Continuum Study of Deconfinement at Finite Temperature

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(Received 4 June 1996)

Deconfinement and chiral symmetry restoration are explored in a confining, renormalizable, Dyson-Schwinger equation model of two-flavor QCD. An order parameter for deconfinement is introduced and used to establish that, in the chiral limit, deconfinement and chiral symmetry restoration are coincident at $T_c \approx 150$ MeV. The transitions are second order and each has the same critical exponent: $\beta \approx 0.3$. The deconfinement transition exhibits sensitivity to the current-quark mass. f_π and m_π change by less than 10% for $T < 0.7T_c$; however, as $T \to T_c$, thermal fluctuations cause the pion bound state contribution to the four-point quark-antiquark correlation function to disappear. [S0031-9007(96)01539-6]

PACS numbers: 12.38.Aw, 11.10.Wx, 11.15.Tk, 24.85.+p

The Dyson-Schwinger equations (DSEs) provide a nonperturbative, renormalizable, continuum framework for analyzing quantum field theories. An important example is the fermion DSE, which has proven useful in the study of confinement and dynamical chiral symmetry breaking (DCSB) [1]. The DSEs form a tower of coupled equations, which must be truncated to arrive at a tractable problem. Truncations that preserve the global symmetries of a field theory are easy to implement. Preservation of a gauge symmetry is more difficult, but there is progress in that direction [2]. The elementary quantities are the Schwinger functions whose analytic properties provide information about confinement and DCSB; e.g., the absence of a Lehmann representation for the two-point dressed-gluon Schwinger function (gluon propagator) entails the absence of asymptotic gluon states; i.e., gluon confinement. The approach is reviewed in Ref. [3].

The finite-T properties of QCD are important in astrophysics and cosmology, and may be explored in a future-generation heavy-ion accelerator program. Theoretical tools that can be employed reliably in the nonperturbative study of deconfinement and chiral symmetry restoration are therefore valuable. In Ref. [4] a one-parameter model dressed-gluon propagator was proposed, which provided a good description of π- and ρ-meson observables. The single parameter in this model is a mass scale, m_t, that marks the point where the nonperturbative, infrared enhancement found in gluon DSE studies [5] becomes dominant. The model gluon propagator has no Lehmann representation. The calculated quark propagator also has no Lehmann representation; therefore both the gluon and quark are confined. With m_t fixed at $T = 0$, one has a renormalizable DSE model of QCD, which manifests both confinement and DCSB, whose finite-T behavior may provide insight into the finite-T properties of QCD.

The DSE for the renormalized dressed-quark propagator at finite T involves a sum over Matsubara frequencies. In nonperturbative DSE studies, where the analytic structure of the dressed-quark propagator is calculated rather than assumed, it is necessary to perform this sum numerically because the usual analytic methods of evaluating it rely upon the quark propagator having a Lehmann representation, which may not be the case for a confined quark. The absence of a Lehmann representation also complicates, if not precludes, a real-time formulation of the finite temperature theory.

Herein we employ a Euclidean space formulation with $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_\mu = \gamma_\mu^\dagger$, $\gamma \cdot p \equiv \sum_{i=1}^3 \gamma_i p_i$, $\omega_k \equiv (2k+1)\pi T$, and $\Omega_k \equiv 2k\pi T$. The DSE for the renormalized quark propagator, $S \equiv [-i\gamma \cdot p\sigma_A(p,\omega_k) - i\gamma_4\omega_k\sigma_C(p,\omega_k) + \sigma_B(p,\omega_k)]$, is

$$S^{-1}(p,\omega_k) \equiv i\gamma \cdot pA(p,\omega_k) + i\gamma_4\omega_k C(p,\omega_k) + B(p,\omega_k) \quad (1)$$

$$= Z_2^A i\gamma \cdot p + Z_2(i\gamma_4\omega_k + m_{\rm bm}) + \Sigma'(p,\omega_k), \quad (2)$$

