Relativistic Hartree-Bogoliubov Description of the Neutron Halo in ^{11}Li

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(Received 4 June 1996)

Relativistic Hartree-Bogoliubov theory in coordinate space is used to describe the chain of lithium isotopes reaching from ^{6}Li to ^{11}Li. Pairing correlations are taken into account by a density dependent force of zero range. In contrast to earlier investigations within a relativistic mean field theory and a density dependent Hartree-Fock theory, where the halo in ^{11}Li could only be reproduced by an artificial shift of the $1p_{1/2}$ level close to the continuum limit, the halo is now reproduced in a self-consistent way, without further modifications, using the scattering of Cooper pairs to the $2s_{1/2}$ level in the continuum. Excellent agreement with recent experimental data is observed. [S0031-9007(96)01531-1]

PACS numbers: 21.60.Jz, 21.10.Gv, 21.10.Pc, 27.20.+n

Since the experimental discovery of neutron halo phenomena in ^{11}Li [1], the study of exotic nuclei has become a very challenging topic in nuclear physics (for a recent review, see Ref. [2] and references given therein). Experimentally, a sudden rise in the interaction cross sections has been observed for light neutron-rich nuclei, specifically while going from ^{9}Li to ^{11}Li, ^{12}Be to ^{14}Be, and ^{15}B to ^{17}B. This sudden increase in the interaction cross sections has been attributed to a relatively large rms matter radius as compared to that expected from the conventional mass dependence $1.2A^{1/3}$.

On the theoretical side very different models have been used to describe these phenomena [2]: Three-body calculations based on an inert core of ^{9}Li and two outside neutrons treat the translational invariance properly, but they neglect polarization effects. Full shell model calculations contain all the configuration mixing, but they are limited to relatively small configuration spaces in the oscillator model. Mean field theories allow the inclusion of very large spaces. Their parameters are adjusted in other areas of the periodic table. However, violation of translational invariance has to be corrected at least partially by one of the various recipes known in the literature [3].

The large radius observed experimentally in ^{11}Li has been interpreted in the mean field description by the fact that, filling in more and more neutrons in the nuclear well, the Fermi surface for the neutrons comes close to the continuum limit in this nucleus. Because of the small single-neutron separation energy, the tails of the two wave functions of the last filled orbital $(1p_{1/2})$ reach very far outside of the nuclear well, and a neutron halo is formed [1].

Although this simple interpretation is based on the mean field picture, several microscopic theoretical investigations within self-consistent mean field models [4–7] have failed. In fact, it would be a strange accident if the last occupied neutron level in ^{11}Li, the $1p_{1/2}$ orbit, would be so close to the continuum limit that the tail of the last two uncorrelated neutron wave functions reaches so far out as would be necessary to reproduce the large experimental radius observed. Therefore, in the nonrelativistic scheme, Bertsch, Brown, and Sagawa [4] and Sagawa [6] introduced an artificial modification to the potential in order to reproduce the small separation energy in their mean field calculations using Skyrme interactions. In this way the authors were able to reproduce qualitatively the observed trends, although some discrepancies still remained.

Koepf *et al.* [5] were the first to investigate the neutron halo in relativistic mean field (RMF) theory for both spherical symmetrical and axially deformed cases. They found large deformations for the lighter Li isotopes, but a spherical shape for ^{11}Li, and, as in the nonrelativistic investigations, the binding energy of the $1p_{1/2}$ was too large so as to reproduce a neutron halo. Zhu *et al.* [7] improved the result of the RMF calculations by applying a similar modification to the potential as in Ref. [6] in order to adjust the proper size of the halo.

Bertsch and Esbensen [8] recognized that pairing correlations play an essential role. Performing a quasiparticle continuum random-phase approximation (RPA) calculation using a core of ^{9}Li and a density dependent interaction of zero range in the pairing channel, they found a large neutron halo ^{11}Li and could reproduce the proper size of the experimental matter radius.

