Self-Similar Barkhausen Noise in Magnetic Domain Wall Motion

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A model for domain wall motion in ferromagnets is analyzed. Long-range magnetic dipolar interactions are shown to give rise to self-similar dynamics when the external magnetic field is increased adiabatically. The power spectrum of the resultant Barkhausen noise is of the form $1/\omega^{\alpha}$, where $\alpha \approx 1.5$ can be estimated from the critical exponents for interface depinning in random media. [S0031-9007(96)01590-6]

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When a domain wall in a ferromagnet moves in response to a change in the externally applied magnetic field, it is known to do so in a jerky, irregular manner. As a result of this irregular motion, the magnetization changes in bursts, leading to the phenomenon of Barkhausen noise. The reason for the unevenness in the motion is that the domain wall is pinned in various places by impurities in the material. The domain wall moves forward by breaking free of the impurities holding it back, only to be obstructed by impurities further ahead. A simple model for the dynamics has been proposed, in which the coordinate of the domain wall is treated as a single dynamical variable [1]. As one might expect, within such a model the temporal fluctuations in the motion of the domain wall (revealed in the Barkhausen noise) yield information about the spatial distribution of the impurities in the material. However, more recent experiments have revealed that this single degree of freedom model for domain wall motion is essentially incomplete [2].

The reason for the inadequacy of the model is that a magnetic domain wall is a *spatially extended* object, with a large number of degrees of freedom. Under slow driving, the dynamics of the domain wall are expected to be governed by the collective behavior of these multiple degrees of freedom. This is reminiscent of "depinning transitions" seen in a variety of driven systems, where close to the transition the dynamics are affected qualitatively by collective behavior [3].

Despite the similarities, there is an important difference between magnetic domain wall motion and conventional depinning transitions. For any value of the external driving force (the magnetic field), a magnetic domain wall reaches a stationary configuration. This stationary configuration appears to be self-similar, a fact inferred experimentally from the power-law correlations in the Barkhausen noise generated when the magnetic field is slowly increased. This is in sharp contrast to conventional depinning transitions, where increasing the external force results in a phase transition from a static to a moving phase at a critical force, and self-similarity is seen only at the transition. The apparent self-similarity in magnetic domain walls achieved without specially adjusting the external magnetic field is reminiscent of the concepts of self-organized criticality [4].

In a recent paper, Urbach *et al.* [5] provide evidence that this departure from conventional depinning transitions is caused by the presence of long-range dipolar interactions in a ferromagnet. These dipolar interactions push the domain wall towards the center of the system. In addition, they also produce long-range effective forces between different parts of the domain wall. Urbach *et al.* numerically solve a model for the dynamics with an approximate treatment of these long-range forces [5]. In one limit, the magnetic force is taken to be infinite ranged, and the numerics indeed yield a power-law distribution for the power spectrum of the resultant Barkhausen noise. However, in the opposite limit, where the magnetic force is taken to be local, self-similar behavior is not seen.

Thus while the tendency of the interactions to push the domain wall towards the center of the system is sufficient to destroy the moving phase commonly seen in such externally driven systems (and thereby the depinning transition leading to it), the exact nature of the interactions is important in determining whether the resultant state is self-similar or not. It is not clear whether an accurate description of the forces induced by the dipolar interactions, which must lie between the two limits considered by Urbach *et al.* [5], will result in self-similar behavior. In this paper I analyze the dynamics of a magnetic domain wall without any approximations for the dipolar interactions, verifying that the resultant behavior is indeed self-similar.

Following Urbach *et al.* [5], I consider a two-dimensional Ising system magnetized perpendicular to the plane. A single domain wall is assumed to run approximately parallel to one of the sides of the system (the transverse direction) and close to its midpoint. The domain wall is characterized by its (small) longitudinal displacement $h(x,t)$ as a function of the transverse coordinate x and time t. [In d dimensions, x is generalized to a $(d-1)$-dimensional vector.] The equation of motion used for the motion of the domain wall is very similar to the one used by Urbach *et al.* [5]:

$$\Gamma \partial_t h(x,t) = k\partial_x^2 h(x,t) + u[h(x,t);x] + H + \int dx' I(x,x') h(x',t) . \quad (1)$$

This equation is obtained by neglecting inertial effects and thermal noise, so that the dynamics are purely relaxational. Γ is a constant characterizing the amount of dissipation. The surface tension of the domain wall gives rise to the first term on the right-hand side, the next term is due to the pinning forces from the impurities, and H is the applied magnetic field. The last term in the equation is obtained by expanding the dipolar interaction energy to second order in $h(x,t)$ as $-\frac{1}{2}\int dx dx' I(x,x') h(x,t) h(x',t)$ and differentiating with respect to h.

