Efficient Co-generation of Seventh-Harmonic Radiation in Cyclotron Autoresonance Acceleration

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(Received 21 March 1996)

It is shown that the lowest $\mathrm{TE}_{s,l}$ mode in a cylindrical waveguide at frequency $s\omega$, with group velocity nearly identical to group velocity for the TE_{11} mode at frequency ω, is that with $s = 7$, $l = 2$. This allows coherent radiation to be generated at the seventh harmonic during electron cyclotron autoresonance acceleration. Conditions are found where such co-generation of 7th harmonic power at 20 GHz is possible with overall efficiency greater than 80%. This mechanism could make possible high efficiency cm-wavelength high power rf sources suitable for driving a future multi-TeV electron-position collider. [S0031-9007(96)01468-8]

PACS numbers: 84.40.Ik, 41.60.Ap, 41.75.Lx, 52.75.Ms

It has not generally been noted that certain TE_{sl} electromagnetic modes of a cylindrical waveguide at frequency $s\omega$ exhibit coincidental near matching between their group velocities and those of the TE_{11} mode at frequency ω. This basic characteristic of the simplest closed guided wave structure may allow the excitation of strong sth harmonic radiation when a fundamental injected wave interacts with a nonlinear medium in the waveguide. Group velocity matching can allow classical parametric interactions to occur [1] but, as will be shown here, it may also allow coupling in which the nonlinear medium supplies and receives significant power to and from the waves. The nonlinear medium for the work presented here is a relativistic gyrating electron beam; the fundamental TE_{11} mode excites cyclotron autoresonance acceleration (CARA) of the beam [2]; and the lowest mode for which a near match in group velocity obtains is the $\mathrm{TE}_{7,2}$ at the seventh harmonic. It will be shown that high efficiency production of multimegawatt power at 20 GHz is possible through this "co-generation" mechanism, when a beam is energized using 2.856 GHz power. Other high power microwave TE-mode fast-wave harmonic interactions may also be enhanced by this matching phenomenon, including the gyro traveling-wave amplifier, gyroklystron, and waveguide free-electron laser [3]. Similar matching phenomena for TM modes may enhance other types of harmonic interactions, such as in the frequency-doubling magnicon [4].

The term co-generation is used here to signify that harmonic power is generated in the self-same structure where CARA acceleration (and deceleration) of the beam occurs. Since the interaction to be discussed occurs in a traveling-wave structure, injected rf power that does not become transformed into the harmonic wave can be recovered, recycled, and perhaps used in a subsequent co-generator stage. Energy in the spent electron beam emerging from the interaction can also be recovered with high efficiency since the energy spread on the beam induced in the acceleration and generation processes can be small. It is these unusual features that permit overall 7th harmonic co-generation efficiencies greater than 80% to be predicted, as is illustrated below.

A prior approach that utilizes CARA for gyroharmonic conversion divides the acceleration in CARA from the region of harmonic generation that occurs in a separate output waveguide [5]. Experiments with CARA with beams up to about 10 MW have shown, in agreement with theory, that the acceleration process can be highly efficient, with efficiency measurements reported as high as 96% [6]. But as a CARA beam drifts (along a rising magnetic field) into an output waveguide where gyroharmonic conversion can occur, the gyrophase distribution of the beam can spread, since strong rf trapping is not present in the drift region to preserve good phase coherence among the beam particles. Nevertheless, 4th harmonic output with conversion efficiency values of over 48% are predicted for this type of gyroharmonic converter; experiments to attempt to confirm this prediction are underway [7]. Improved particle phase coherence might be expected if the strong rf trapping forces acting within CARA are present during emission of the harmonic radiation, and also because of the resulting short interaction length.

The requirement for group velocity matching for efficient power transfer between two guided waves that interact with an electron beam near gyroresonance and its harmonics can be easily appreciated. Below are written the resonance conditions that must be satisfied approximately for simultaneous acceleration due to the CARA mechanism, and for deceleration due to the generation of radiation near the sth gyroharmonic, namely,

