Giant Radiation Linewidth of Multifluxon States in Long Josephson Junctions

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Millimeter-wave band radiation measurements of flux-flow states in long Josephson junctions are reported. At low fluxon density, the radiation linewidth is found by factor of 10^4 larger than that of a lumped Josephson oscillator. With increasing the fluxon density, a sharp crossover is observed to a much smaller linewidth approaching the lumped oscillator limit. Numerical simulations using perturbed sine-Gordon model show that the observed behavior is related to internal degrees of freedom in the moving fluxon chain. [S0031-9007(96)01509-8]

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A time-dependent response of a distributed nonlinear oscillator is a signature of its intrinsic spatial dynamics. A long Josephson junction is a well-known example of such an oscillator. The external magnetic field penetrates into the junction in the form of Josephson vortices, or fluxons, each of them carrying one magnetic flux quantum Φ_0. Fluxons can be moved by a current flowing through the junction, and their motion leads to an electromagnetic radiation. The frequency f of the radiation is given by the Josephson relation $f = V/\Phi_0$, where V is a dc voltage induced by the fluxon motion.

In general, the linewidth Δf of the Josephson radiation is determined by thermal fluctuations of current passing through the junction. Assuming a Nyquist noise spectrum, for a current-biased lumped (short) Josephson tunnel junction the full linewidth at half power is given by expression [1,2]

$$\Delta f = \frac{4\pi k_B T}{\Phi_0^2}\,\frac{R_D^2}{R_S}\,, \tag{1}$$

where k_B is Boltzmann's constant, T is the temperature. The linewidth depends on the differential resistance $R_D = dV/dI$ at the junction bias point and the static resistance $R_S = V/I$, where I is the bias current through the junction.

There are two main regimes which characterize the fluxon motion in long junctions. First, a shuttlelike *resonant fluxon motion* gives rise to zero-field steps (ZFSs) in the dc current-voltage (I-V) characteristics of the junction. In this regime fluxons and antifluxons undergo reflections from the junction boundaries and the radiation frequency is determined by the junction length L and the fluxon velocity v as $f_{\mathrm{ZFS}} = v/2L$. Second, in a magnetic field, the so-called *flux-flow regime* occurs and is manifested by a flux-flow step (FFS) on the I-V curve. In this regime fluxons are created at one boundary of the junction and annihilate at the other boundary. The radiation frequency $f = v/d_{\mathrm{fl}}$ is determined by the spacing between moving fluxons d_{fl}. In general, for both ZFS and FFS regimes, the radiation linewidth Δf should be related to thermal fluctuations of the fluxon velocity vs Joergensen *et al.* [3] obtained a striking general result that, in spite of a different nature of the phase slippage in small and long junctions, the linewidth of the resonant single-fluxon radiation in a long junction is given by the same Eq. (1), except for a missing factor of 4 due to the modified Josephson relation $f = V/2\Phi_0$ at the ZFS. Experiment [3] showed a reasonable agreement with that theory, though some excess linewidth broadening has been seen.

It can be argued that internal degrees of freedom in the moving fluxon chain may give a significant contribution into Δf. Here, in contrast to the resonant single-fluxon case, local variations of the fluxon spacing d_{fl} change the radiation frequency. Recent FFS radiation linewidth measurements by Koshelets *et al.* [4] showed the scaling of Δf_{FF} as predicted by Eq. (1) but with an effective temperature T_{eff} being by a factor of 8 larger than the physical temperature of their experiment. The reason for the excess noise was not resolved.

In this Letter we report radiation measurements of flux-flow states in long Josephson junctions which show the radiation linewidth Δf by a factor of 10^4 larger than that predicted by Eq. (1). With increasing the fluxon density, a sharp crossover to much smaller Δf is observed. We present numerical simulations which consistently show that the experimentally observed broadband radiation can be related to self-induced chaotic oscillations in the moving fluxon chain.

