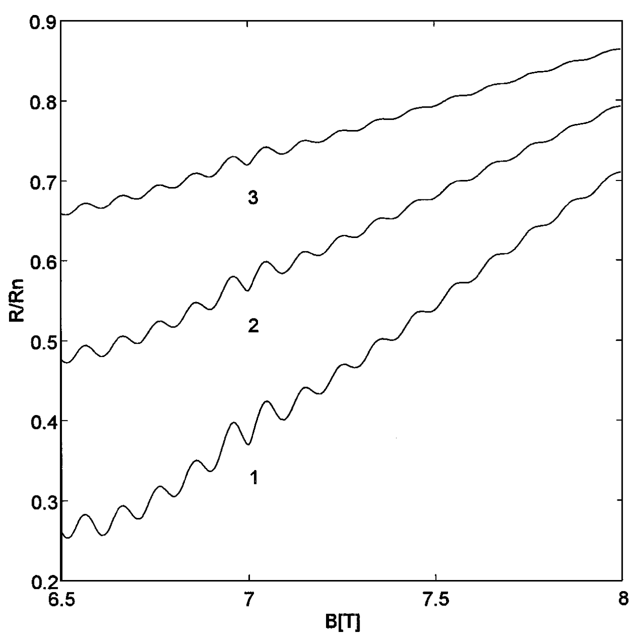

FIG. 3. $R(H)$ calculated numerically from Eq. (7) for $m = 5$, $\gamma = 0$, $\epsilon = 0.2$, $j_m = j_1(1-b)/m$, $j_q = j_0(1-b)^2$, $H_{c2} = 10$ T, and $R_f = R_nH/H_{c2}$, where R_n is the normal state resistance. For $d = 1$ μm, $l = 35$ nm, and $\lambda = 2500$ Å, we get $j_0 = cl\phi_0/32\pi\lambda^2d^2\epsilon = 5\times10^3$ A/cm^2 which is about 1% of a typical NbTi value $j_1 \approx 5\times10^5$ A/cm^2 [15]. The $R(H)$ curves are plotted for $j_1 = 100j_0$ and $j = 0.1j_1(1)$, $j = 0.12j_1(2)$, $j = 0.2j_1(3)$.

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