$$\omega = \frac{\Omega}{\gamma} + ck_{z,11}\beta_z \,, \tag{1}$$

and

$$s\omega = \frac{s\Omega}{\gamma} + ck_{z,sl}\beta_z \,. \tag{2}$$

In Eqs. (1) and (2), ω is the radian frequency at which CARA is operated, $\Omega = eB_0/m$ is the rest gyrofrequency for electrons of mass m and charge e in a magnetic field B_0, γ and $c\beta_z$ are the relativistic energy factor and the axial velocity for the beam electrons, and $k_{z,sl} = (s\omega/c)(\nu_{g,sl}/c) = (s\omega/c)n_{sl}$ is the axial wave number, itself proportional to the normalized group velocity n_{sl} for each wave; for CARA, $s = l = 1$. Clearly, Eqs. (1) and (2) cannot be satisfied simultaneously unless

$$\frac{k_{z,sl}}{s} = k_{z,11} \quad \text{or} \quad n_{sl} = n_{11}\,, \tag{3}$$

namely, equal group velocities for the two modes, when the frequency of the TE$_{sl}$ mode is s times that of the TE$_{11}$ mode. This is consistent with the selection rule for gyroharmonic conversion with an axis-encircling beam in a cylindrical waveguide, namely, that power at the sth harmonic can flow cumulatively from the beam only into TE$_{sl}$ modes [5]. A measure of the degree to which TE$_{sl}$ modes satisfy Eq. (3) can be seen by substituting into it the relation $n_{sl} = \sqrt{1 - (j'_{sl}c/s\omega R)^2}$, where j'_{sl} is the lth zero of the Bessel function derivative $(d/dx)J_s(x)$, and where R is the waveguide radius. When $n_{sl} \approx n_{11}$, one can derive the approximate relationship

$$\frac{n_{sl}}{n_{11}} \approx 1 + (1 - r_{sl})\left(\frac{1}{n_{11}^2} - 1\right), \tag{4}$$

where $r_{sl} = j'_{sl}/sj'_{11}$. In Fig. 1 are plotted values of r_{sl} for modes with l (the radial index) up to 8, and s (the azimuthal index) up to 30. It is seen that r_{sl} is within the range 1.00 ± 0.005 for the TE$_{7,2}$, TE$_{13,3}$, TE$_{24,5}$ and TE$_{30,6}$ modes. This suggests that one might find conditions where a prescribed magnetic field profile allows efficient CARA acceleration and simultaneously allows strong radiation at one or more of the 7th, 13th, 24th, and 30th harmonics. Since successively higher beam energies are required to obtain significant coupling as s increases, there is reason to expect that conditions might be found where only one of these modes will be excited at a time. For a beam with finite initial velocity spread, small detunings from precise resonance in Eqs. (1) and (2) are introduced so as to maximize acceleration in CARA and to achieve maximum gyroharmonic conversion. These detunings can exceed detuning caused by group velocity mismatch for the modes identified above, so that the mismatch may be relatively inconsequential. In a rectangular waveguide, group velocity matching is much more prevalent, occurring (for example) between the TE$_{01}$ and all TE$_{0s}$ modes at the sth harmonics. This situation could lead to serious mode competition at high harmonics; moreover, the aforementioned selection rule for axis-encircling beams does not apply for rectangular waveguides. Thus only cylindrical waveguides are appropriate for co-generation.

In this Letter, we examine co-generation at the 7th harmonic, but possible competing modes are also considered. We take the rf source frequency to be 2.856 GHz, corresponding to the frequency of the 65 MW klystrons

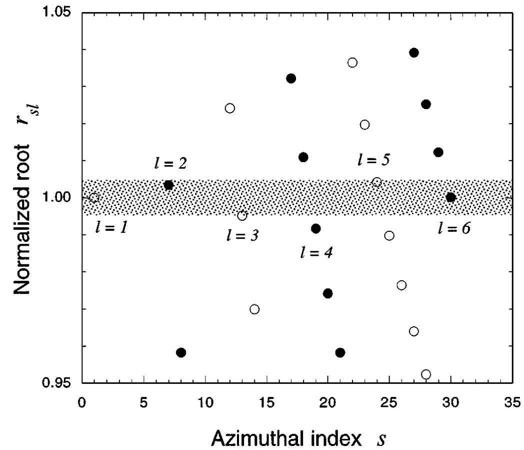

FIG. 1. Ratio of Bessel function roots r_{sl} for $1 \le s \le 30$ and $1 \le l \le 6$. In the crosshatched region, $0.995 \le r_{sl} \le 1.005$, and near matching of group velocities occurs.