The samples were fabricated using a standard niobium-based process on thermally oxidized silicon wafers. The investigated long Josephson junctions of the size $L \times W = 400 \times 5\ \mu\mathrm{m}^2$ had the critical current density j_c of about 100 $\mathrm{A/cm^2}$ which corresponds to the Josephson penetration depth $\lambda_J \approx 35\ \mu$m and plasma frequency $f_p \approx 30$ GHz. The sample layout is schematically shown in the inset in Fig. 1. Each junction had overlap geometry and was integrated in a microstripline ending with a finline antenna [5] inserted into a slit in a waveguide. The magnetic field H parallel to the junction plane and perpendicular to its larger dimension L was provided by a control current I_H through the bottom Nb electrode. Measurements were performed at 4.2 K. Radiation at 80–120 GHz was measured with a room temperature superheterodyne receiver with an effective noise figure of

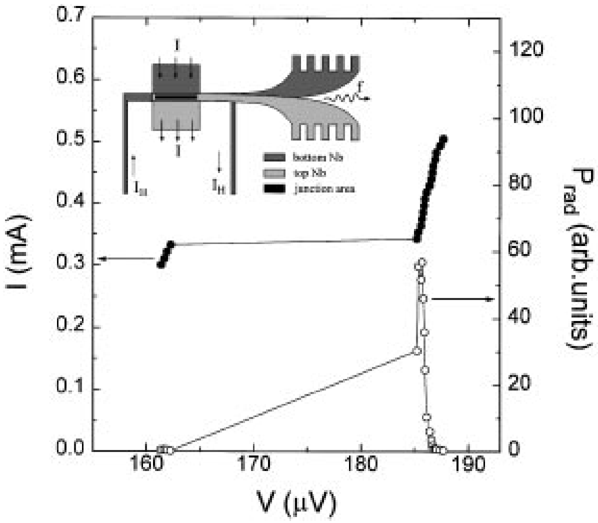

FIG. 1. I-V characteristic of two Fiske steps (FSs) measured at $I_H = 5.5$ mA. I-V data (solid symbols) are shown together with the detected radiation power (open symbols) at $f_{\rm LO} =$ 86.0 GHz. The inset shows schematic top view of a Josephson junction with a finline antenna.

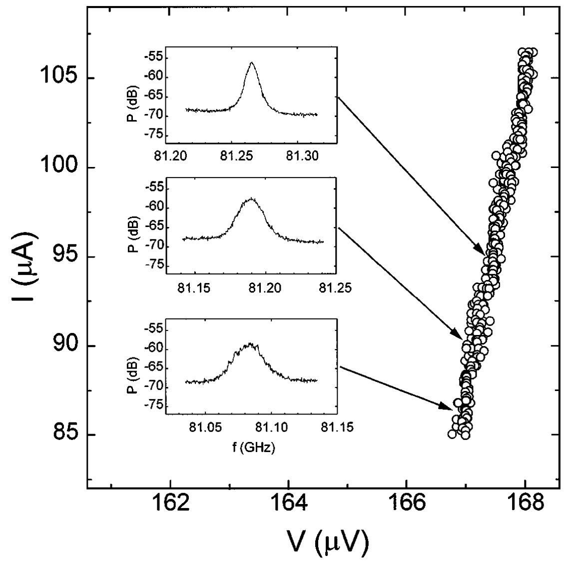

FIG. 2. Part of a high-order Fiske step (open circles) together with radiation spectra (insets) measured at indicated I-V states.

about 13 dB referred to the receiver input (including isolation). Typical radiation power at the receiver input was about 1 pW. The receiver was operated either as a spectrometer or as a radiometer. In the spectrometer mode the radiation spectrum in the intermediate frequency (IF) band $0.1 < f_{\rm IF} < 1.5$ GHz was detected using a spectrum analyzer. In this case a high spectral resolution was achieved using an externally phase-locked local oscillator with an effective linewidth smaller than 2 kHz. The alternative, input-modulated radiometer mode is characterized by a much higher sensitivity $\delta P \approx 6 \times 10^{-15}$ W but with a much lower spectral resolution limited by the double-side IF band of the receiver $\Delta f_{\rm IF} \approx 3$ GHz.