We now evaluate the last term in Eq. (1) above. For the perpendicular Ising system considered here, the interaction between two dipoles at $\mathbf{r}_1$ and $\mathbf{r}_2$ is isotropic, and proportional to $|\mathbf{r}_1 - \mathbf{r}_2|^{-3}$. Translational invariance requires that (neglecting edge effects) $I(x,x') = I(x - x')$. From the scale invariance of $|\mathbf{r}_1 - \mathbf{r}_2|^{-3}$, and power counting, the dipolar interaction energy expanded to second order in h must be of the form $-\frac{1}{2}\int dq\, h(q) h(-q) [f(qL)/L^2]$, where L is the linear extent of the system. By considering a uniform displacement, $h(x)$ independent of x, it is possible to verify that $f(qL)$ has a finite $q = 0$ limit. Thus the $q = 0$ limit yields an effective restoring force that drives the domain wall towards the center of the system ($h = 0$), while for $qL \gg 1$ one obtains an effective reduction in the surface tension k, which has no qualitative effect.

If the applied external magnetic field increases at a slow constant rate, the domain wall moves forward at, on the average, a constant rate. The deviation from this uniform motion obeys an equation that can be obtained from Eq. (1). Apart from the last term, the resultant equation is the same as for conventional interface depinning [6].

The crucial feature of the restoring force obtained from the dipolar interactions is that it is *small* for large systems. Thus although the $[kq^2 + f(qL)/L^2] h(q)$ term in Eq. (1) has a "mass term," and therefore a cutoff to the self-similar behavior, this cutoff diverges with system size. In fact, since $1/L^2$ scales in the same manner as q^2 under the renormalization group (and neither term receives loop corrections) [6], the only cutoff to the scaling of quantities like $\langle h(q) h(-q)\rangle$ is the standard finite size cutoff. (There is another cutoff if the external magnetic field is increased at a finite rate instead of adiabatically.)

The scaling of the wavelength and frequency dependent fluctuations in h for the domain wall is obtained from the corresponding interface result:

$$\langle h(q,\omega) h(-q,-\omega)\rangle = L q^{-(2\zeta+1+z)} \Phi(qL, q^z/\omega) , \quad (3)$$

where ζ is the roughness exponent of the interface and z the dynamic exponent. [This form can be seen to be correct by integrating over ω and expressing $\langle h(q,t) h(-q,t)\rangle$ in terms of $\langle h(x,t) h(x',t)\rangle$, which scales as $|x - x'|^{2\zeta}$.] The Barkhausen noise measures the temporal fluctuations in the rate of change of the total magnetization of the system. These are proportional to the fluctuations in the spatially averaged velocity of the domain wall. Thus the power spectrum of the Barkhausen noise is obtained from the velocity-velocity correlation function of the domain wall. Multiplying the right-hand side of Eq. (2) by ω^2 and taking the $q \to 0$ limit yields for the correlations in the spatially averaged velocity

$$L^{-2}\langle \partial_t h(\omega) \partial_t h(-\omega)\rangle = F(\omega L^z)/[L\omega^{(2\zeta+1)/z-1}] . \quad (3)$$

Ignoring the finite size cutoff which occurs at very low frequencies, this has a power-law form. Numerical estimates for the critical exponents in two dimensions [7] indicate that $(2\zeta + 1)/z - 1$ is close to 1.5. This is lower than the exponent of 2 for the power spectrum of the Barkhausen noise obtained in early experiments [8], but is not inconsistent with recent experimental results [9].

It is clear why the short-range model of Urbach *et al.* did not yield self-similar behavior, since the mass term in the equation of motion (and thus the cutoff) is finite. The mean field limit that they consider is even simpler: Since $f(qL) = 0$ for $q \neq 0$, expressing Eq. (1) in terms of $h(x,t) - \overline{h}(t)$ results in exactly the same equation as for conventional interface depinning. The interface velocity is replaced by the domain wall velocity. Since this tends to zero when the external magnetic field is increased adiabatically, the system is at its critical point.

The analysis above is easily generalized to other dimensions, since in all dimensions the interaction energy has a scaling form and a $q = 0$ limit proportional to $1/L^2$. The power law for the Barkhausen noise power spectrum will, however, be different in different dimensions, most notably for $d = 1$, where the exponent of the power law should be zero. This is in contrast to the single degree of freedom model originally proposed for the dynamics [1], where a $1/\omega^2$ dependence is predicted independent of dimension. Extending the results obtained above to systems with multiple domain walls remains an open issue.

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