α Decay of Deformed Actinide Nuclei

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α decay through a deformed potential barrier produces significant mixing of angular momenta when mapped from the nuclear interior to the outside. Using experimental branching ratios and either semiclassical or coupled-channels transmission matrices, we have found that there is a set of internal amplitudes which is essentially constant for all even-even actinide nuclei. These same amplitudes also give good results for the known anisotropic α-particle emission of the favored decays of odd nuclei in the same mass region. [S0031-9007(96)00550-9]

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The phenomenon of "tunneling in the presence of an environment" is of considerable interest in many branches of physics and chemistry [1], for example, in the tunneling of Cooper pairs through a Josephson junction [2]. In the domain of nuclear physics, the study of the subbarrier fusion of heavy nuclei has made significant contributions to this problem over recent years in that the experimental distribution of fusion barriers has been shown to be intimately related to the environment consisting of the target and projectile excited states [3]. In this problem the incident wave is known and all the transmitted flux ends up in the single fusion channel. The phenomenon of α decay is potentially more difficult, since the incident wave (α-particle wave function in the nuclear interior) is unknown but potentially much more rewarding since (a) for even-even nuclei the transmitted flux may end up in different daughter states for which the individual fluxes, i.e., branching ratios, can be measured and (b) since for odd nuclei, the presence of different angular momenta in each daughter state may lead to a measurable anisotropy in the α emission.

The α decay of deformed nuclei may be divided into two distinct parts: the formation of an α particle in the nuclear interior, followed by its penetration through the α daughter deformed Coulomb barrier. There are various approaches to the formation problem. One of these assumes a preformed α particle (or at least a spatially correlated four-nucleon cluster with the appropriate quantum numbers), which moves in the deformed field of the daughter nucleus [4,5]. Fröman [6] assumes a constant α particle probability on the deformed nuclear surface. Mang *et al.* [7] and, more recently, Delion, Insolia, and Liotta [8] have considered the deformed single-particle (Nilsson) states in the vicinity of the Fermi surface and taken the overlap of the correlated neutron and proton BCS wave functions with an α particle at the nuclear surface.

Whatever the formation mechanism, the decay proceeds by penetration through the Coulomb barrier. If this barrier is assumed to be spherical, there will be no mixing of orbital angular momentum states during the tunneling and, for an even-even nucleus, the observed branching ratios to different rotational states of the daughter ($I = 0^+, 2^+, 4^+, \ldots$) will be determined by the L admixture in the nuclear interior, modified by the transmission factors for the different centrifugal barriers evaluated at slightly different α-particle energies due to the excitation energies ϵ_I^* of the daughter. If one takes account of the deformation of the barrier, there will be additional mixing during the tunneling [6,9]. Such effects have been considered in the fusion of heavy nuclei [10] and more recently confirmed experimentally [11,12]. In this Letter, we take known branching ratios and calculate the mixing in the barrier to obtain the internal amplitudes near the nuclear surface. No model of internal dynamics (α preformation or distribution) is required, and indeed one does not know what preformation factors have to be fitted unless the barrier penetration problem is first solved. We then systematically survey the internal amplitudes for the even-even actinide α emitters for various choices of relative phases. A surprising result that emerges from our analysis is that the relative internal amplitudes may possibly be constant over a wide range of actinide nuclei and this in itself gives a very strong indication of the type of nuclear model for the preformation factor which might be the most appropriate.

The fine structure of the α particle energy spectrum determines the branching ratios to states of known spins I in the daughter. Since, for an even-even nucleus, the orbital angular momentum of the α particle must be $L = I$, we shall set up our problem in terms of these outgoing L waves. In the intrinsic frame of the axially symmetric deformed daughter, one may write an asymptotic outgoing wave in the form

$$\psi^{\text{outside}} = \sum_L C_L \, \mathcal{O}_L^{\text{outside}}(k_L, \mathbf{r})$$
$$= \sum_L C_L \, [G_L(k_L, r) + iF_L(k_L, r)]Y_{L0}(\hat{\mathbf{r}})\chi_{L0},$$

(1)

where F_L and G_L are the regular and irregular Coulomb wave functions. The $\mathcal{O}_L^{\text{outside}}$ represent outgoing α particles with orbital angular momentum L coupled to total angular momentum zero with a state χ_{L0} of the daughter of spin $I = L$. The wave numbers k_L differ due to the different ϵ_I^*. With such an outgoing wave, one can perform a coupled-channels calculation to obtain the wave function on the other side of the barrier. This must have both incoming and outgoing components

$$\psi^{\text{inside}} = \sum_L [A_L \, \mathcal{O}_L^{\text{inside}}(k_L, \mathbf{r}) + B_L \, \mathcal{I}_L^{\text{inside}}(k_L, \mathbf{r})].$$

(2)

The corresponding currents are shown schematically in Fig. 1.

