# Giant Quantum Oscillations of Acoustoelectric Effect in Nanostructures

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We work out a theory of the acoustoelectric effect in nanostructures under the ballistic conductance regime. The ultrasonic wavelength is assumed to be much smaller than the longitudinal dimension of the microstructure. We predict giant quantum oscillation of the acoustoelectric current under gate voltage variation. By this we mean that the maxima of the oscillatory part far exceed the minima. The effect can be used for the investigation of the electron spectrum of microstructures. [S0031-9007(96)01436-6]

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The wave propagating in the semiconductor creates a net drag of the electrons and hence a dc acoustoelectric current $J$ or, if the circuit is disconnected, a dc acoustoelectric potential difference $V$. Up to now this effect has been considered in the classical transport regime of Drude conductivity where the relaxation plays a crucial role. In this Letter we consider the opposite limit—the ballistic transport. In the ballistic case the conductance is a steplike function of the Fermi level. Each step corresponds to the inclusion of a new mode of transverse quantization to the conduction process and has a height of $G_0 = 2e^2/h$ multiplied by transmission probability.

In the present Letter we are interested in a case of low temperatures and high ultrasonic frequencies when the interaction of ultrasound with electrons can be treated as a direct absorption and emission of ultrasonic phonons by the electrons of the nanostructure. Using the physical picture developed by Landauer, Büttiker, and Imry [1,2] we consider a semiconductor quantum nanostructure connected to two reservoirs, each in independent equilibrium. We calculate the voltage (or current) brought about by a traveling acoustic wave [3].

We assume that the nanostructure has a form of quantum wire with a constant cross section, with the $x$ axis parallel to the wire. We also assume that the direction of the phonon propagation is parallel to the wire, i.e., to the $x$ axis. Then the energy and quasimomentum conservation law reads

$$\epsilon_n(p) + \hbar\omega_{\mathbf{q}} = \epsilon_n(p + \hbar q). \tag{1}$$

Here $\epsilon_n(p) = \epsilon_n(0) + p^2/2m$ is the energy of the electron belonging to the one-dimensional (1D) subband (channel), $m$ is the electron effective mass, $n$ is the quantum number of transverse quantization, $p$ is the $x$ component of the electron quasimomentum, while $\omega_{\mathbf{q}} = wq$ is the frequency of phonons with wave vector $\mathbf{q}$ and $w$ is the sound velocity.

Equation (1) gives $p = mw - \hbar q/2$, $p + \hbar q = mw + \hbar q/2$. This means that in the course of sound absorption there is a quasimomentum transfer from phonons to electrons which should bring about an acoustoelectric current, $J$.

We consider as a typical example electron gas near the GaAS-GaAlAs interface. The elastic properties of both materials are assumed to differ slightly. Therefore the front of the acoustic wave near the interface is distorted slightly as well. Moreover, because of the translational symmetry along the $x$ axis, $q_x$ remains to be a good quantum number.

These equations give a single value for the quasimomentum of electrons that take part in the transitions, which happen if the initial electron state is either within the thermal layer near the Fermi level (if $\hbar\omega_{\mathbf{q}} \ll k_BT$) or within a layer of width $\hbar\omega_{\mathbf{q}}$ between $\mu_n - \hbar\omega_{\mathbf{q}}$ and $\mu_n$ (assuming that $\hbar\omega_{\mathbf{q}} \gg k_BT$). When in the course of gate voltage variation an initial electron state with $p = mw - \hbar q/2$ disappears from such a layer the current drops. With further change in the gate voltage an initial electron state belonging to another subband $n'$ appears now in the layer between $\mu_{n'} - \hbar\omega_{\mathbf{q}}$ and the $\mu_{n'}$, where $\mu_n = \mu - \epsilon_n(0)$ (see Fig. 1) which leads to an increase of the acoustoelectric current. Consequently, one observes what may be called *giant quantum oscillation* of acoustoelectric current as a function of gate voltage. This phenomenon is similar to the giant quantum oscillations of ultrasonic absorption in a magnetic field [4] (as well as in the nanostructures [5]) where due to the system of Landau levels analogous oscillations can be observed in the course of magnetic field variation.

