Thermal Activation above a Dissipation Barrier: Switching of a Small Josephson Junction

D. Vion,[1] M. Götz,[1,2] P. Joyez,[1] D. Esteve,[1] and M. H. Devoret[1]
[1]*Service de Physique de l'Etat Condensé, CEA-Saclay, F-91191 Gif-sur-Yvette, France*
[2]*Institut für Festkörperphysik, Friedrich-Schiller-Universität, D-07743 Jena, Germany*
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An unshunted Josephson tunnel junction switching out of its zero-voltage state is a model system for thermal activation out of a metastable state. For small-capacitance low-critical-current junctions, this thermal activation process follows a generalized Arrhenius law involving dissipation directly in its exponent. This escape over a dissipation barrier can be computed exactly for a junction connected to a *RC* impedance providing large damping. The diffusion branch and the switching histograms measured for such a junction are in agreement with theory. [S0031-9007(96)01408-1]

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Thermal activation out of a metastable state is a process of fundamental importance in nonequilibrium statistical mechanics of dissipative systems [1]. Placed initially in the neighborhood of a metastable attractor A_0 (see Fig. 1), such systems follow, at finite temperature T, diffusive trajectories in phase space. They eventually cross a separatrix Σ and fall into the basin of a lower energy attractor A_1. In the usual case of the escape from a potential well [shown in Fig. 1(a)], A_0 corresponds to a static state and Σ has a saddle point S corresponding to an unstable static equilibrium state. The thermal escape rate Γ then follows the ubiquitous Arrhenius law: $\Gamma = a \exp(-\Delta U/k_B T)$. In this expression, ΔU is simply the potential energy difference $U_S - U_{A_0}$ between the attractor and saddle points. The prefactor a depends weakly on the dissipation parameters [1,2]. In many systems of interest, however, A_0 corresponds to a dynamical state [Fig. 1(b)]. Energy must be dissipated to reach Σ, and escape now occurs over a "dissipation barrier." The escape rate is here given by a generalized Arrhenius law: $\Delta U/k_B T$ is replaced by an exponent $\mathcal{B}$ which depends on dissipation as well as temperature [3–5]. In this Letter we present an experiment on a system for which the dependence of $\mathcal{B}$ on dissipation is given by a closed form expression, and we compare measured escape rates above the dissipation barrier with theoretical predictions.

The system, whose schematics is given in Fig. 2, consists of a Josephson tunnel junction, characterized by its critical current I_0 and capacitance C_0, connected through a resistance R to a current source I in parallel with a capacitance C. This biasing circuit constitutes the simplest, well-characterized electrical environment in which a junction unshunted at dc [6] can be realistically embedded. As in the well-studied RCSJ model [7], our system is equivalent to a particle whose position corresponds to the phase difference δ across the junction and which moves in a tilted washboard potential. However, here, the particle is submitted to a frequency dependent friction [8] which vanishes when the velocity of the particle remains constant. The dc voltage V across the junction, which corresponds to the average velocity of the particle, is measured as a function of I, which corresponds to the tilt of the potential. At $T = 0$, and for $0 < I < I_0$, the system has two distinct states: a metastable "zero-voltage" state ($V = 0$) for which the particle is trapped in one of the wells of the washboard and a stable "voltage" state ($V \simeq 2\Delta/e$) for which the particle runs down the washboard at a limit velocity (this velocity corresponds to the onset of the breaking of Cooper pairs not represented in Fig. 2). At $T \neq 0$, under the influence of thermal fluctuations, the system can switch from the zero-voltage state to the voltage state. Two regimes must be considered depending on whether I is above or below I_1, the minimal current corresponding to the tilt of the washboard for which the particle starting from one maximum with zero velocity can run down the washboard without getting trapped in the next well because of frictional losses [8,9]. For $I > I_1$, the particle escaping out of a potential well directly accelerates down the washboard (runaway). In this regime, usually observed in large area Josephson junctions, the switching process falls in the usual class of escape over a potential barrier [10]. For $I < I_1$ the thermal escape from one well does not warrant runaway: The particle will hop diffusively from well to well down the washboard until fluctuations raise the velocity above