(type 5045) that drive the Stanford Linear Collider (SLC), so that the 7th harmonic is at 19.992 GHz. A calculation could be made for any rf source frequency, but we purposely have chosen 2.856 GHz in anticipation of conducting experiments using an existing rf source, and in conceiving of the rf driver system for a future multi-TeV electron-positron collider being built using existing mature rf technology, including type 5045 klystrons. Thus to examine in detail the co-generation of harmonic radiation within CARA, particle simulation studies were carried out using methods described previously [5], for examples with rf and injected beam powers corresponding to Yale/Omega-P experimental capabilities, namely for an rf source power level at 2.856 GHz of 10.0 MW, and an injected beam of 100 kV, 25 A.

Results to illustrate the principle of co-generation are shown in Fig. 2 for an ideal "cold" beam, i.e., a beam with no initial axial momentum spread or guiding center spread. Figure 2(a) shows the evolution along the axis in CARA of rf power in the TE$_{11}$ and TE$_{7,2}$ modes at 2.856 GHz and 19.99 GHz in a waveguide of radius 3.65 cm ($n_{11} = 0.5374$ and $n_{7,2} = 0.5329$); Figure 2(a) also shows the variation in imposed resonant axial magnetic field, according to Eq. (1). A pronounced recurrence phenomenon is observed, with rf power alternately rising and falling for the fundamental and 7th harmonic waves. At $z = 84.0$ cm, where the 7th harmonic power has reached 3.32 MW, the fundamental power is 6.22 MW, and the sum of the two is only 5% less than the 10.0 MW fundamental power injected initially. Figure 2(b) shows the ensemble-average beam energy factor $\langle\gamma\rangle$, the relative rms energy spread $\delta\gamma/\langle\gamma\rangle$, and the ratio of the ensemble-average gyration radius to waveguide radius $\langle r_g/R\rangle$. The beam energy at $z = 84.0$ cm is 100.4 kV, so the beam is seen to have acted essentially as a catalyst by exchanging power with the rf modes up to this point, but returning nearly to its original state. At $z = 167.2$ cm, the fundamental rf power level has risen to 9.45 MW, and

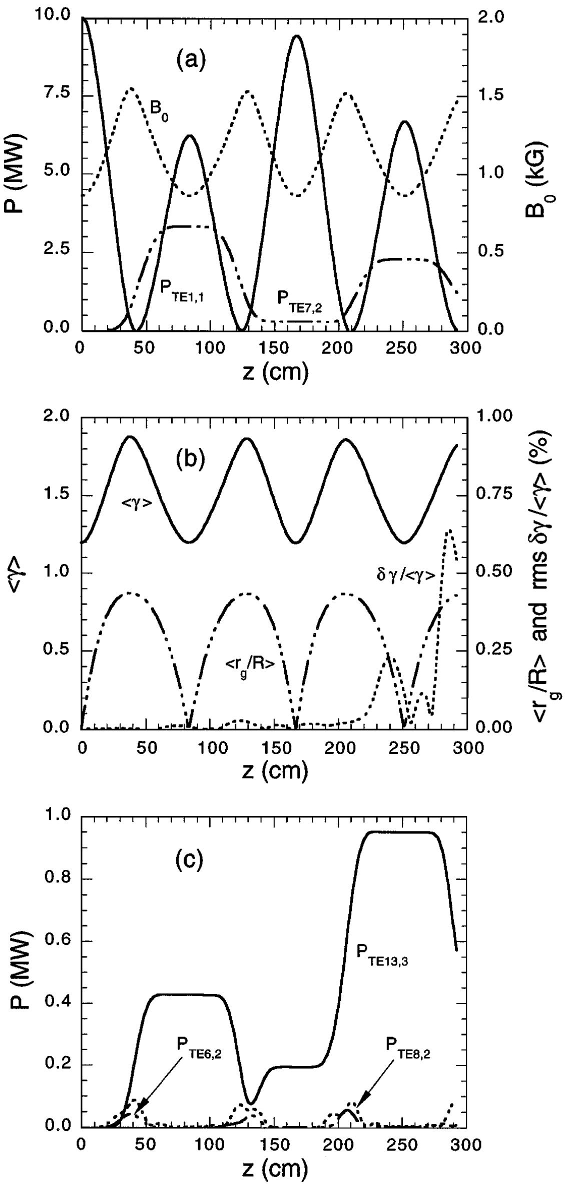

FIG. 2. Example of 7th harmonic co-generation for an initially cold beam. (a) Spatial evolution of fundamental power at 2.856 GHz, 7th harmonic power at 19.992 GHz, and resonant axial magnetic field B_0. (b) Spatial evolution of average beam energy factor, average ratio of gyration radius to waveguide radius, and percentage rms energy spread. (c) Spatial evolution of power in 6th, 8th, and 13th harmonics, in the $TE_{6,2}$ $TE_{8,2}$, and $TE_{13,3}$ modes, respectively.