The present investigation is based on a similar idea. Pairing correlations and the scattering of Cooper pairs into the continuum are taken into account in a fully self-consistent way using a continuum Hartree-Bogoliubov formalism [9] in the framework of relativistic mean field theory. A consistent treatment of pairing correlations within the framework of a generalized relativistic mean field approximation has been developed by Kucharek and Ring [10]. However, applications of this theory to nuclear matter clearly show that a quantitative description of pairing correlations in the nuclear many-body system cannot be achieved in this way with the presently used parameter sets of relativistic mean field theory. The behavior of the meson exchange forces entering this theory is simply not properly adjusted at large momentum transfer. In principle, these forces have finite range, and kinematical factors guarantee the convergence of the relativistic gap equation. The large masses of the σ- and the ω-mesons, however, do not yield a realistic cutoff.

Therefore, the situation is similar to that of Skyrme forces with zero range: The short range of the relativistic forces produces no problem in the Hartree case, where only momenta up to the Fermi surface are involved, but it causes severe problems in the description of pairing, which allows scattering to very high momentum states.

The present work is based on the idea of combining the advantages of a relativistic description in the framework of RMF theory in the Hartree channel with that of a phenomenological force in the pairing channel. In a recent investigation by Gonzalez-Llarena *et al.* [11], a similar concept has been used for the solution of the relativistic Hartree-Bolgoliubov equation in an oscillator basis with a finite range force of Gogny's type in the pairing channel. At present, the application of finite range forces in the pairing channel of continuum Hartree-Bogoliubov calculations is technically not yet feasible. We therefore use, as did the authors of Ref. [8], a density dependent δ force of the form

$$V(\mathbf{r}_1, \mathbf{r}_2) = V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{1}{4}(1 - \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2) \left[1 - \frac{\rho(r)}{\rho_0} \right]. \tag{1}$$

The starting point is the RMF Lagrangian which describes the nucleons as Dirac spinors moving in meson fields:

$$\mathcal{L} = \bar{\psi}[\not{\partial} - g_\omega \not{\omega} - g_\rho \vec{\not{\rho}} \vec{\tau} - \tfrac{1}{2} e(1 - \tau_3) \not{A} - g_\sigma \sigma - M_N]\psi + \tfrac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \tfrac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \tfrac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \tfrac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \tfrac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{2}$$

where M_N is the bare nucleon mass and ψ is its Dirac spinor. We have, in addition, the scalar meson (σ), isoscalar vector mesons (ω), isovector vector mesons ($\vec{\rho}$), and the photons A^μ, with the masses m_σ, m_ω, and m_ρ and the coupling constants $g_\sigma, g_\omega, g_\rho$. The field tensors for the vector mesons are given as $\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and, by similar expressions, for the ρ-meson and the photon. For simplicity we neglect in the following these fields; however, they are taken fully into account in the calculations. For a realistic description of nuclear properties a nonlinear self-coupling $U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$ for the scalar mesons has turned out to be crucial [12].

Using Greens function techniques it has been shown in Ref. [10] how one can derive a relativistic Hartree-Fock-Bogoliubov theory from such a Lagrangian: After a full quantization of the system, the mesonic degrees of freedom are eliminated and, in full analogy to the nonrelativistic case, the higher order Greens functions are factorized in the sense of Gorkov [13]. Finally, neglecting retardation effects in the Fock term, as is mostly done in relativistic mean field theory, one ends up with relativistic Dirac-Hartree-Bogoliubov (RHB) equations,

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}_k = E_k \begin{pmatrix} U \\ V \end{pmatrix}_k , \tag{3}$$

where E_k are quasiparticle energies and the coefficients U_k and V_k are four-dimensional Dirac spinors normalized in the following way:

$$\int U_k^+ U_{k'} + V_k^+ V_{k'} \, d^3 r = \delta_{kk'} . \tag{4}$$

The average field

$$h = \boldsymbol{\alpha p} + g_\omega \omega + \beta(M + g_\sigma \sigma) - \lambda \tag{5}$$

contains the chemical potential λ adjusted to the proper particle number and the meson fields σ and ω determined self-consistently from the Klein Gordon equations:

$$\{-\Delta + m_\sigma^2\}\sigma = -g_\sigma \rho_s - g_2 \sigma^2 - g_3 \sigma^3, \tag{6}$$

$$\{-\Delta + m_\omega^2\}\omega = g_\omega \rho_B \tag{7}$$

with the scalar density $\rho_s = \sum_k \bar{V}_k V_k$ and the baryon density $\rho_B = \sum_k V_k^+ V_k$. The sum over k runs only over all the particle states in the *no-sea approximation.*