The influence of nonequilibrium phonons on the electrons of a microcontact has been considered by Kozub

  

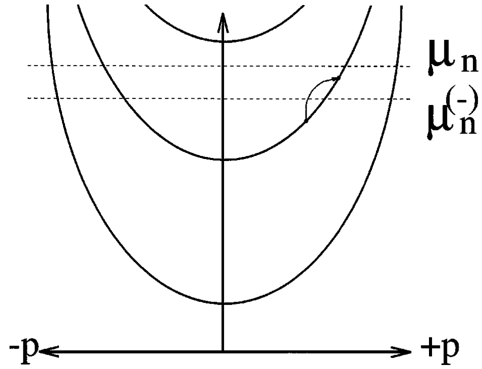


FIG. 1. Horizontal dashed lines define a layer of width $\hbar\omega_{\mathbf{q}}$ between $\mu_n^{(-)} = \mu_n - \hbar\omega_{\mathbf{q}}$ and $\mu_n$ which is wider than the thermal layer if $\hbar\omega_q \gg k_B T$. When an initial electron state with $p = mw - \hbar q/2$ is not within such a band, the transitions are forbidden and no current can flow.

and Rudin [6]. They have investigated thermopower of the quantum point contacts as a consequence of thermal phonon drag of electrons (see also Ref. [7]). With electron-phonon interaction taking place mainly in the reservoirs. In contrast, we are interested in situations where the main effect of the phonon drag on electrons comes from the electron-phonon interaction in the nanowire region, while the effects of phonon drag in the reservoirs can be ignored. For instance, the geometry of a wire and reservoirs can be arranged in such a way that the ultrasound beam is directed along much of the wire while it is practically perpendicular to the large leads that form reservoirs. In such a situation the phonon drag in the reservoirs does not contribute at all to the acoustoelectric current.

We treat the acoustic wave as a flux of phonons. The distribution function of the electrons, $f_n(p)$, in the absence of sound is the Fermi function, $f^{(F)}(\epsilon,)$ where $\epsilon = \epsilon_n(p)$ is the electron energy. Adding a weak interaction of electrons with the ultrasonic phonons we have $f = f^{(F)} + \Delta f$ with $\Delta f$ satisfying the equation

$$v \frac{\partial \Delta f}{\partial x} = I[f] + e \frac{\partial \Delta \phi}{\partial x} \frac{\partial f^{(0)}}{\partial p}\,. \qquad (2)$$

Here $v = \partial \epsilon_n / \partial p$ is the electron velocity (which does not depend explicitly on the quantum number $n$) and $\Delta\phi$ is the time averaged electrostatic potential which may be brought about by the ultrasound wave. This term will be omitted below as it gives no contribution to the current. $I[f]$ represents the electron-phonon collision term. For $p > 0$ ($p < 0$) the solution of Eq. (2) is

$$\Delta f^{\gtrless}(x) = (x \pm L/2) I[f]/v\,. \qquad (3)$$

Here we have assumed that the zero of the coordinate lies at the midpoint of the conductor of a total length $L$ and that the boundary condition $\Delta f^{\gtrless} = 0$ is satisfied at $x = \mp L/2$. The explicit form of the collision term reads

$$\begin{aligned} I[f] &= \int \frac{dp'}{2\pi\hbar} \int \frac{d^2 q_\perp}{(2\pi)^2} W (b^{(+)} + b^{(-)})\,, \\ b^{(\pm)} &= [f'(1-f)(N + 1/2 \pm 1/2) \\ &\quad - f(1-f')(N + 1/2 \mp 1/2)] \\ &\quad \times \delta(\varepsilon' - \varepsilon \mp \hbar\omega_{\mathbf{q}})\,, \end{aligned} \qquad (4)$$

where $N$ is the phonon distribution function and $W$ is the coefficient of electron-phonon interaction (see below). The total current is given by

$$\begin{aligned} J &= 2eL \sum_n \int_0^\infty \frac{dp}{2\pi\hbar} I[f] \\ &\quad - 2eL \sum_n \int_{-\infty}^0 \frac{dp}{2\pi\hbar} I[f] \\ &\quad + 4ex \sum_n \int_{-\infty}^\infty \frac{dp}{2\pi\hbar} I[f]\,. \end{aligned} \qquad (5)$$

The last term in the square brackets vanishes since collisions do not change the total number of electrons.