Flux-flow behavior was observed in I-V characteristics of the junctions in the applied magnetic field. In the intermediate field range (corresponding to $5 < I_H <$ 10 mA), the FFS was split into a series of Fiske steps (FSs). The FSs correspond to cavity resonances of the Josephson generation frequency on the junction length with the voltage spacing between them $\Delta V_{\rm FS} = \Phi_0 \bar{c}/2L$, where $\bar{c}$ is the Swihart velocity. Typical measurements of the FS regime in the radiometer mode are shown in Fig. 1. The radiation power $P_{\rm rad}$ around fixed local oscillator frequency $f_{\rm LO}$ was detected while recording the I-V curve. The voltage width ΔV of the emission peak corresponds to the double-side band of the receiver $\Delta f_{\rm IF}$. Figure 2 presents spectrometer mode measurements of a FS which indicate that the linewidth of FS radiation Δf is much smaller than $\Delta f_{\rm IF}$. Typically, Δf of about several MHz was measured for various FSs. Within 1 order of magnitude, this Δf agrees with that calculated from Eq. (1) using experimental values for R_D and R_S.

In the lower magnetic field range we observed another distinct flux-flow feature of long Josephson junctions, the so-called *displaced linear slope* (DLS) on I-V characteristics. DLS received a considerable interest in the past and was found in distributed junctions of various geometries [6]. In spite of many observations, the physical nature of DLS remained obscure until now. In long quasi-one-dimensional junctions, DLS is typically seen at magnetic fields near the critical field H_{C1} of the junction [7]. The DLS does not have a pronounced resonant shape and, in a limited range, its voltage can be smoothly tuned by the magnetic field. Compared to FSs, we found substantially lower radiation power and much broader linewidth for DLS regimes. Typical measurements of DLS regime in the radiometer mode are shown in Fig. 3. When increasing the magnetic field from zero, the critical current of the junction decreases and at $I_H \approx 4.4$ mA a smooth flux-flow branch (DLS) appears on the I-V curve. As shows the inset in Fig. 3, with increasing the field DLS branch shifts approximately linearly with I_H towards higher voltages. The differential resistance R_D of DLS is almost constant as a function of I and I_H. At $I_H \approx I_H^{\rm cross} \approx 5.1$ mA the DLS changes dramatically and splits into several FSs. Such data for the same junction at $I_H > I_H^{\rm cross}$ are presented in Fig. 1. The radiation linewidth in the DLS regime at low fields ($I_H < I_H^{\rm cross}$) was found to be larger than $\Delta f_{\rm IF}$. Thus, it was impossible to measure the DLS linewidth in the spectrometer mode. In the radiometer mode, the half-power radiation linewidth $\Delta f = \Delta V/\Phi_0 - \Delta f_{\rm IF}$ estimated from the voltage width ΔV of the emission peak is as large as 13 GHz, for both DLSs shown in Fig. 3. Such a giant Δf, ranging from about 10 to 20 GHz, was systematically observed for DLS branches in all measured samples, using various $f_{\rm LO}$ frequencies. In contrast to that, as illustrated in Fig. 1, FSs appearing at $H > H_{\rm cross}$ always showed $\Delta V/\Phi_0 \approx \Delta f_{\rm IF}$. The large discrepancy between the radiation linewidth of DLS and FSs cannot be explained by the difference between

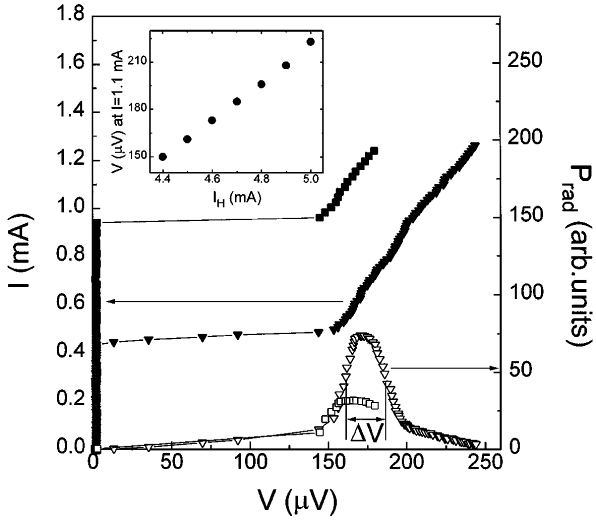

FIG. 3. I-V characteristics of displaced linear slope (DLS) measured at $I_H = 4.5$ mA (squares) and $I_H = 5.0$ mA (triangles). I-V data (solid symbols) are shown together with the detected radiation power (open symbols) at $f_{\mathrm{LO}} = 89.2$ GHz.

their differential resistances R_D through Eq. (1). The ratio $R_D^{\mathrm{DLS}}/R_D^{\mathrm{FS}}$ varies for different states between 3 and 5, while their $R_S \sim 0.4\text{--}0.5\ \Omega$ remains about the same. For DLS Eq. (1) predicts $\Delta f^{\mathrm{DLS}} \sim 3$ MHz which is by a factor of nearly 10^4 smaller than that found in experiment.

