Logarithmic Temperature Dependence of Conductivity at Half Filled Landau Level

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(Received 6 March 1996)

We study the temperature dependence of the longitudinal conductivity at half filled Landau level by means of the theory of composite fermions in the weakly disordered regime ($k_F l \gg 1$). At low temperatures we find the leading $\ln T$ correction resulting from the interference between impurity scattering and gauge interactions of the composite fermions. The prefactor appears to be strongly enhanced as compared to the standard Altshuler-Aronov term in agreement with recent experimental observations. [S0031-9007(96)00613-8]

PACS numbers: 73.40.Hm

Since 1990 a number of strong experimental evidences of the existence of compressible metal-like states of two-dimensional (2D) electrons in a strong magnetic field at even denominator fractions were obtained [1]. Remarkably, the results of these experiments can be interpreted in terms of a simple semiclassical picture of spinless fermionic excitations forming a Fermi surface at $k_F = (4\pi n_e)^{1/2}$ and experiencing only the difference magnetic field $\Delta B = B - 4\pi q n_e$ in the vicinity of $\nu = 1/2q$. The appropriate theoretical framework implied by this picture was elaborated by Halperin, Lee, and Read [2] on the basis of the idea of composite fermions (CF) viewed as $2q$ gauge flux quanta attached to spin-polarized electrons [3].

The mean field theory developed in [2] allows a qualitatively successful understanding of the observations made in [1] in terms of a nearly Fermi liquid behavior of CF. However, more recent experimental studies revealed features which cannot be readily explained by the picture of weakly interacting fermions.

These puzzling data include a divergency of a CF effective mass extracted from measurements of magnetoresistivity at incompressible fractions $\nu = N/(2N \pm 1)$ converging towards $\nu = 1/2$ and unusual $\sim 1/(\Delta B)^4$ scaling of the corresponding Dingle plot at low temperatures [4]. Moreover, temperature dependence of the resistivity at $\nu = 1/2$ and $3/2$ exhibits the leading $\ln T$ behavior with an unusually large coefficient [5].

These striking features are not unexpectable on general grounds because in contrast to the case of weakly interacting fermion gas the formation of the Fermi surface at even denominator fractions results from a minimization of the interaction rather than the kinetic energy. Therefore, one could well expect this system to provide a genuine example of 2D non-Fermi-liquid (NFL).

On the theoretical side, the NFL behavior stems from long-ranged retarded gauge interactions between CF in the presence of disorder [2]. Both gauge interactions and impurity scattering of CF are more singular than their counterparts in the original electron representation. So far all theoretical studies were concentrated either on the effects of dynamic gauge interactions in a pure system or on a noninteracting problem of impurity scattering of CF which translates into a problem of static random magnetic field (RMF). These problems, arising in other contexts as well, are certainly of a great interest, but in the real system provided by $GaAs/Al_xGa_{1-x}As$ heterojunction the number of strong Coulomb impurities (ionized donors) is essentially equal to the number of electrons. Therefore, impurities have to be considered as an inherent element of the system and one should expect a variety of interference effects which were extensively studied in the case of zero (or weak) magnetic field over a decade ago [6,7].

In the present Letter we estimate the low temperature conductivity $\sigma_{xx}(T)$ at exactly half filled Landau level proceeding along the lines of the previous analysis at $B = 0$ [6]. The calculable quantity is a conductivity of CF which is related to the physical conductivity as $\sigma^{\mathrm{CF}} = \sigma_{xx}^2 + \sigma_{xy}^2/\sigma_{xx} \approx (e^2/2h)^2 \sigma_{xx}^{-1}$.

In the Coulomb gauge $\mathrm{div}\vec{A} = 0$ the gauge interaction between CF at $\omega\tau_{\mathrm{tr}} \ll 1$ and $ql \ll 1$ is described by the inverse of the matrix [2]:

$$D_{\mu\nu}^{-1}(\omega, q) = \begin{pmatrix} \frac{m}{2\pi}\frac{Dq^2}{Dq^2 - i\omega} & -i\frac{q}{4\pi} \\ i\frac{q}{4\pi} & -i\gamma_q\omega + \chi_q q^2 \end{pmatrix}, \quad (1)$$

where $D = \frac{1}{2}v_F^2\tau_{\mathrm{tr}}$ is a diffusion coefficient, $\chi_q = 1/12\pi m + V_q/(4\pi)^2$ is an effective diamagnetic susceptibility given in terms of the pairwise electron potential V_q, $\gamma_q = n_e l/k_F$ is proportional to the CF mean free path (MFP) $l = v_F\tau_{\mathrm{tr}}$, and m stands for the CF effective mass.