The above problem may be solved numerically to obtain the outside coefficients C_L in terms of those inside, A_L. One may thus define a transmission matrix M

$$C_L = \sum_{L'} M_{LL'} A_{L'}.$$

(3)

Since we wish to undertake a systematic study of α emitters in the actinide region, we shall first obtain the transmission matrix using the semiclassical method outlined below. We thus express the outer amplitudes c_L (lower case indicates WKB solutions) in terms of the amplitudes a_L in the nuclear interior by

$$c_L = \sum_{L'} \mathcal{K}_{LL'} a_{L'}$$
$$= \sum_{L'} \langle Y_{L0} | \exp[-I_L(\cos\theta)] | Y_{L'0} \rangle a_{L'},$$

(4)

where I takes the usual WKB form

$$I_L(\cos\theta) = \int_{r_1(\cos\theta)}^{r_2} \left[\frac{2\mu}{\hbar^2} [V_L(r, \cos\theta) - E_\alpha] \right]^{\frac{1}{2}} dr$$

(5)

(see Fig. 1). The potential $V_L(r, \cos\theta)$ comprises the Coulomb field due to the deformed charge distribution of the daughter, a deformed Woods-Saxon potential and a centrifugal term. We consider both β_2 and β_4 terms in the deformation. The angular integrals were evaluated using the technique of Kermode and Rowley [13]. The matrix $\mathcal{K}_{LL'}$ is the analog of $M_{LL'}$ in the coupled-channels formalism.
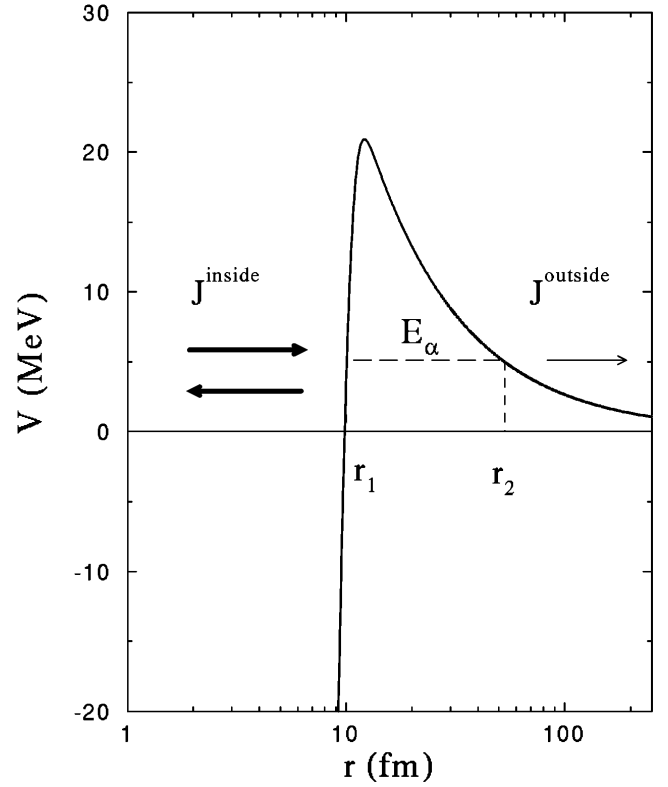

FIG. 1. Schematic diagram of the potential V and the incoming and outgoing currents. The classical turning points r_1 and r_2, and the energy of the α particle E_α are used in the semiclassical calculation of the transmission matrix; see Eq. (5).

The above approach is essentially an exact calculation of the semiclassical transmission coefficients introduced by Fröman [6] and used, for example, in Ref. [8]. Indeed the earlier expressions of Fröman may be obtained from Eq. (4) by ignoring the hexadecapole deformation, taking the nuclear potential to be a θ dependent sharp cutoff, and by making a first-order expansion in β_2. Approximations similar to these have also been employed in a coupled-channels formalism [14,15].