For the deformation potential DP interaction we have $W = \pi\Lambda^2 q^2/\rho\omega_{\mathbf{q}}$, where $\Lambda$ is the DP constant for the phonon branch in consideration and $\rho$ is the mass density. For the unscreened piezoelectric interaction

$$W_a = (\pi/\rho\omega_{\mathbf{q}})[4\pi e \beta_{q,lq} \nu_l(\mathbf{q}, a)/\varepsilon_{qq}]^2. \qquad (6)$$

Here $\beta_{i,ln}$ is the tensor of piezoelectric moduli which is symmetric in the last two indices (see, for instance, Ref. [8]), $\varepsilon_{il}$ is the tensor of dielectric susceptibility, and $\nu_l(\mathbf{q}, a)$ is the polarization vector (i.e., a unit vector along the elastic displacement $\mathbf{u}$) of the phonon with wave vector $\mathbf{q}$, belonging to the branch $a$. Index $q$ indicates the projection of a tensor on the $\mathbf{q}$ direction.

The frequency dependence of the piezoelectric interaction differs from that of the DP interaction. If the symmetry allows the piezoelectric interaction, it should be predominant for comparatively small values of $\omega_{\mathbf{q}} = wq$. The integrations in Eq. (4) are in fact over the three components of the phonon wave vector. The phonon distribution function $N_{\mathbf{k}}$ can be presented in the form $N_{\mathbf{k}} = [(2\pi)^3 S/\hbar w^2 k]\delta^{(3)}(\mathbf{k} - \mathbf{q})$, where $S$ is the sound intensity. One can discard 1 as compared to $N_{\mathbf{k}}$ in Eq. (4). Then for the electron distribution function determined by Eq. (3) one has

$$\begin{aligned} \Delta f &= \left(x \pm \frac{L}{2}\right) \frac{Sm^2 W}{p(\hbar\omega_{\mathbf{q}})^2} \\ &\quad \times \left\{ (f_{p+\hbar q} - f_p)\delta\left[p - \left(mw - \frac{\hbar q}{2}\right)\right] \right. \\ &\quad \left. + (f_{p-\hbar q} - f_p)\delta\left[p - \left(mw + \frac{\hbar q}{2}\right)\right]\right\}. \end{aligned} \qquad (7)$$

Inserting $N_{\mathbf{k}}$ into Eq. (4) one finally gets for $\hbar q > 2mw$

$$J = \frac{emSWL}{2\pi\hbar^3 \omega_{\mathbf{q}}^2} [f^{(F)}(\epsilon^{(-)} - \mu_n) - f^{(F)}(\epsilon^{(+)} - \mu_n)]\,, \qquad (8)$$

where $\epsilon^{(\pm)} = (\hbar q/2 \pm mw)^2/2m$.

For $\hbar q < 2mw$, one cannot use the approach developed above to calculate a nonvanishing current $J$. This is a consequence of the ballistic nature of transport, and it is due to the current conservation in combination with the charge conservation in the course of electron-phonon collisions. To consider this issue in detail we calculate the rate of variation of 1D electron density $n$ due to electron-phonon collisions. It can be presented as

$$\left[\frac{\partial n}{\partial t}\right]_{\text{coll}} = 2\int_0^\infty \frac{dp}{\pi\hbar} I[f] + 2\int_{-\infty}^0 \frac{dp}{\pi\hbar} I[f].$$