Depending on the structure of a concrete device, the electron Coulomb interaction potential $V_q = 2\pi e^2/q$ can be screened by image charges induced in a ground plate. Therefore we shall consider both cases of Coulomb and short-range ($V_q \approx V_0 = 2\pi e^2/\kappa$ where κ stands for a screening constant) potentials.

Below we shall see that m (as well as details of the interaction potential V_q) drop out of final expressions for the conductivity. In fact, our calculations do not require a detailed knowledge of the frequency dependence of the CF Green function in the metallic regime but only its momentum dependence.

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Each charged impurity placed on distance d_s away from 2D electron gas creates a scalar potential with 2D Fourier transform given by $A_0^{(0)}(q) = 2\pi e^2 e^{-qd_s}/q$. Because of 2D screening by gapless CF this potential gets renormalized and also acquires a vector component corresponding to the gauge flux located at the impurity position. In the random phase approximation the renormalized potential has a form [2]

$$A_\mu(q) = 2\pi e^{-qd_s} \begin{pmatrix} \frac{1}{m} \\ \frac{2i}{q} \end{pmatrix}. \quad (2)$$

After averaging over positions of impurities with concentration n_i the vector disorder appears to be equivalent to RMF correlated as $\langle B_q B_{-q} \rangle = (4\pi)^2 n_i e^{-2qd_s}$. In what follows we put impurity concentration equal to the electron density n_e.

The RMF problem received a lot of attention over the last few years and is now believed to have localization properties described by the unitary random ensemble [8]. Although a total scattering rate $1/\tau = m \int \frac{d\theta}{2\pi} (\frac{v_F}{q})^2 \langle B_q B_{-q} \rangle$, where $q = 2k_F \sin\theta/2$ governing a nongauge invariant single particle Green function $G_\pm(\epsilon, \vec{p}) = [\epsilon - \xi(\vec{p}) - \Sigma(\epsilon) \pm i/2\tau]^{-1}$ appears to be divergent, it is a finite momentum relaxation rate $1/\tau_{\rm tr} = m \int \frac{d\theta}{2\pi} (1 - \cos\theta) (\frac{v_F}{q})^2 \langle B_q B_{-q} \rangle = \frac{4\pi n_i}{m} (k_F d_s)^{-1}$ which enters all physical quantities. Therefore one can safely operate with $1/\tau$ as if it were finite [8]. The CF self-energy $\Sigma(\epsilon)$ due to gauge fluctuations depends mostly on energy ϵ and not on momentum [2]. It guarantees that drastic effects of gauge fluctuations on the CF Green function resulting in a divergency of CF effective mass [2] do not affect our results. To put it another way, the following calculations do not rely on the validity of the quasiparticle picture [although it can be justified in the $T = 0$ Coulomb case with no impurities where $\Sigma'(\epsilon) \sim \epsilon \ln\epsilon \gg \Sigma''(\epsilon) \sim \epsilon$ [9]] and they can also be performed by means of a more general approach of quantum kinetic equation [10] generalized onto the case with impurities [11].

The RMF transport time $\tau_{\rm tr}$ determines a classical value of CF conductivity $\sigma_0^{\rm CF} = \frac{e^2}{2h}(k_F d_s) \gg \frac{e^2}{h}$ found in [2]. Note that the scalar component of (2) gives a contribution to $\sigma_0^{\rm CF}$ which is $(k_F d_s)^2$ times smaller.

The corresponding MFP of CF $l \sim d_s$ is small compared with its value at zero magnetic field $l_0 \sim k_F^{-1}(k_F d_s)^3$ [2]. Nevertheless, a metallicity parameter $k_F l$ can still be appreciably large (typically, $k_F d_s \sim 15$ [2]), which allows one to estimate corrections to $\sigma_0^{\rm CF}$ by means of a standard perturbative calculation of a linear current response [6].