The pairing potential Δ in Eq. (3) is given by

$$\Delta_{ab} = \frac{1}{2} \sum_{cd} V_{abcd}^{pp} \kappa_{cd} . \tag{8}$$

It is obtained from the pairing tensor $\kappa = U^* V^T$ and the one-meson exchange interaction V_{abcd}^{pp} in the pp channel. More details are given in Ref. [10]. As mentioned above, these forces are not able to reproduce even in a semiquantitative way the proper pairing in the realistic nuclear many-body problem. We therefore replace V_{abcd}^{pp} in Eq. (8) by the density dependent two-body force of zero range given in Eq. (1).

For a zero range force in the pairing channel, the RHB equations (3) are a set of four coupled differential equations for the HB Dirac spinors $U(r)$ and $V(r)$. They are solved in a self-consistent way by the shooting method and the Runge-Kutta algorithm with a step size of 0.1 fm using proper boundary conditions in a spherical box of radius $R = 20$ fm. The details will be published elsewhere. The results do not depend on the box size for $R > 15$ fm. For a radius $R = 40$ fm, we found the same results within an accuracy of 1%. Since we use a pairing force of zero range (1), we have to limit the number of continuum levels by a cutoff energy. For each spin-parity channel, 20 radial wave functions are taken into account, which corresponds for $R = 20$ fm roughly to a cutoff energy of 200 MeV. For fixed cutoff energy and for fixed box radius R, the strength V_0 of the pairing force (1) for the neutrons is determined by a calculation in the nucleus ^{7}Li, adjusting the corresponding pairing energy $-\frac{1}{2} Tr \Delta \kappa$ to that of a RHB calculation in an oscillator basis [11] using the finite range part of the Gogny force D1S of Ref. [14] in the pairing channel. For ρ_0 we use the nuclear matter density 0.152 fm^{-3}. In order not to miss any bound state, the cutoff energy has to be larger than the depth of the potential. But as long as this is the case, and as long as the interaction strength is properly renormalized, the results of this investigation stay practically unchanged: Reasonable variations of 10% in the interaction strength V_0 change the binding energy by only 0.5% and the rms radius by only 3%.

We use here the nonlinear Lagrangian parameter set NL2 which was designed in Ref. [15] for light nuclei: $M_N = 938$, $m_\sigma = 504.89$, $m_\omega = 780$, $m_\rho = 760$ MeV, and $g_\sigma = 10.444$, $g_\omega = 12.945$, $g_\rho = 4.383$, $g_2 = 6.6099$ fm^{-1}, $g_3 = 13.783$. Pairing is neglected for the three protons, and the strength V_0 of the pairing force (1) for the neutrons is determined by a calculation in the nucleus ^{7}Li adjusting the corresponding pairing energy $-\frac{1}{2}Tr\Delta\kappa$ to that of a RHB calculation in an oscillator basis using the finite range part of the Gogny force D1S of Ref. [14] in the pairing channel. In Fig. 1 we show the calculated binding energies E_B and the rms matter radii for the Li isotopes with mass numbers $A = 6$ to $A = 11$ and compare them with experimental values. In order to correct for the center of mass motion, a spurious energy of a harmonic oscillator $E_{\rm cm} = 0.75\hbar\omega_0$ with $\hbar\omega_0 = 41A^{-1/3}$ is subtracted. The calculated values show a small underbinding, and the odd-even staggering is somewhat exaggerated by our blocking calculations, but, in general, the agreement is very satisfactory and in full agreement with experiment. The matter radius shows a considerable increase when going from the nucleus ^{9}Li to ^{11}Li. In contrast to the earlier mean field calculations of Refs. [4,6,7], these results are obtained without any artificial modifications of the potential.

In Fig. 2 we show the corresponding density distribution for the neutrons in the isotopes ^{9}Li and ^{11}Li. It is clearly seen that the increase of the matter radius is caused by a large neutron halo in the nucleus ^{11}Li. Its density distribution is in very good agreement with the experimental density of this isotope, shown with its error bars by the shaded area.