only 0.31 MW resides in the 7th harmonic wave; again the beam energy is back to 99.9 kV. In the course of this one full recurrence cycle, the magnetic field varies quasi-sinusoidally between 0.86 and 1.55 kG. Figure 2(c) shows the rf power at the 6th harmonic in the $TE_{6,2}$ mode, at the 8th harmonic in the $TE_{8,2}$ mode, and at the 13th harmonic in the $TE_{13,3}$ mode. The maximum total of 6th and 8th harmonic power is 0.13 MW, while the 13th harmonic power is seen to reach 0.427 MW at $z = 84.0$ cm, and 0.949 MW at $z = 251$ cm. The fact that the $TE_{13,3}$ mode is the only serious competitor to the $TE_{7,2}$ enforces the claim made above that group velocity matching is a major factor in co-generation, since $n_{13,3} = 0.5438$. More power appears at the 13th harmonic during the second recurrence cycle because of injection of finite power from the first cycle. Were the device to be terminated at $z = 84.0$ cm, rf power could be extracted with about one-third at the 7th harmonic and two-thirds at the fundamental; the beam power could in principle be largely recovered using a depressed collector. The 6.22 MW of extracted fundamental power could, in principle, be used (with an additional 3.78 MW) to drive a second CARA identical to the first, and the sequence continued indefinitely. Such a system would have a nearly ideal conversion efficiency, the net effect of which is to convert rf power from the fundamental to the 7th harmonic. Losses in this system arise from finite conductivity in the waveguide walls, from imperfect beam power recovery at a depressed collector, and from power converted to competing harmonics. But these can each be small fractions of the total 12.5 MW in circulation. Of course, an injected beam with zero axial velocity spread is an idealization that cannot be met in practice.

Results of a computation for an injected beam with a finite rms axial velocity spread of 0.10% are shown in Fig. 3, with the various quantities displayed as in Fig. 2. The basis for employing this value of axial velocity spread is a design study using the DEMEOS code [8] for the moderate convergence, perveance $K = 1.0 \times 10^{-6}\ \mathrm{A\,V^{-3/2}}$ Pierce-type electron gun that is in operation on the Yale/Omega-P CARA [6]. Two significant additional features are present in this example, namely a detuning of the axial magnetic field from exact resonance, and a finite amplitude injected signal at the 7th harmonic. Without magnetic field detuning, co-generation is found to be severely weakened by velocity spread. The optimum detuning parameter $\Delta = \Omega/\omega\gamma\beta_z - 1/\beta_z + n_{11}$ for this case was found to be -0.02, a magnitude well in excess of the group velocity mismatch $n_{11} - n_{7,2} = 0.0045$. Injection at the CARA input of 0.10 MW of 7th harmonic power in the $TE_{7,2}$ mode was found to yield a 7th harmonic power level at $z = 82$ cm of 1.99 MW, shown in Fig. 3(a). To achieve this result, it was necessary to optimize the phase of the injected initial 7th harmonic power. If the injected 7th harmonic power level is increased from 0.1 to 1.0 MW, the 7th harmonic output increases to 3.9 MW, for gain of an additional 1.0 MW. Injection can be accomplished by recirculating to the input a portion of the 7th harmonic output power. For higher initial axial velocity spreads, namely 0.15% and 0.20%, the maximum 7th harmonic powers for the parameters as in Fig. 3 were found to be 1.61 MW and 1.26 MW; this result adds emphasis to the need for carefully designed injector gun. If a device based on the results shown in Fig. 3 were terminated at $z = 82$ cm, one could extract 1.99 MW of 7th harmonic power, 6.47 MW of fundamental harmonic power, and 0.098 MW of power in competing harmonics. The efficiency for utilization of rf power then follows as $\eta_{\mathrm{rf}} = (1.99 + 6.47)/10.10 = 83.8\%$. If energy in the