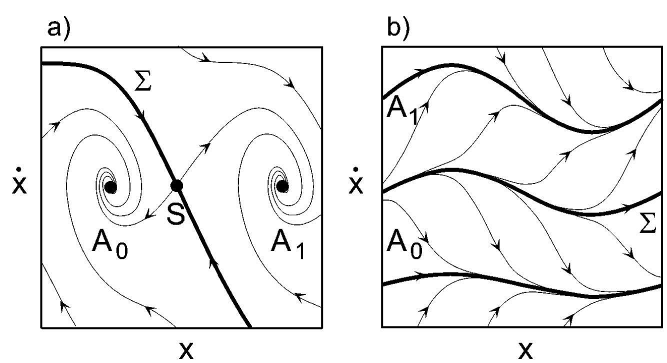

FIG. 1. Schematic flows in phase space for two different types of dissipative systems. In both systems there is a metastable attractor A_0 and a stable attractor A_1, but in (a) the attractors correspond to static states, whereas in (b) they correspond to dynamical states. The separatrix between the two attractors is labeled Σ.

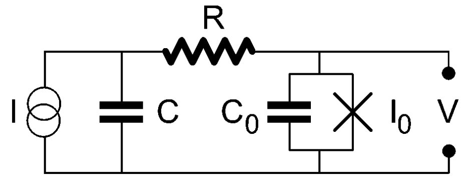

FIG. 2. Schematic of circuit implementing the dissipation barrier of Fig. 1(b). It is based on a Josephson junction with critical current I_0 and capacitance C_0 connected to a RC impedance. The voltage V across the junction is measured as the bias current I is increased.

the value for which runaway occurs [8,11,12]. Provided that the particle escapes from the wells frequently enough during the measurement of V, one will observe prior to switching not a true zero-voltage state but a diffusion state with $0 < V \ll 2\Delta/e$ analogous to a diffusion along A_0 of Fig. 1(b). The switching process in this regime belongs to the class of escape over a dissipation barrier, the role of the stable dynamical state A_1 of Fig. 1(b) being played by the voltage state. This process is amenable to detailed theoretical predictions as we will now show.

The circuit of Fig. 2 is described by three dynamical variables: δ, $\dot{\delta}$, and u, the ratio of the voltage across C to the characteristic voltage RI_0. These variables are treated here classically. The parameters of the system can be combined to form three independent quantities: the Josephson frequency $\omega_J = RI_0/\varphi_0$ and the damping factors $\alpha_0 = \varphi_0/R^2 I_0 C_0$ and $\alpha = R^2 I_0 C/\varphi_0$, where φ_0 denotes $\hbar/2e$. For junctions so small that $\alpha_0 \gg 1$, the current in C_0 is negligible. Thus, neglecting $\ddot{\delta}/\alpha_0$ terms, the time evolution of the circuit is governed by the set of dimensionless equations,

$$\frac{d\delta}{d\tau} = u - \sin\delta + \epsilon(\tau), \tag{1}$$

$$\frac{du}{d\tau} = \alpha^{-1}(s - \sin\delta). \tag{2}$$

Here the reduced parameters are $\tau = \omega_J t$, $s = I/I_0$. The reduced thermal noise ϵ obeys $\int_0^\infty \langle \epsilon(u,0)\varepsilon(u,\tau)\rangle \times \exp(i\omega\tau)d\tau = \Theta$, where $\Theta = k_B T/\varphi_0 I_0$ is the reduced temperature. For damping so large that $\alpha \gg 1$, the time evolution of u is much slower than the time evolution of δ. This separation between characteristic time scales allows one to use an adiabatic approximation: The stochastic Eq. (1) is first solved with u being kept constant, to get the time average expression $S(u)$ of $\sin\delta$ and the diffusion coefficient $\mathcal{D}(u) = \int_0^\infty \eta(u,0)\eta(u,\tau)d\tau$ associated with the fluctuations $\eta(u,\tau)$ of $\sin\delta$ around its average value [13]:

$$S(u) = \mathrm{Im}(x_1), \tag{3}$$

$$\mathcal{D}(u) = \frac{2}{\alpha^2}\,\mathrm{Im}\left\{\sum_{n=1}^{\infty}\left[(-1)^n\left(\frac{S(u)-u}{n} - i\Theta\right)x_n^2\right]\right\}, \tag{4}$$