In order to understand the microscopic structure of this halo, we show in Fig. 3 the mean field $S(r) + V(r) = g_\sigma\sigma + g_\omega\omega \pm g_\rho\rho + eA$ for the protons and neutrons together with the energy levels $\epsilon_n = \langle n|h|n\rangle$ in the canonical basis [3]. The Fermi level for the neutrons is very close to the continuum limit in close vicinity to the $\nu 1p_{1/2}$ and to the $\nu 2s_{1/2}$ level. The length of these energy levels in Fig. 3 is proportional to the corresponding occupation. We clearly see that pairing correlations cause a partial occupation of both the $\nu 1p_{1/2}$ and the $\nu 2s_{1/2}$ level, i.e., a scattering of Cooper pairs to the continuum.

Figure 4 shows the single particle levels in the canonical bases for the isotopes with an even neutron number as a function of the mass number together with the contribution of the different spin-parity channels to the matter radius

$$\langle r\rangle_{lj} = \left(\int r^2\rho_{lj}\, d^3r\right)^{1/2}. \qquad (9)$$

The total rms matter radius is obtained from this quantity as

$$\langle r^2\rangle^{1/2} = \left[\frac{1}{A}\sum_{lj}\langle r\rangle_{lj}^2\right]^{1/2}. \qquad (10)$$

Going from $A = 5$ to $A = 11$, we observe a continuous increase of the contribution of the $p_{1/2}$ channel to the total rms matter radius and, in addition, a sudden increase of the contribution of the $s_{1/2}$ channel. This means that

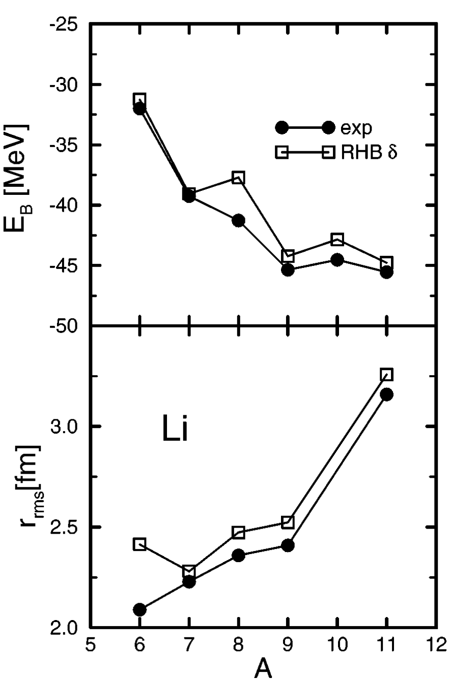

FIG. 1. Binding energies (upper part) and matter radii (lower part) for Li isotopes: RHB calculations (empty squares) are compared with experimental values (full dots).

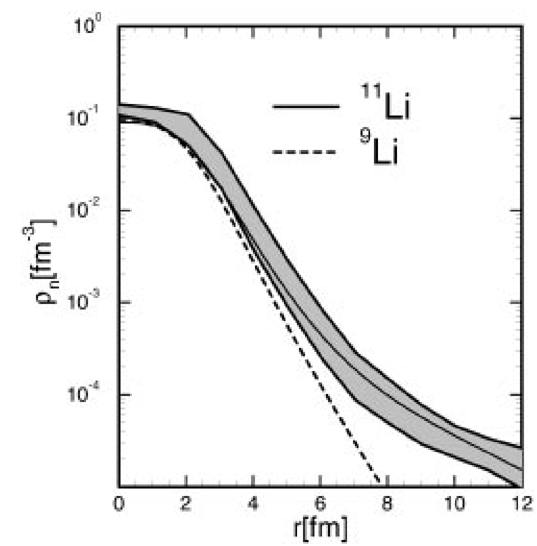

FIG. 2. Calculated and experimental density distribution in ^{11}Li and ^{9}Li. The solid line shows the result of ^{11}Li, while the dashed line corresponds to the calculation of ^{9}Li. The shaded area gives the experimental results with error bars.