The current is proportional to the difference of the same integrals

$$J = 2eL\int_0^\infty \frac{dp}{2\pi\hbar} I[f] - 2eL\int_{-\infty}^0 \frac{dp}{2\pi\hbar} I[f].$$

When $\hbar q < 2mw$ and therefore both $mw - \hbar q/2$ and $mw + \hbar q/2$ are positive the $\delta$ functions in Eq. (7) do not contribute to the second integral so we are left with

$$\left[\frac{\partial n}{\partial t}\right]_{\text{coll}} = \frac{J}{eL} = 2\int_0^\infty \frac{dp}{2\pi h} I[f]. \tag{9}$$

The integral Eq. (9) should vanish as the collisions conserve the concentration of electrons. Therefore the current which is proportional to the same integral should also vanish. This means in fact that one can expect an abrupt change of the acoustoelectric current at $\hbar q = 2mw$. When other collisions, not related to the ultrasound wave, are taken into account the above considerations are not valid and the current $J$ due to the ultrasound is no longer zero for $\hbar q < 2mw$. One can show, however, that in this case too an abrupt change of the acoustoelectric current as a function of a dimensionless parameter $\eta = \hbar q/2mw$ is still expected at $\hbar q/2mw = 1$.

For $\hbar q > 2mw$ one can consider two limiting cases, i.e., $\hbar\omega_q \gg k_BT$

$$J = (emSWL/2\pi\hbar^3\omega_{\mathbf{q}}^2)\sum_n \theta(\mu_n - \epsilon^{(-)})\theta(\epsilon^{(+)} - \mu_n) \tag{10}$$

and $\hbar\omega_q \ll k_BT$

$$J = \sum_n \frac{emSWL}{8\pi\hbar^2\omega_{\mathbf{q}}k_BT\cosh^2[(\hbar^2\omega_q^2/8mw^2 - \mu_n^{(1)})/2k_BT]}, \tag{11}$$

where $\mu_n^{(1)} = \mu - \epsilon_n(0) - mw^2/2$. Both of these limits are illustrated in Fig. 2. It is worth mentioning that the peak-to-valley ratio is either infinite or exponentially large in this theory.

The current $J$ generated by the ultrasound wave of any branch $a$ in the piezoelectric media can be obtained from equations above with the following substitution: $\Lambda^2 \to \Lambda_a^2 + [4\pi e\beta_{q,lq}\nu_l(\mathbf{q},a)/q\varepsilon_{qq}]^2$. Here $\Lambda_a$ denotes the DP constant for the acoustic wave belonging to branch $a$.

Let us consider as an example a piezoelectric interaction. Then for $\hbar\omega_q/k_BT \gg 1$, assuming that $q = 3 \times 10^6\ \text{m}^{-1}$ and for the effective mass $m = 0.07m_0$, the piezoelectric coupling coefficient [9] $4\pi\beta^2/\varepsilon\rho w^2 = 6 \times 10^{-4}$, the velocity of sound $w = 5 \times 10^5\ \text{cm/sec}$ and mass density $\rho = 5\ \text{g/cm}^3$, we have $J \approx 2 \times 10^{-7}$ A/W and $V \approx 2$ mV/W. Such an effect could be used as an experimental tool for the detection of high frequency ultrasound. It can also be used to investigate the electron spectrum of the microstructure, particularly the positions of levels of transverse quantization.