In order to find an origin of the experimentally observed anomalous radiation properties, we performed full numerical simulations of the fluxon dynamics in the relevant magnetic field range. The dynamics of a long one-dimensional Josephson junction is described by the perturbed sine-Gordon equation [8],

$$\varphi_{xx} - \varphi_{tt} = \sin\varphi + \alpha\varphi_t - \beta\varphi_{xxt} + \gamma\,. \qquad (2)$$

The spatial coordinate x is normalized to the Josephson penetration depth λ_J, and the time t is normalized to the inverse plasma frequency $\omega_p^{-1} = (2\pi f_p)^{-1}$. The coefficients α and β correspond to the damping arising from the quasiparticle tunneling and the surface losses in the electrodes, respectively. In simulations we assumed a spatially independent normalized bias current density $\gamma = I/I_c$, where I_c is the critical current of the junction. Eq. (2) was solved together with the boundary conditions,

$$\varphi_x(0) + \beta\varphi_{xt}(0) = \varphi_x(\ell) + \beta\varphi_{xt}(\ell) = h\,, \qquad (3)$$

where the normalized field $h = 2\pi H\lambda_J\Lambda/\Phi_0 \propto I_H$ and the normalized junction length $\ell = L/\lambda_J$. For simplicity, we neglect the surface losses taking $\beta \equiv 0$. For moderate fluxon velocities the effective damping can be accounted by $\alpha_{\mathrm{eff}} \sim \alpha + \beta/3$ [8]. Simulations were performed using $\ell = 10$ and $\alpha = 0.1$. The calculated average voltage V is normalized to the voltage of the first FS. Current-voltage characteristics were calculated for the magnetic field range around the critical field H_{C1} of the junction, which corresponds to the normalized field $h \sim 2$. Because of complicated dynamics found in some parameter ranges, the voltage integration was made over long time intervals, up to 20 000 time units.

Figure 4 shows the numerically calculated current-voltage (γ vs V) characteristics for $h = 1.7$, 1.9, and 2.2. In good qualitative agreement with experiment shown in Fig. 3, starting from low magnetic fields ($h = 1.7$) a DLS-like branch first appears in γ-V characteristics. Increasing of the field ($h = 1.9$ and 2.2) induces several steep cavitylike resonances corresponding to low Fiske steps. One can see that the DLS preserves at larger bias currents ($\gamma > 0.2$) and, in agreement with experiment, smoothly shifts towards high voltages with increasing h.

We found that voltage oscillations in the junction are extremely nonperiodic in the DLS regime. In order to simulate the expected radiation spectrum from the junction, we calculated Fourier power spectra of the voltage oscillations $\partial\varphi/\partial t$ at one of the junction boundaries $x = \ell$ using FFT. Figure 5 illustrates such spectra for two selected points P1 and P2 of the current-voltage characteristics shown in Fig. 4. Point P1 corresponds to periodic oscillations at one of the FS modes. In contrast, point P2 shows a typical response of the states along all DLS branches: oscillations are strongly irregular and the radiation spectrum resembles chaotic state. One may notice that at P2 there is a visible harmonic amplitude modulation with the angular frequency spacing $\Delta\omega \approx 0.025$. This value accounts for the fundamental cavity mode of the junction. More or less pronounced, this behavior was found along various DLS. Thus, irregular fluxon oscillations at DLS regime resemble that of a broadband noise source pumping a cavity resonator given by the junction length. Comparing oscillation spectra for various bias points, we found that with increasing bias current a typical scenario of the transition from regular (type P1) to chaotic (type P2) oscillations takes place via period doubling and generation of subharmonics.