$m_{\rm bm}$ is the bare mass and the regularized self energy is

$$\Sigma'(p,\omega_k) = i\gamma \cdot p\Sigma'_A(p,\omega_k) + i\gamma_4\omega_k\Sigma'_C(p,\omega_k) + \Sigma'_B(p,\omega_k), \quad (3)$$

with

$$\Sigma'_{\mathcal{F}}(p,\omega_k) = \int_{l,q}^{\bar\Lambda} \tfrac{4}{3}g^2 D_{\mu\nu}(p-q,\omega_k-\omega_l)\,\tfrac{1}{4}\,{\rm tr}[\mathcal{P}_{\mathcal{F}}\gamma_\mu S(q,\omega_l)\Gamma_\nu(q,\omega_l;p,\omega_k)], \quad (4)$$

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where $\mathcal{F} = A, B, C$; $\mathcal{P}_A \equiv -(Z_1^A/p^2) i\gamma \cdot p$, $\mathcal{P}_B \equiv Z_1$, $\mathcal{P}_C \equiv -(Z_1/\omega_k) i\gamma_4$, and $\int_{l,q}^{\bar{\Lambda}} \equiv T \sum_{l=-\infty}^{\infty} \int^{\bar{\Lambda}} d^3q/(2\pi)^3$. In Eq. (4), $\Gamma_\nu(q, \omega_l; p, \omega_k)$ is the renormalized dressed quark-gluon vertex and $D_{\mu\nu}(p, \Omega_k)$ is the renormalized dressed-gluon propagator.

In renormalizing we require that

$$S^{-1}(p, \omega_0)|_{p^2+\omega_0^2=\mu^2} = i\gamma \cdot p + i\gamma_4\omega_0 + m_R\,, \quad (5)$$

which entails that the renormalization constants are given by $Z_2^A(\mu, \bar{\Lambda}) = 1 - \Sigma_A'(\mu, \omega_0; \bar{\Lambda})$, $Z_2(\mu, \bar{\Lambda}) = 1 - \Sigma_C'(\mu, \omega_0; \bar{\Lambda})$, $m_R(\mu) = Z_2 m_{\rm bm}(\bar{\Lambda}^2) + \Sigma_B'(\mu, \omega_0; \bar{\Lambda})$, and the renormalized self energies are

$$\mathcal{F}(p, \omega_k; \mu) = \xi_{\mathcal{F}} + \Sigma_{\mathcal{F}}'(p, \omega_k; \bar{\Lambda}) - \Sigma_{\mathcal{F}}'(p, \omega_0; \bar{\Lambda})\,, \quad (6)$$

$\mathcal{F} = A, B, C$, $\xi_A = 1 = \xi_C$, and $\xi_B = m_R(\mu)$.

It is invalid to neglect $A(p, \omega_k; \mu)$ and $C(p, \omega_k; \mu)$ and their dependence on their arguments. In studying confinement the (p, ω_k) dependence of A and C is qualitatively important since it can conspire with that of B to eliminate free-particle poles in the quark propagator [6].

The finite-T gluon propagator in Landau gauge is

$$g^2 D_{\mu\nu}(p, \Omega) = P_{\mu\nu}^L(p, \Omega)\Delta_F(p, \Omega) + P_{\mu\nu}^T(p)\Delta_G(p, \Omega)\,, \quad (7)$$

$$P_{\mu\nu}^T(p) \equiv \begin{cases} 0; & \mu \text{ and/or } \nu = 4\,, \\ \delta_{ij} - \frac{p_i p_j}{p^2}; & \mu, \nu = 1, 2, 3\,, \end{cases} \quad (8)$$

with $P_{\mu\nu}^T(p) + P_{\mu\nu}^L(p, p_4) = \delta_{\mu\nu} - p_\mu p_\nu / \sum_{\alpha=1}^4 p_\alpha p_\alpha$; $\mu, \nu = 1, \ldots, 4$. A "Debye-mass" for the gluon appears as a T-dependent contribution to Δ_F.