It was pointed out in [2,12] that weak-localization corrections $\delta\sigma_{\rm wl}^{\rm CF} \sim \frac{1}{\epsilon_F \tau_{\rm tr}} \ln T\tau_{\rm tr}$ are negligible because as a consequence of broken time-reversal symmetry there is no pole in the particle-particle (Cooperon) channel of the two-particle Green function. However, the pole in the particle-hole channel (diffusion)

$$\Gamma(\epsilon, \omega, q) = \frac{1}{m\tau^2} \frac{1}{Dq^2 - i\omega} \quad (3)$$

is still present at $\omega\tau_{\rm tr} \ll 1$, $ql \ll 1$, and $\epsilon(\epsilon + \omega) < 0$ since it is due to the particle number conservation only. In [8,13] the occurrence of the diffusion pole had been shown explicitly by summing the particle-hole ladder diagrams.

A complete treatment of the CF problem requires a simultaneous account of singular gauge interactions $D_{\mu\nu}$ and impurity ladders responsible for the diffusion pole Γ in the particle-hole amplitude. Similar to the case of disordered electrons in zero magnetic field [6] the first interaction corrections to conductivity (Altshuler-Aronov type terms) arise from exchange diagrams of Fig. 1.

As in the $B = 0$ case [6] the three first diagrams representing self-energy, vertex, and impurity scattering rate corrections cancel each other in the leading approximation independently of the nature of interaction. At finite $T < 1/\tau_{\rm tr}$ the residual terms representing inelastic scattering from thermal gauge fluctuations give power-law corrections which scale as $T^{3/2}$ in the short-range case and as T^2 in the Coulomb one in accordance with predictions made in [2].

To estimate the contributions of the last two diagrams of Fig. 1 one first has to notice an important difference between impurity renormalization of scalar (density) and vector (current) vertices coupled by the gauge propagator $D_{\mu\nu}(\omega, q)$.

Each scalar vertex $\Lambda^{(0)} = 1$ gets dressed by an impurity ladder which introduces an extra diffusion propagator into the integrand $\Lambda(\omega, q) = \frac{1}{\tau} \frac{1}{Dq^2 - i\omega}$ [13]. Then, exchange by the scalar component of the gauge propagator $D_{00}(\omega, q) = \frac{2\pi}{m} \frac{Dq^2 - i\omega}{Dq^2}$ gives the same contribution as in the case of zero field $\delta\sigma_{\rm sc,exch}^{\rm CF} = \frac{e^2}{\pi h} \ln T\tau_{\rm tr}$ [6,7]. A more complete analysis which accounts for Hartree terms in addition to the exchange contribution yields [15]

$$\delta\sigma_{\rm sc}^{\rm CF} = \frac{e^2}{\pi h} (2 - 2\ln 2) \ln T\tau_{\rm tr}. \quad (4)$$

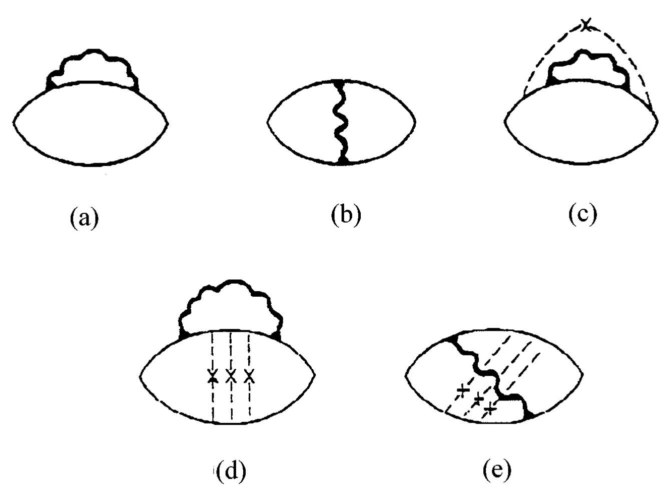

FIG. 1. Conductivity diagrams.

On the contrary, a transverse vector vertex $\vec{\Lambda}^{(0)} = \frac{\vec{p}}{m}$ acquires only an additional factor $\frac{\tau_{\rm tr}}{\tau}$ [13], while a diffusion pole appears in the longitudinal part proportional to $\vec{q}$ and does not contribute to gauge invariant observables. Then, one can easily see that the contribution of the off-diagonal (Chern-Simons) component $D_{01}(\omega, q)$ vanishes. It should be noted, however, that at finite residual field ΔB an exchange by the Chern-Simons component provides a mechanism of additional skew scattering which modifies both longitudinal and Hall conductivities of CF in the vicinity of $\nu = 1/2$ [11].