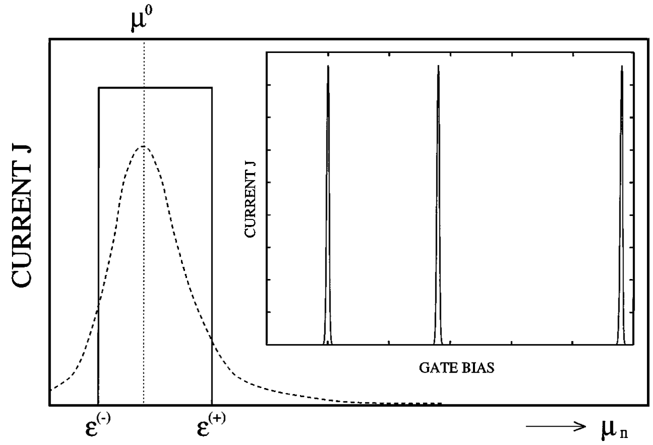


FIG. 2. Plotted is the acoustoelectric current (in units of $emSWL/8\pi\hbar^2\omega_q k_BT$) as a function of the chemical potential $\mu_n = \mu - \epsilon_n(0)$ (which itself is controlled by gate voltage) for a quantum wire of constant cross section. The solid line represents the $\hbar\omega_q \ll k_BT$ case described by Eq. (10). The dashed line represents the $\hbar\omega_q \gg k_BT$ case described by Eq. (11), with peaks at $\mu^0 = (\eta^2 + 1)mw^2/2$, where $\eta = \hbar q/2mw$. Inset: a schematic plot of the giant oscillations of the acoustoelectric current as a function of the gate voltage for $\hbar\omega_q \ll k_BT$ in the limit of $\hbar q > 2mw$ described by Eq. (11). Both $J$ and gate bias are given in arbitrary units.

So far we have assumed that the reservoirs give no contribution to the current. Let us now investigate the contribution from the reservoirs. To do so, it is natural to consider a general case of a nanostructure of a variable cross section which consists of two reservoirs connected by a so-called "wire" region.

Following Glazman *et al.* [10] we shall represent the electrostatic confining potential $\phi(x,y)$ as $\phi(x)$ along the direction of current and the width of an "aperture" $D(x)$. We explicitly assume such form of the gates that one can represent the problem in this way. The transverse dimensions widen adiabatically from the wire region to the reservoirs; namely, $D(x)$ has a minimum $D_0$ at the midpoint $x = 0$ while $D(x) \to \infty$ for $x \to \pm\infty$, i.e., deep within the reservoirs. Both $D(x)$ and the electrostatic potential $e\phi(x,y)$ change slowly on the scale of the de Broglie wavelength $\lambda$ [10]. This is what one refers to as the *adiabatic transport*. We also assume that the width $D_0$ is of the order of the de Broglie wavelength $\lambda_F$. Since $D(x)$ and $\phi(x)$ change with $x$ adiabatically, only the electrons with total energy above the threshold of propagation at $x = 0$ will be transmitted, all others are totally reflected. We shall further assume that such a wire region is of length $L$ and of essentially constant effective confinement width along $x$, the orientation of a wire.

Turning our attention to the reservoirs we first consider a case of a wire so narrow that the two reservoirs are effectively decoupled. The ultrasound wave brings about the acoustoelectric effect in each reservoir, so that one can write for the total current density in a reservoir $\mathbf{j} = -(\sigma/e)\nabla(\mu + e\phi) + \mathbf{j}^{(\mathrm{ac})}$ where the acoustoelectric current $\mathbf{j}^{(\mathrm{ac})}$ is proportional to the sound intensity $\mathbf{S}$: $\mathbf{j}^{(\mathrm{ac})} = \alpha\mathbf{S}$ and where $\alpha$ depends in particular on the scattering rate within the reservoir. Under the assumptions that $\mathbf{j} = 0$ and that the sound is weakly absorbed over the length of a reservoir $L_r$ one can write for the difference of electrochemical potentials over $L_r$, $\Delta(\mu + e\phi) = e(\alpha/\sigma)SL_r$ . In such a case the reservoirs act as two batteries connected in series and determine the current in a short circuited situation.

Since the wire is significantly narrower than the reservoirs, the difference given will be practically unaffected by the tiny current in the wire. The difference $\Delta(\mu + e\phi)$ is one of the sources which contribute to the acoustoelectric current through a nanowire. On the one hand, this source could be excluded in the experimental setup (by arranging the geometry of the nanostructure). On the other hand, if one chooses to investigate this contribution, this difference in the electrochemical potential will drive the ballistic current though a nanostructure much in the same way as an ordinary voltage bias in the absence of ultrasound. If one measures $\alpha$ independently, one can then measure the intensity of sound near the nanostructure.