Herein we employ a one-parameter model dressed-gluon propagator. The value of this parameter, $m_t = 0.69$ GeV, was fixed in the $T = 0$ studies of Ref. [4]. Our extension of this model to finite T, which involves no additional parameters, is defined with $\Delta_F(p, \Omega) \equiv \mathcal{D}(p, \Omega; m_D)$ and $\Delta_G(p, \Omega) \equiv \mathcal{D}(p, \Omega; 0)$;

$$\mathcal{D}(p, \Omega; m) \equiv 4\pi^2 d \left[\frac{2\pi}{T} m_t^2 \delta_{0n} \delta^3(p) + \frac{1 - e^{[-(p^2+\Omega^2+m^2)/4m_t^2]}}{p^2 + \Omega^2 + m^2} \right], \quad (9)$$

where $d = 12/(33 - 2N_f)$ and the "Debye mass" is $m_{\rm D}^2 = \bar{c}T^2$, $\bar{c} = 4\pi^2 dc$, $c = (N_c/3 + N_f/6)$, which is included in a manner analogous to that in Ref. [7]. There is no T-dependent mass in Δ_G.

The first term in Eq. (9) is an integrable, infrared singularity [8] that generates long-range effects associated with confinement [5]. The second term ensures that, neglecting quantitatively unimportant logarithmic corrections, the propagator has the correct perturbative behavior at large spacelike arguments. In this model the large-distance, confining effects associated with $\delta_{0n}\delta^3(p)$ are completely canceled at small distances and one recovers the perturbative result. $\mathcal{D}(p, \Omega; m)$ has no Lehmann representation and hence represents a confined particle, since this ensures the absence of gluon production thresholds in S-matrix elements describing color-singlet to singlet transitions.

Equation (9) is an Ansatz based on $T = 0$ studies augmented by finite-T perturbation theory. Solving the gluon DSE at finite T would provide additional insight, but no such studies are currently available. Without further information; for example, solving the coupled gluon-quark DSE system or lattice estimates of the two-point gluon Schwinger function, we cannot be sure that the T dependence we have introduced provides a qualitatively accurate representation of the temperature evolution of the gluon propagator. The paucity of relevant experimental data means that one cannot presently use experiment to constrain this extrapolation. This is the primary source of uncertainty in our study, and we therefore advocate a cautious interpretation of our results near the transition temperature.

The model of Ref. [4] is recovered with two further simplifying specifications. The approximation

$$Z_1 = Z_2 \quad \text{and} \quad Z_1^A = Z_2^A \quad (10)$$

is used. Gauge invariance in finite-T QED entails these Ward identities. In QCD the analogous identities are more complicated, and this is only a simplifying truncation, which has proven qualitatively and quantitatively reliable in Landau gauge, $T = 0$ studies. In nonperturbative DSE studies [4,9] Z_2 and Z_2^A are finite and ≈ 1.

In addition, the rainbow approximation

$$\Gamma_\mu(q, \omega_l; p, \omega_k) = \gamma_\mu \quad (11)$$

is used. In $T = 0$ studies this has proven to be reliable in Landau gauge; i.e., an efficacious phenomenology with a more sophisticated Ansatz only requires a small quantitative modification of the parameters that characterize the small-k^2 behavior of the gluon propagator [6,10]. To verify our assumption that this remains true at finite T, one must repeat the development of our model using one of the more sophisticated vertex Ansätz discussed in Refs. [3,11] or employ the systematic procedure presented in Ref. [12].

Confinement can be investigated by studying the analytic properties of the two-point Schwinger function of a given excitation. The confinement test proposed in Ref. [11], and used to good effect in Ref. [1], is appropriate to this task. Consider $\Delta_{B_0}(x, 0) \equiv (T/2\pi x) \times \sum_{k=0}^\infty \Delta_{B_0}^k(x)$

$$\Delta_{B_0}^k(x) \equiv \frac{2}{\pi} \int_0^\infty dp\, p \sin(px) \sigma_{B_0}(p, \omega_k)\,, \quad (12)$$

where the subscript "0" denotes a quantity calculated in the chiral limit, $m_R \to 0$, in this case B.