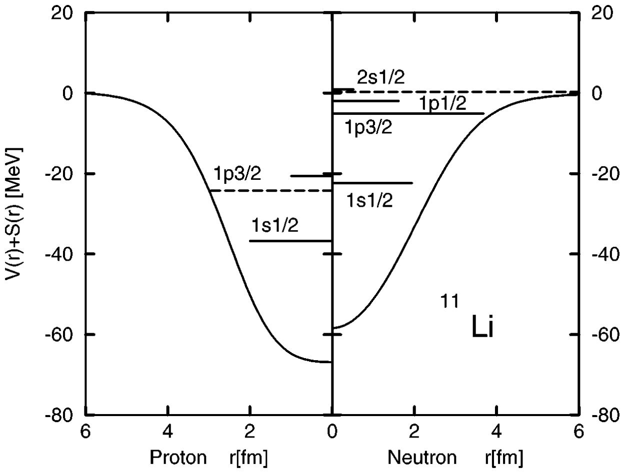

FIG. 3. The mean field potential $S + V$ for protons (left-hand side) and neutrons (right-hand side). The chemical potential is given by a dashed line. The energy levels in the canonical basis are indicated by horizontal lines with various lengths proportional to the occupation of the corresponding orbit.

the halo in ^{11}Li is formed by the occupation of the $2s_{1/2}$ level, which approaches the Fermi surface for this mass number, as seen in the upper part of Fig. 4.

Summarizing, we can say that we have solved for the first time the Hartree-Bogoliubov equations in the continuum [9] in the framework of relativistic mean field theory. A density dependent force of zero range has been used in the pairing channel, whose strength is adjusted

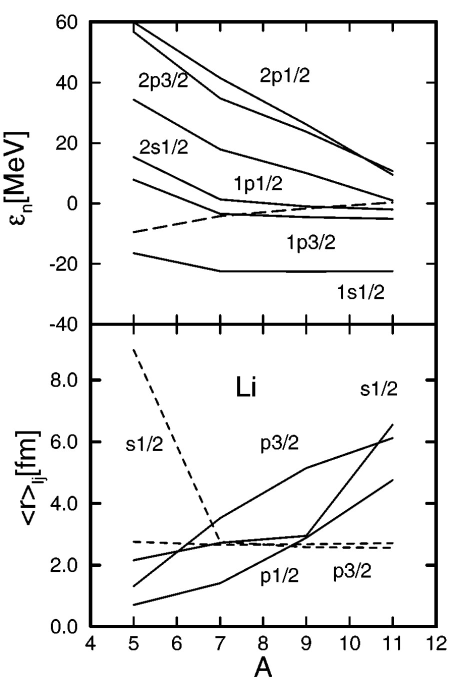

FIG. 4. Single particle energies for neutrons in the canonical basis (upper part) and contributions to the matter radius for various spin-parity channels as a function of the mass number. In the upper part the dashed line corresponds to the chemical potential, and in the lower part the dashed lines correspond to proton contributions.

for the isotope ^{7}Li to a similar calculation with Gogny's force D1S in the pairing channel. Good agreement with experimental values is found for the total binding energies and the radii of the isotope chain ^{6}Li to ^{11}Li. In excellent agreement with the experiment, we obtain a neutron halo for ^{11}Li without any artificial adjustment of the potential, as was necessary in earlier calculations. In contrast to these investigations, the halo is not formed by two neutrons occupying the $1p_{1/2}$ level very close to the continuum limit, but is formed by Cooper pairs scattered mainly in the two levels $1p_{1/2}$ and $2s_{1/2}$. This is made possible by the fact that the $2s_{1/2}$ comes down close to the Fermi level in this nucleus, and by the density dependent pairing interaction coupling the levels below the Fermi surface to the continuum. In contrast to the previous explanation which uses the accidental coincidence that one single particle level is so close to continuum threshold so that the tail of its wave function forms a halo, this is a much more general mechanism, which could possibly be observed in other halo nuclei also. One needs only several single particle levels with small orbital angular momenta and correspondingly small centrifugal barriers close to, but not directly at, the continuum limit.

Obviously, the experimentally observed long tail of the density distribution in the halo can be reproduced in this way without including explicitly the contributions from the one-pion-exchange potential, which is often assumed to be responsible for peripheral phenomena such as a halo.

We would like to express our gratitude to R. R. Hilton for a careful reading of the manuscript. This work was supported by the Alexander von Humboldt Foundation and by the Bundesministerium für Forschung und Technologie under Project No. 06 TM 743.

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