where $x_n = I_{n-iu\beta}(\beta)/I_{-iu\beta}(\beta)$, $I_\nu(z)$ being the modified Bessel function and $\beta = \Theta^{-1}$. Then, substituting $\sin\delta$ in Eq. (2), we get the Langevin-like equation $du/d\tau = \mathcal{F}(u) + \eta(u,\tau)$, where $\mathcal{F}(u) = [S(u) - s]/\alpha$. The original problem is now transformed into the problem of a particle with position u and diffusion coefficient $\mathcal{D}(u)$, escaping out of an effective potential well given by $\mathcal{F}(u)$. Using Kramer's large friction limit result [2] for the prefactor, we find the escape rate,

$$\Gamma(s) = \frac{1}{2\pi}\,\omega_J \mathcal{D}_{\mathrm{top}}\sqrt{(\mathcal{F}/\mathcal{D})'_{\mathrm{bot}}(\mathcal{F}/\mathcal{D})'_{\mathrm{top}}}\,\exp(-\mathcal{B}), \tag{5}$$

with $\mathcal{B} = \int_{u_{\mathrm{bot}}}^{u_{\mathrm{top}}}(\mathcal{F}/\mathcal{D})du$ (bot and top stand, respectively, for the bottom and top of the well). The main result of our calculation is that $\mathcal{B} \propto \alpha$. Our experiment tests the predictions of Eq. (3) for the voltage in the diffusion state and of Eq. (5) for the switching rate. It is performed on a sample consisting of two circuits implementing Fig. 2 with α differing by a factor of 60.

The sample fabrication involved four steps. First, a gold ground plane forming one plate of the capacitors C was deposited on a Si wafer and covered by a silicon nitride insulating layer. Then two different resistors R were made by optical lithography and evaporation of an AuCu alloy. The other plates of the C capacitors and the junction pads involved another optical lithography step and evaporation of pure Au. Finally, two nominally identical Al-AlOx-Al Josephson junctions were fabricated using e-beam lithography and double angle shadow mask evaporation [14]. We estimate the capacitances $C_0 = 8 \pm 2$ fF of the junctions from their area. The capacitance $C = 0.15$ nF was measured at room temperature. The sample was mounted in a copper box thermally anchored to the mixing chamber of a dilution refrigerator. The electrical wiring for the bias and voltage leads was made using coaxial lines with miniature cryogenic filters [15]. The resistances R and the superconducting energy gap of the junctions were measured on the I-V characteristics at 30 mK in zero magnetic field. The junction critical currents were obtained from the Ambegaokar-Baratoff relation [16] using the measured tunnel resistances in the normal state. The parameters characterizing the two circuits referred to in the following as #1 and #2 were $I_0 = 40.1$ nA, $R = 70\ \Omega$, $\alpha_0 \simeq 160$, $\alpha = 83$ and $I_0 = 37.5$ nA, $R = 540\ \Omega$, $\alpha_0 \simeq 3$, $\alpha = 5100$, respectively. The bias current was ramped at constant reduced speed $\dot{s}$. We show in Fig. 3 a typical I-V characteristic, obtained for circuit #1 at 40 mK. The branch corresponding to the diffusion state appears vertical on this large scale. It is interrupted at the switching current I_S which fluctuates from one ramp cycle to another. A histogram of I_S is shown in the inset. In Fig. 4 we show diffusion branches measured using a lock-in technique for both circuits and for different temperatures. At a given current bias, the voltage across the junction,

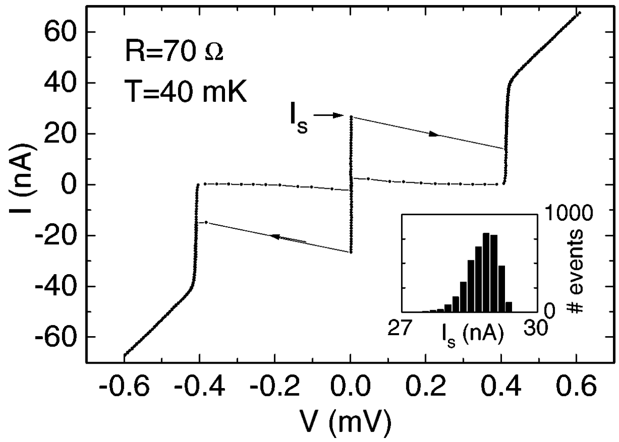

FIG. 3. Large scale I-V characteristic of a Josephson junction corresponding to the circuit of Fig. 2. The switching at current I_s from the diffusion branch (vertical branch in the center of the characteristic) to the quasiparticle branch of the junction is a random process. Inset shows histogram of I_S measured at $dI/I_0dt = 8.5$ s^{-1} for circuit #1.