For a free fermion of mass m, $\sigma_B(p,\omega_k) = m/[\omega_k^2 + p^2 + m^2]$, hence $\Delta_B^k(x) = m\exp(-x\sqrt{\omega_k^2 + m^2})$. This illustrates that the $k = 0$ term dominates the sum. In this case, $M(x;T) \equiv -(d/dx)\ln|\Delta_B^0(x)| = \sqrt{\pi^2 T^2 + m^2}$, a finite constant. Finite-T effects become important for $T \sim m/\pi$. In DSE studies, m is a mass-scale characteristic of dynamical mass generation, $M_E^{u/d} \approx 300$ MeV; hence we expect finite-T effects to become noticeable at $T \sim 100$ MeV.

A Schwinger function with complex-conjugate poles, with $\Re(p^2) < 0$ (timelike) and $\Im(p^2) \propto b$, does not have a Lehmann representation; i.e., it represents a confined excitation with no associated asymptotic state. This is the nature of the quark Schwinger function obtained in Ref. [4]. In such cases $\Delta_{B_0}^0(x)$ has at least one zero. Denote the position of the first zero in $\Delta_{B_0}^0(x)$ by $r_0^{z_1}$, which is inversely proportional to b [11]. Define $\kappa_0 \equiv 1/r_0^{z_1}$, then $\kappa_0 \propto b$, and deconfinement is observed if, for some $T = T_c$, $\kappa_0(T_c) = 0$; at this point thermal fluctuations have overwhelmed the confinement scale parameter and the poles have migrated to the real axis.

We solve the DSE for $S(p,\omega_k)$ numerically with

$$m_t = 0.69 \text{ GeV}, \qquad m_R(\mu) = 1.1 \text{ MeV}, \qquad (13)$$

which were fixed in Ref. [4] by requiring a best χ^2 fit to a range of π-meson observables at $T = 0$. We use $d = 4/9$ in Eq. (9) and renormalize at $\mu = 9.47$ GeV [4]. With these choices our current study has no free parameters. We use $\bar{\Lambda}/\mu = 1$, $|\omega_{k_{\max}}|/|\vec{p}|_{\max} \approx 1$, $n_{|p|} = 64$ points in the $|p|$ array. Our results are cutoff independent. At $T = 5$ MeV the results in Table I of Ref. [4] are reproduced to within 6%.

At finite T the pion mass is given by [4]

$$m_\pi^2 N_\pi^2 = \langle m_R(\mu)\,(\bar{q}q)_\mu \rangle_\pi ; \qquad (14)$$

$$\langle m_R(\mu)\,(\bar{q}q)_\mu \rangle_\pi \equiv 8N_c \int_{k,p}^{\bar{\Lambda}} B_0(\sigma_{B_0} - B_0[\omega_k^2\sigma_C^2 + p^2\sigma_A^2 + \sigma_B^2]), \qquad (15)$$

which vanishes linearly with $m_R(\mu)$, and

$$N_\pi^2 = 2N_c \int_{k,p}^{\bar{\Lambda}} B_0^2 \{ \sigma_A^2 - 2[\omega_k^2\sigma_C\sigma_C' + p^2\sigma_A\sigma_A' + \sigma_B\sigma_B'] - \tfrac{4}{3}p^2(\{\omega_k^2[\sigma_C\sigma_C'' - (\sigma_C')^2] + p^2[\sigma_A\sigma_A'' - (\sigma_A')^2] + \sigma_B\sigma_B'' - (\sigma_B')^2\})\}, \qquad (16)$$

with $\sigma_B' \equiv \partial\sigma_B(p^2,\omega_k)/\partial p^2$, etc. N_π is the canonical normalization constant for the Bethe-Salpeter amplitude in ladder approximation. The pion decay constant is obtained from

$$f_\pi N_\pi = 4N_c \int_{k,p}^{\bar{\Lambda}} B_0\{\sigma_A\sigma_B + \tfrac{2}{3}p^2(\sigma_A'\sigma_B - \sigma_A\sigma_B')\}. \qquad (17)$$

Equation (14) accurately estimates the mass obtained in solving the dressed-ladder pion Bethe-Salpeter equation [4]. Equations (15)–(17) neglect O(m_π^2) corrections, which are unimportant (<3%) for all values of T; i.e., m_π is dominated by a linear response to m_R at all T.