To calculate the contribution of the transverse gauge fluctuations governed by $D_{11}(\omega, q) = \frac{1}{-i\gamma_q\omega + \chi_q q^2}$, we first do themomentumsum in the CF Green functions. This sum factorizes into two parts on either side of the impurity ladder. By contrast to the case of the density-density coupling considered in [6], each of these two factors is essentially independent of $\vec{q}$ and proportional either to $M_{ij} = \sum_{\vec{p}} G_+^2(\vec{p}) G_-(\vec{p}) \frac{p_i p_j}{m^2} \frac{\tau_{\rm tr}}{\tau} = -i\epsilon_F \tau_{\rm tr} \tau \delta_{ij}$ or to its complex conjugate depending on the signs of frequencies ϵ and ω carried by the fermionic and gauge lines.

In the case of $T = 0$ and finite external frequency $\Omega > 0$, the two last diagrams of Fig. 1 do not cancel out only in the domain $\epsilon < -\Omega < \omega + \epsilon < 0$. Together with two similar diagrams with the gauge interaction dressing of the particle and the hole line interchanged they combine into

$$\delta\sigma_{\rm vec}^{\rm CF}(\Omega) = \frac{ie^2}{2\pi} \int_{\Omega}^{1/\tau_{\rm tr}} \frac{d\omega}{2\pi} \int \frac{d\vec{q}}{(2\pi)^2} \frac{M_{ik} M_{kj}^* (\delta_{ij} - q_i q_j/q^2)}{m\tau^2 [Dq^2 - i(\omega + \Omega)](-i\omega\gamma_q + \chi_q' q^2)}, \quad (5)$$

where $\chi_q' = \chi_q + 1/8\pi m$. Notice that the impurity scattering lifetime τ as well as the CF effective mass m drop out of the formula (5). At finite temperature $T \gg \Omega$ the calculation of (5) can be done by using imaginary frequencies and analytic continuation. At $k_F l \gg 1$ and $T \ll 1/\tau_{\rm tr}$ the integral in (5) can be estimated with logarithmic accuracy. In the case of short-range interaction we obtain

$$\delta\sigma_{\rm vec}^{\rm CF}(T) = \frac{e^2}{2\pi h} (\ln T\tau_{\rm tr})(\ln k_F l). \quad (6)$$

As compared to the conventional scalar contribution (4) the $\ln T$ correction (6) appears to be enhanced by a large factor $\ln k_F l$ which results from the momentum integration in the range $\omega < Dq^2 < \omega(k_F l)^2$. Also, by contrast to the conventional Coulomb problem at zero field [6] the static limit of the transverse gauge interaction remains singular $[D_{11}(0, q) \sim 1/q^2]$ and Hartree terms do not reduce the result in the leading approximation.

The negative transverse gauge field contribution (6) diverges as T tends to zero and manifests a breakdown of the perturbation theory at $T \sim 1/\tau_{\rm tr} \exp(-\frac{\pi k_F l}{\ln k_F l})$ suggesting a localization length $L_{\rm loc} \sim l \exp(\pi k_F l/\ln k_F l)$ which is shorter than that implied by the scalar term (4) only.

In the unscreened Coulomb case we obtain the result containing double-logarithmic terms

$$\delta\sigma_{\rm vec}^{\rm CF}(T) = \frac{e^2}{2\pi h} (\ln T\tau_{\rm tr})[\ln k_F l + \tfrac{1}{4}\ln T\tau_{\rm tr}], \quad (7)$$

which holds at $T > T_0 \sim \epsilon_F/(k_F l)^3$ since the $\ln T$ term can emerge only from the momentum integration in the interval $\omega < Dq^2 < \omega^2 (k_F l)^3/\epsilon_F$. In the range of temperatures $T_0 < T < 1/\tau_{\rm tr}$ the coefficient in square brackets reduces by a factor of 2. Because of a singular behavior of $D_{11}(0, q) \sim 1/q$ the leading logarithms in (7) are not affected by Hartree terms either.

At $T < T_0$ the divergency in (7) is cut off but at temperatures of order $T_{\rm cr} = \frac{\kappa^2}{m} k_F l$ one may expect a crossover to a short-range regime due to the screening by a ground plate placed at a distance $\sim\kappa^{-1}$ apart from 2D electron gas.