We associate the reason for extremely large radiation linewidth in DLS regime with a complicated spatiotemporal dynamics of fluxons, possibly a chaotic state, which develops in the junction. A hypothesis that such a state may

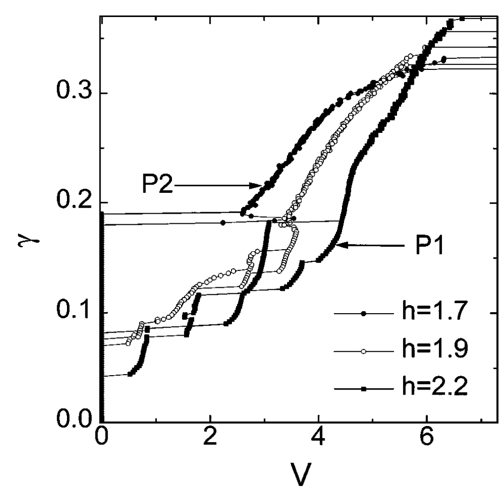

FIG. 4. Numerically simulated current-voltage characteristics of the junction. The arrows show points where the voltage spectra shown in Fig. 5 are calculated.

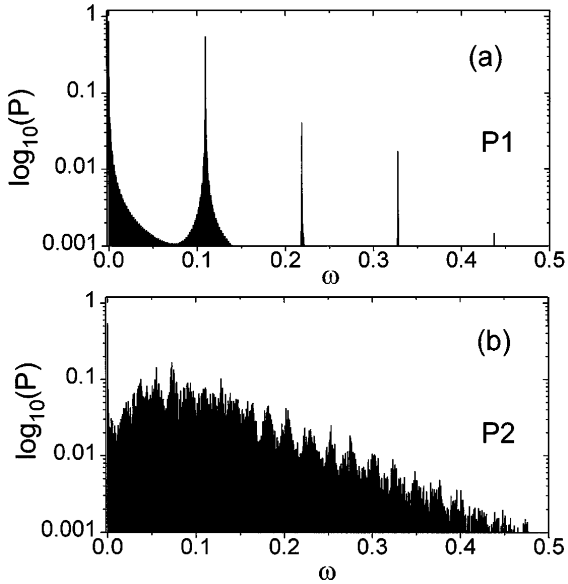

FIG. 5. Fourier power spectra of voltage oscillations for two selected points P1 and P2 shown in Fig. 4: (a) point P1; (b) point P2.

occur was suggested some years ago [7] intuitively based on dc properties of DLS. Here we presented strong arguments in favor of self-induced chaotic states based on our radiation measurements and numerical simulations. The DLS occurs at magnetic fields $H \sim H_{C1}$ where the fluxon density is rather low ($d_{\mathrm{fl}} \approx \pi \lambda_J$) and their mutual interaction is weak. Such a soft fluxon chain strongly interacts with junction boundaries which excites internal oscillation modes of the chain. With increasing H the fluxon spacing decreases, the chain becomes more stiff, and the dynamics turns more regular. A similar effect is achieved by increasing losses in the junction with temperature, as we observed in experiments.

Some examples of chaotic soliton dynamics without an external rf driving force have been earlier reported in numerical simulations [9–11] and indirectly indicated experimentally [12,13] from dc current-voltage characteristics. One particular sort of a self-induced chaotic state in the form of spatiotemporal intermittency between Fiske steps (FS1 and FS2) has already been demonstrated numerically for long junctions [9] and its indication has also been seen in experiment [12]. More recently, energy estimates using the perturbation theory have been proposed and qualitative agreement with simulations has been found for the parameter range of switching between FS1 and FS2 [14]. A variety of chaotic behaviors such as period doubling, intermittency, and quasiperiodic motion at Fiske steps reported by Yeh *et al.* [10] can be also found in the dynamics of DLS reported here.

Long Josephson junctions biased in the flux-flow mode have been recently used as local oscillators for integrated submillimeter-wave receivers [15]. Here we argue that intrinsic oscillation modes in the fluxon chain might be responsible for the observed excess radiation linewidth of such oscillators [4]. For their best performance, such flux-flow oscillators should be operated at high enough magnetic fields in order to avoid the DLS region. The DLS regime itself can possibly be used as a broadband noise generator for the millimeter wave range.

In summary, we reported radiation detection experiments and numerical simulations for multifluxon states in long Josephson junctions. Depending on the applied magnetic field, we were able to detect radiation from both displaced linear slope and Fiske steps in current-voltage characteristics. Our conclusion is that the displaced linear slope for long junctions is characterized by intrinsically chaotic fluxon dynamics.

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