In deriving these formulas one assumes the pion Bethe-Salpeter amplitude $\Gamma_\pi = i\gamma_5 B_0$, which is a manifestation of Goldstone's theorem [12]. Since $B_0 \not\equiv 0$ only if chiral symmetry is dynamically broken, these formulas are not valid above any chiral symmetry restoration temperature, T_c^χ.

All calculated observables are quantitatively sensitive to A and C; e.g., in the present study we find $A(0,\omega_0) \sim 1.5$ and $C(0,\omega_0) \sim 1.5-1.8$ (increasing with T) and they return to 1 at the renormalization point—neglecting this variation entails $f_\pi \to 2f_\pi$, which one may not be able to compensate by readjusting a model's parameters.

We employ the simplest order parameter for DCSB,

$$\chi \equiv B_0(p = 0, \omega_0). \qquad (18)$$

We plot $\chi(T)$ and $\kappa_0(T)$ in Fig. 1. The curves, fitted on $T \in [120, 150]$ MeV, are of the form $\alpha(1 - T/T_c)^\beta$ with $T_c = 150$ MeV and α, β given in Table I. The transitions are coincident and $\beta_\chi = \beta_{\kappa_0}$, within errors, ~10%.

N_π^2, $N_\pi f_\pi$, and $\langle m_R(\mu)\,(\bar{q}q)_\mu \rangle$ behave similarly and the parameters characterizing their behavior at the transition

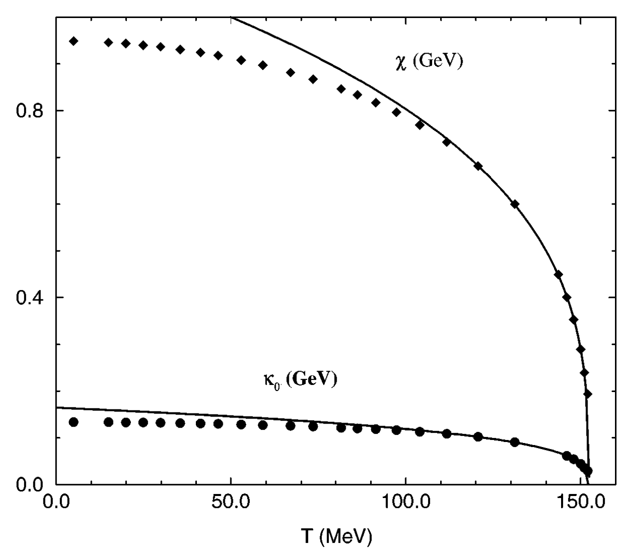

FIG. 1. The order parameters for chiral symmetry restoration [$\chi(T)$, diamonds] and deconfinement [$\kappa_0(T)$, circles] both vanish at $T_c = 150$ MeV. The parameters for the fitted curves are presented in Table I.

TABLE I. Parameters characterizing the behavior of the listed quantities, fitted to $\alpha(1 - T/T_c)^\beta$, near $T_c = 150$ MeV.

	α	β
χ	1.1 GeV	0.33
κ_0	0.16 GeV	0.30
N_π^2	$(0.18\ \text{GeV})^2$	1.1
$f_\pi N_\pi$	$(0.15\ \text{GeV})^2$	0.93
$\langle m_R(\bar{q}q)\rangle$	$(0.15\ \text{GeV})^4$	0.92
m_π	0.12 GeV	−0.11
f_π	0.12 GeV	0.36

are presented in Table I. To avoid round-off errors associated with division near $T = T_c$, we used the results for these quantities to determine the parameters describing the behavior of m_π and f_π near the transition, which are listed in Table I. We plot $m_\pi(T)$ and $f_\pi(T)$ in Fig. 2.