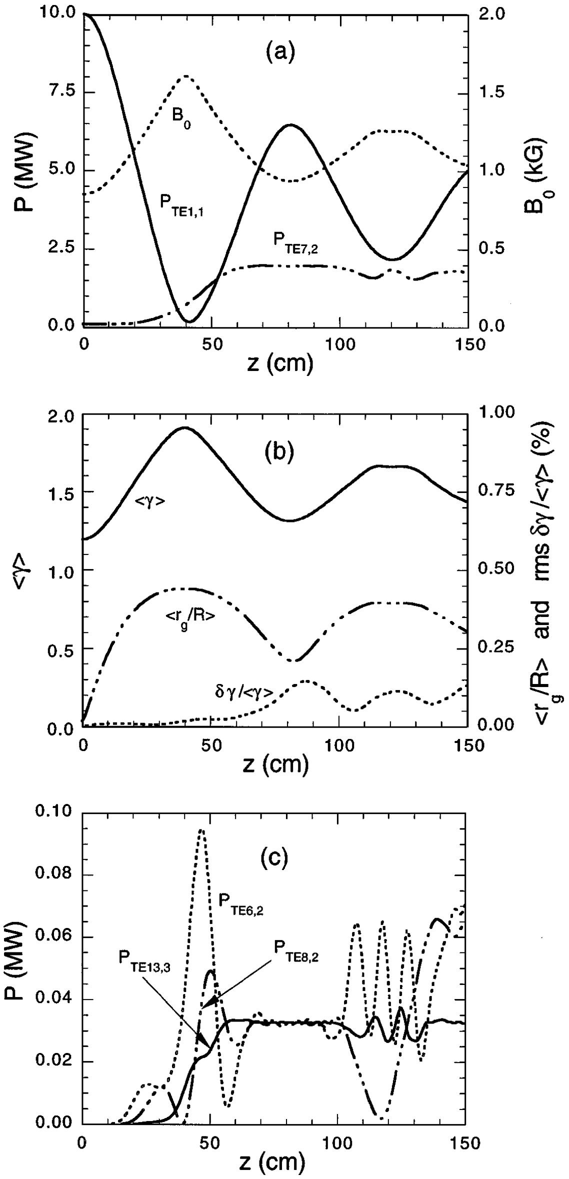

FIG. 3. Example of 7th harmonic co-generation for a beam with an initial axial velocity spread of 0.10%. Quantities depicted are as in Fig. 2, except that axial scale is shortened to 150 cm. Here, magnetic field detuning with $\Delta = -0.02$ is imposed, and 0.10 MW of 7th harmonic power is injected.

spent electron beam is included (3.86 MW), one would have $\eta_{\text{total}} = (8.46 + 3.86\eta_{\text{rec}})/12.6$, where η_{rec} is the efficiency for recovery of beam power using a depressed collector. One can define $\eta_{\text{rec}} = (\gamma_r - 1)/(\langle\gamma_s\rangle - 1)$ as an ideal recovery efficiency for a single-stage depressed collector, where $\gamma_r = 1 + e|V_r|/mc^2$ is the energy factor corresponding to the collector retarding potential V_r, and where $\langle\gamma_s\rangle$ is the average energy factor of the spent beam particles. To avoid undesirable beam reflection at the collector, γ_r is taken to be the lowest value of beam energy factor in the spent beam. For the beam at $z = 82$ cm in Fig. 3, computations have been performed that yield $\eta_{\text{rec}} = 63.0\%$. Using this gives $\eta_{\text{total}} = 86.4\%$. This figure embodies several idealizations, but it seems high enough to provide strong motivation for further study of co-generation.

The results described in this Letter suggest that it may be possible to design a train of high power 20 GHz co-generators with a total power efficiency of greater than 80%. This train of co-generators could be driven using existing SLC klystrons and associated modulators. Of course, the examples presented in this Letter are for a 10 MW rf driver, namely, at the level where experimental tests are planned. Further studies are required to optimize a system using 65 MW input pulses as provided by type 5045 klystrons. But it does not seem unreasonable for each co-generator in such a system to provide 50–60 J output pulses at 20 GHz with an overall rf system efficiency, including the klystron drivers, that would exceed 50%. This system could constitute the sought-after rf source for a future multi-TeV electron-positron collider. Co-generation interactions with other fast wave interactions should also be possible, as shown by preliminary experiments on 7th harmonic generation in a waveguide FEL amplifier [3]. Needless to say, the potential and practicality of co-generation will not be fully confirmed until additional experiments and design studies of this new mechanism are carried out.

This research was sponsored in part by the U.S. Department of Energy and in part by the U.S. Office of Navel Research. Fruitful discussions with B. Hafizi are acknowledged.

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