Remarkably, in both cases the logarithmic corrections are nonuniversal, although only weakly dependent on details of interaction potential V_q. In fact, they depend only on k_F and l, both of which can be extracted from the experimental data of surface acoustic wave propagation [1].

In agreement with general expectations the correction $\delta\sigma_{\rm vec}^{\rm CF}(T)$ is stronger in the short-range case when effects of gauge forces, which physically correspond to local electron density fluctuations, are more pronounced [2].

In terms of physical observables the negative corrections (4), (6), and (7) to $\sigma^{\rm CF}(T)$ increase both the longitudinal conductivity $\sigma_{xx}(T) \approx (\frac{e^2}{2h})^2 [\sigma_0^{\rm CF} + \delta\sigma^{\rm CF}(T)]^{-1}$ and the resistivity $\rho_{xx}(T) = [\sigma_0^{\rm CF} + \delta\sigma^{\rm CF}(T)]^{-1}$ at half filled Landau level at temperatures $T < 1/\tau_{\rm tr}$. The first order corrections (6) and (7) have the right tendency to increase with both electron density and mobility. This enhancement, however, is not as strong as that observed in the experiments [5].

In order to make a direct comparison with the available experimental data we invoke the parameters of the samples used by Rokhinson *et al.*: $n_e = (0.4 - 1.2) \times 10^{11}$ cm^{-2} and $d_s = 120$ nm. At these parameter values we estimate the effective Fermi energy as $\epsilon_F \sim 14$–23 K and the transport scattering rate $1/\tau_{\rm tr}$ as 0.4–0.7 K which agrees reasonably well with the threshold temperature below which strong although sample-dependent $\ln T$ corrections were observed. The cutoff temperature $T_0 \sim 6$–9 mK appearing in the Coulomb case is sufficiently high to distinguish between the two behaviors described by formulas (6) and (7). An essentially temperature-independent slope of the $\ln T$ term reported in [5] is better consistent with the short-range case (6).

By using our first order result (6) we can estimate the coefficient λ in front of the $\ln T$ term at three different values of the bare (classical) CF conductivity $\sigma_0^{\rm CF} = 5.4e^2/h, 16.0e^2/h, 39.0e^2/h$ as $\lambda_{\rm th} = 0.27, 0.44, 0.58$. The corresponding experimental values $\lambda_{\rm exp} = 0.1, 0.4, 1.6$ found in Ref. [5] appear to exhibit a faster growth with $\sigma_0^{\rm CF}$.

This situation is somewhat similar to that with the CF effective mass $m(\Delta B)$ as extracted from the Shubnikov–de Haas oscillations [4] which shows a much stronger divergency than any current theoretical predictions [2,9,14]. Therefore, at this stage one can only conclude that the existing CF theory is, at least, in a qualitative agreement with experimentally observed features. The lack of a better quantitative agreement suggests the necessity of a more accurate account of disorder and/or CF gauge interactions.

A more complete account of the gauge interactions beyond the first order can be achieved by means of the renormalization group analysis of the scale dependent conductivity $\sigma_{xx}(L)$ [15]. It is not clear at this point if double-logarithmic terms appearing in the Coulomb case spoil the renormalizability of the noninteracting theory of 2D fermions in the RMF. Work in this direction is in progress.

We also note that in the presence of a thermal gradient the gauge field corrections to other kinetic coefficients can be found in a similar way. In particular, we expect an enhanced nonlinear correction to the Seebeck coefficient $\delta S_{xx}(T) \sim -\frac{T}{e\epsilon_F k_F l}(\ln T\tau_{\rm tr})(\ln k_F l)$. The existing experimental techniques [16] should in principle allow the observation of such an effect at temperatures below 100 mK, where the phonon drag contribution varying approximately as T^4 [17] becomes negligible. A detailed discussion of the effects of transverse gauge fluctuations on thermoelectric transport properties will be presented elsewhere [11].

In conclusion, we study the combined effect of disorder and interactions on the longitudinal conductivity at half filled Landau level in the framework of the CF theory [2]. A general consistency of our results with the recently observed strong nonuniversal $\ln T$ corrections to the conductivity at even denominator fractions [5] supports the idea of the CF and their transverse gauge interactions.

The author is indebted to B. I. Halperin, P. A. Lee, G. Kotliar, and L. P. Rokhinson for valuable discussions.

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