The magnitudes of the coefficients c_L are proportional to the square roots of the branching ratios for the angular momenta L. They may be taken to be real and can, in principle, be either positive or negative. Their reality follows from the requirement that the imaginary part of the wave function at the nuclear surface should be small [4]. In this Letter, we restrict our calculations to $\{L\} = \{0, 2, 4\}$ since few nuclei have measured branches to the 6^+ state. (We have, however, considered the inclusion of 6^+ and 8^+ states and have found that our conclusions are essentially unaffected.) We consider four possibilities for the relative signs of the $\{c_L\}$, i.e., $\{+ + +\}, \{+ + -\}$, $\{+ - +\}$, and $\{+ - -\}$. For all the decays we consider, the Sommerfeld parameters are large and the Coulomb phases then ensure that for the case $\{+ + +\}$ the spherical harmonics in Eq. (1) are in phase along the symmetry axis. Thus if one considered the amplitudes in Eq. (1) to add coherently, the outgoing flux would be axial. In

the even-even system, of course, the currents are not added coherently and the outgoing flux is always isotropic for each L, after integrating over all orientations of the daughter. However, we shall see below that the above consideration is important for odd-even systems which may be polarized to yield anisotropic α decay. For each of the above phase combinations, we have determined the amplitudes a_L from Eq. (4) using deformation parameters from Möller *et al.* [16]. Figure 2 shows the four sets of a_L (normalized to unity) for actinide nuclei with daughter mass (atomic number) $220 \leq A_d \leq 248$ ($88 \leq Z_d \leq 96$).

To test the accuracy of the WKB approximation, we have performed exact coupled-channels calculations for three of the above nuclei. The results were in good agreement with those shown in this figure. For example, for ^{238}U with the phase choice $\{c_L\} = \{+ - -\}$, the coupled channels give $\{a_L\} = \{0.84, -0.54, 0.08\}$ compared with the semiclassical values $\{a_L\} = \{0.83, -0.55, 0.10\}$. The importance of the L mixing under the barrier is demonstrated by the corresponding results $\{a_L\} = \{0.70, -0.68, -0.19\}$ for the *spherical* case.

Since the deformation of the daughter varies smoothly with A_d, one might expect that just one set of the solutions shown in Fig. 2 corresponds to the physical amplitudes. One particularly interesting solution is that obtained with the combination $\{+ - -\}$ [Fig. 2(d)] since the coefficients a_L are practically nucleus independent even though β_2 varies from around 0.10 to 0.24 and the α particle energies vary from around 4 to 7 MeV over this mass region. It is also the only one to yield amplitudes with $|a_0| > |a_2| > |a_4|$ for all nuclei (see the discussion below).

We note, however, that for the constant solution a_2 is always negative. At first sight, this implies that if
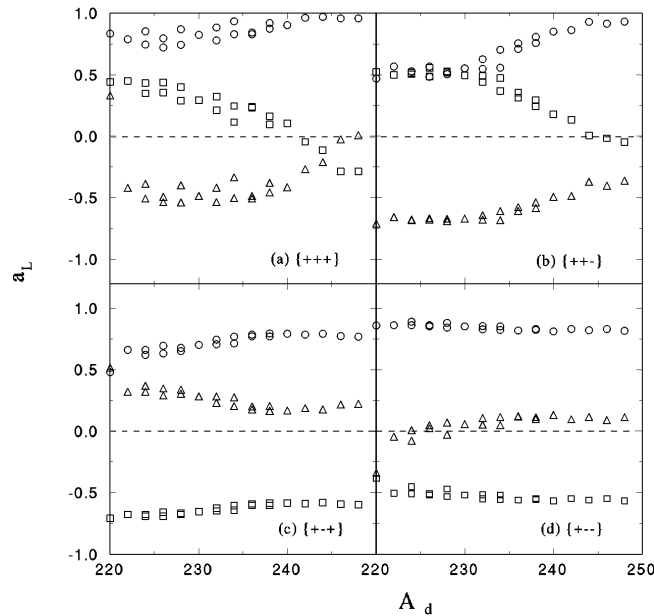

FIG. 2. The amplitudes a_L for even-even actinide nuclei with the four choices of external phase indicated. The circles, squares, and triangles represent $L = 0$, $L = 2$, and $L = 4$, respectively.

the same solution prevailed for odd-even nuclei, then α emission takes place mainly in the equatorial plane—a result which would not be consistent with measured anisotropies. Consider, however, the favored α decay of an aligned nucleus with $K = M = J_p = J_d$, where K is the spin projection of the nucleus along the symmetry axis, M the projection of the parent spin J_p along the space-fixed z axis, and J_d the daughter spin. For this case the angular distribution of the emitted α particles may be written (see, for example, [6,13])

$$W(\theta) \propto \sum_m \left| \sum_L \langle J_d L M - m\, m | J_p M \rangle \right.$$
$$\left. \times \langle J_p L K 0 | J_d K \rangle c_L Y_{Lm}(\theta, 0) \right|^2, \quad (6)$$

where the Clebsch-Gordan coefficients arise out of the transformation from the body-fixed to the laboratory frame. We define the anisotropy by $W(0)/W(\frac{1}{2}\pi)$. Equation (6) describes the favored decay $K_p = K_d = K$ angular distribution only. A contribution from the decay $K_p = K \to K_d = -K$ is suppressed (see, for example, p. 50 of [6] or pp. 272 and 273 of [9]).