The third contribution to the acoustoelectric effect is due to the fact that the reservoirs and the regions where the reservoirs converge into a wire can also contribute to the ballistic transport. This contribution, however, is not always present. For instance, in the temperature range where the electron-electron scattering is significant, and therefore results in energy relaxation of electrons, the effects of the phonon drag in the reservoirs are of little importance. At the same time, the possibility of electron-electron collisions in the microstructure itself is severely restricted (cf. with Ref. [11]). Indeed, one can easily check that the conditions for the energy and quasimomentum conservation in the course of electron-electron collisions cannot be satisfied due to the 1D nature of electron dynamics.

The electrochemical potential of the electrons injected into the wire region from the right and the left is determined by the electrochemical potential of the regions of finite extent located roughly where the reservoirs converge into a wire. This region is determined by the condition $\epsilon_n(0,x) - \epsilon_{n'}(0,x) < k_B T$ for adjacent values of $n$ and $n'$. This difference decreases with an increasing $D(x)$ as $|x|$ increases. An injection of electrons into the wire region, and their subsequent absorption of a quasimomentum from the ultrasound, takes place predominantly for those electrons which are injected from $x$ that do not satisfy this condition. For this reason the integration over $x$ in Eq. (5) is not performed from $-\infty$ to $+\infty$ but rather from some $-L/2$ to $+L/2$.

In summary, we have calculated acoustoelectric current in a nanostructure under the ballistic conductance regime. We predict giant oscillations of the current as a function of the gate voltage. We predict also an abrupt variation of the acoustoelectric effect as a function of the ultrasonic frequency at $\omega_{\mathbf{q}} = 2mw^2/\hbar$. It is pointed out that the effect can be used for the detection of ultrasound as well as for the investigation of the electron spectrum of a microstructure.

We are grateful to V. I. Kozub for a number of illuminating discussions, particularly of the physical meaning of parameter $\hbar q/2mw$, and to Yu. Galperin for sending us a preprint [3] and for critical reading of this Letter with many useful suggestions which we have incorporated. We acknowledge support from NATO linkage grant which made possible numerous collaborative visits. V. L. G. is pleased to acknowledge support by the Russian National Fund of Fundamental Research (Grant No. 95-02-04109-a). V. B. P. gratefully acknowledges support of the National Research Council and the U.S. Army Research Office.

[1] R. Landauer, IBM J. Res. Dev. **1**, 233 (1957); **32**, 306 (1989).

[2] Y. Imry, in *Directions in Condensed Matter Physics*, edited by G. Grinstein and G. Mazenko (World Scientific, Singapore, 1986), p. 101; M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).

[3] Giant quantum oscillation of acoustoelectric current in nanostructures has also been treated in a paper by H. Totland and Yu. Galperin (unpublished). Direct comparison of our results is impossible as they considered as an important relaxation mechanism the scattering of electrons by impurities which is assumed to be negligibly small in our case.

[4] V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **40**, 786 (1961) [Sov. Phys. JETP **13**, 552 (1961)].

[5] C. Rodrigues, A. L. A. Fonseca, and O. A. C. Nunes, Phys. Status Solidi B **189**, 117 (1995).

[6] V. I. Kozub and A. M. Rudin, Phys. Rev. B **50**, 2681 (1994).

[7] P. Streda, J. Phys. Condens. Matter **1**, 1025 (1989).

[8] V. L. Gurevich, *Transport in Phonon Systems* (North-Holland, Amsterdam, 1986).

[9] J. M. Shilton *et al.*, J. Phys. Condens. Matter **7**, 7675 (1995); A. Wixforth *et al.* Phys. Rev. B **40**, 7874 (1989).

[10] L. I. Glazman, G. B. Lesovik, D. E. Khmel'nitskii, and R. I. Shekhter, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 218 (1988) [JETP Lett. **48**, 238 (1988)].

[11] J. P. Leburton, Phys. Rev. B **45**, 11 022 (1992).