The massive quark is not deconfined at the same temperature. However, denoting by κ_m the deconfinement order parameter evaluated with $m_R \neq 0$, we observe that the behavior of κ_m undergoes a qualitative change at $T_c^{\kappa_0} = T_c^\chi$, where no contribution to $B(p, \omega_k)$ remains that does not vanish as $m_R \to 0$. Fitting $\kappa_m = \alpha(1 - T/T^{\kappa_m})^\beta$ we find

$$\begin{array}{lcccc} & \alpha\ (\text{GeV}) & \beta & T^{\kappa_m}\ (\text{GeV}) & \\ T \leq T_c^\chi: & 0.15 & 0.23 & 0.16\,, & (19) \\ T > T_c^\chi: & 0.26 & 0.71 & 0.18\,. & \end{array}$$

For $T > T_c^\chi$, a fit with $\beta_{\kappa_m} = 0.23$ has a 5-times larger standard deviation. The u/d quark in this model is therefore deconfined at $T \sim 180$ MeV, suggesting that the deconfinement temperature increases with m_R.

We explored the finite-T properties of a renormalizable, confining, DSE model of QCD [4]. Introducing an order parameter for confinement we found that, in the chiral limit, a deconfinement transition at $T \approx 150$ MeV is accompanied by the coincident restoration of chiral symmetry. The single model parameter, fixed at $T = 0$, is appropriate for two-flavor QCD: the transitions are second order and are not described by mean-field critical exponents. Similar results have been obtained in recent numerical simulations of lattice QCD at finite T [13]. Some aspects of the non-mean-field behavior obtained in Ref. [13] are discussed in Ref. [14].

f_π and m_π are weakly sensitive to T for $T < 0.7T_c^\chi$. However, as T approaches T_c^χ, the mass eigenvalue in the pion Bethe-Salpeter equation moves to increasingly larger values, as thermal fluctuations overwhelm attraction in the channel, until at $T = T_c^\chi$ there is no solution and $f_\pi \to 0$. This means that the pion-pole contribution to the four-point, quark-antiquark correlation function disappears; i.e., there is no quark-antiquark pseudoscalar bound state for $T > T_c^\chi$. This may have important consequences for a wide range of physical observables [15], if borne out by improved studies; e.g., this T dependence of f_π and m_π would lead to a 20% reduction in the $\pi \to \mu\nu_\mu$ decay widths at $T \approx 0.9T_c^\chi$.

D. B. is grateful for the hospitality of the Physics Division at ANL; C. D. R. is grateful for the hospitality of the Max-Planck Group at the University of Rostock and for a stipend from the Max-Planck-Gesellschaft. This work was supported by the U.S. Department of Energy, Nuclear Physics Division, under Contract No. W-31-109-ENG-38 (A. B. and C. D. R.) and the Deutsche Forschungsgemeinschaft under Contract No. 436 RUS 17/168/95 (Y. K.), and benefited from the resources of the National Energy Research Supercomputer Center.

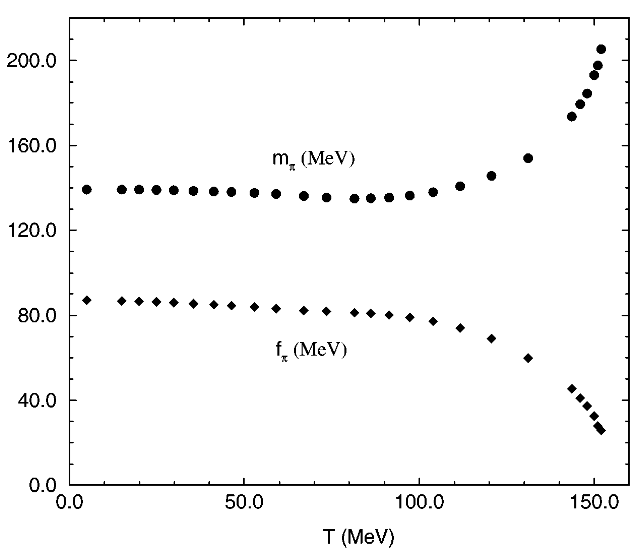

FIG. 2. Temperature dependence of the pion mass [$m_\pi(T)$, circles] and pion weak-decay constant [$f_\pi(T)$, diamonds].

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