A simple model for the odd-even nucleus would be to assume that it consists of an even-even core plus a spectator nucleon. Indeed, Fröman [6] employed the above formula using c_L extracted from the neighboring even-even branching ratios, which implicitly uses the same transformation (4) as for the even-even case. Delion, Insolia, and Liotta [8] have also used Eq. (4) but derive their amplitudes a_L from a BCS calculation. The expression (4) may not, however, be appropriate for odd-even systems, where the α-particle energy is determined by the daughter spin J_d rather than by the orbital angular momentum L. We have applied Eq. (4) to the four odd-even actinide nuclei for which anisotropies have been measured (see Table I), using the amplitudes a_L from Fig. 2. We find that Eq. (4) cannot predict anisotropies both greater than and less than one for these nuclei, for any of the four sets of a_L. This is because the sign of c_2/c_0 is either positive [anisotropy >1, i.e., Figs. 2(a) and 2(b)] or negative [anisotropy <1, i.e., Figs. 2(c) and 2(d)].

If the energies of the excited states of the daughter were high, then barrier penetration would filter out the components of the wave function corresponding to configurations other than its ground state. In that case we obtain

$$c_L \approx \sum_{L'} \mathcal{K}_{LL'} \langle J_p L' K 0 | J_d K \rangle^2 a_{L'}. \quad (7)$$

We note that Bohr and Mottelson [9] suggest a similar J_d-decoupled equation for the leading order transition rates. Equation (7) has also been applied to the four nuclei in Table I, using the internal amplitudes of Fig. 2(d).

The values of the predicted anisotropies agree well with the experimental data. In particular, we emphasize the prediction of anisotropies both less than and greater than 1; i.e., it is possible to have more α particles emitted along the symmetry axis than equatorially, despite the fact that $a_2 < 0$. The reason is that the external amplitude c_2 may

TABLE I. Anisotropies $[W(0)/W(\frac{1}{2}\pi)]$, for the favored decay of four odd-even nuclei. Deformation parameters correspond to the daughter nucleus.

Parent	J_p	β_2	β_4	Theory $\{+ \ - \ -\}$	Experiment	Ref.
^{221}Fr	$\frac{5}{2}$	0.039[a]	0.028	0.77	0.37(2)	[17]
^{227}Pa	$\frac{5}{2}$	0.147[a]	0.110	2.66	3.55(28)	[17]
^{241}Am	$\frac{5}{2}$	0.215	0.102	4.26	>2.7	[18]
^{253}Es[b]	$\frac{7}{2}$	0.235	0.040	3.70	>3.8	[19]

[a]Möller *et al.* give a nonzero value for β_3.
[b]Includes $L = 6$.

become positive, since the Clebsch-Gordan coefficients of Eq. (7) attenuate the effect of the internal amplitude a_2. We note that this effect is not possible if we use the amplitudes from Fig. 2(a) or 2(b). The solutions of Poggenburg *et al.* and of Delion *et al.* give anisotropies always greater than one. However, the excited states of the daughter lie at relatively low excitation energies and, in the present model, their coupling to the ground state is not sufficiently attenuated by barrier penetration for the reversal of sign between a_2 and c_2 to take place. The success of Eq. (7), could, however, suggest some other dynamical element through which the α-particle orbital angular momenta are mixed in exactly the same way as for even-even nuclei, but the coupling to different daughter states is absent.

We have shown that it is possible to describe all known branching ratios of even-even actinide nuclei with an α-particle wave function near the nuclear surface which is practically nucleus independent. This model has a certain aesthetic appeal in itself and, with the assumption that the same L-mixing matrix $\mathcal{K}_{LL'}$ is present in the favored decay of an odd system, is capable of reproducing the known anisotropies in this mass region. This, of course, begs the question as to what physical model could generate such constant amplitudes. The best candidate would appear to be the notion that the α-particle amplitudes should be projected from the pair-correlated neutron and proton Nilsson-model states. In this mass region, the level density is high and the pair forces lead to a rather diffuse Fermi surface. One might then expect that the correlated ground-state wave function should vary rather slowly with the Fermi energy, or in other words with the nucleon numbers of the system. Since pairing mainly takes place through the two-body angular momentum $J = 0$, this model would also be expected to give amplitudes with the property $|a_0| > |a_2| > |a_4|$, as found in Fig. 2(d).

[1] See, e.g., *Proceedings of the 4th International Symposium on the Foundations of Quantum Mechanics, Tokyo, 1992,* edited by M. Tsukada, S. Kobayashi, S. Kurihara, and S. Nomura (Publication Office, Japanese Journal of Applied Physics, Tokyo, 1993) (dedicated to the problem of quantal tunneling).
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