which measures phase diffusion, increases with temperature and is larger for circuit #2 than for circuit #1. We also show in Fig. 4 the curves $I(V) = I_0 S(u) + I_{QP}(V)$, where $u = V/RI_0 - S(u)$, predicted by our model using the measured parameters. The correction $I_{QP}(V)$ due to quasiparticles was calculated using BCS theory [17]. Its relative importance attains only 20% for the highest temperature. The agreement between experimental and calculated curves is quantitative for circuit #1 and only qualitative for circuit #2. By varying I_0 with a small magnetic field, we checked that the discrepancy at low temperature between theory and experiment for circuit #2 could not be explained by some remaining external noise on the sample or Joule heating in the resistor. We attribute the discrepancy to the fact that circuit #1 fully satisfies the hypotheses of our calculation ($\alpha_0 \gg 1$ and $\alpha \gg 1$) while for circuit #2, $\alpha_0 \simeq 3$. However, agreement is recovered at high temperature by performing numerical simulations including C_0 (data not shown). At low temperature, quantum fluctuations of the phase lower the $S(u)$ curves [18] and could be taken into account to make a more accurate theoretical prediction [19]. Note that even when δ can fluctuate quantum mechanically because α_0 is not large enough, u remains a classical variable and the switching is an entirely classical process.

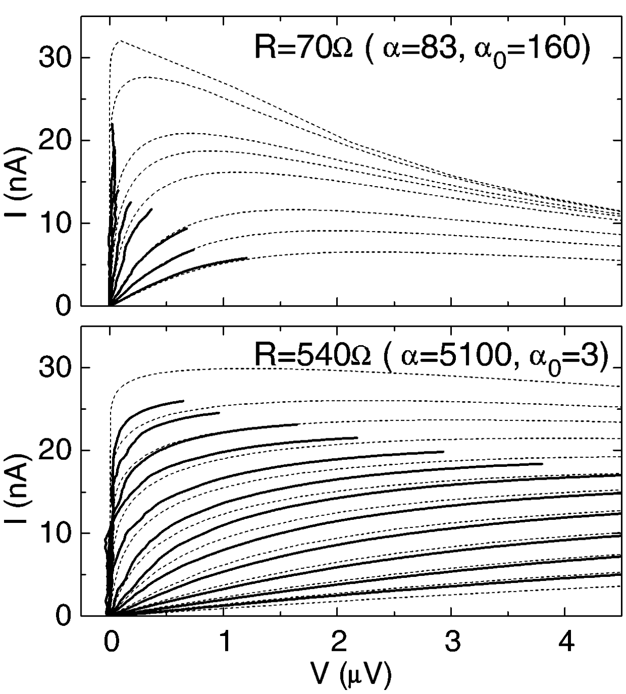

FIG. 4. Experimental (solid lines) and theoretical (dotted lines) diffusion branches of two circuits of the type in Fig. 2. Top: circuit #1 at $T = 47$, 110, 330, 422, 598, 700, and 809 mK (from top to bottom). Bottom: circuit #2 at $T = 47$, 100, 140, 193, 253, 312, 372, 448, 535, 627, 718, and 813 mK (from top to bottom).

Histograms of the current I_S obtained from 8000 switching events were measured as a function of temperature in order to test the predictions of Eq. (5). The measured histograms were first converted into $\ln \Gamma(s)$ sets of data points by the method of Fulton and Dunkleberger [10]. For a given temperature, these data points fall on a single curve independent of $\dot{s}$ (data not shown). It is convenient to characterize the current dependence of the rate at a given temperature by two values: the average switching current $\langle I_S \rangle$ and the standard deviation ΔI_S. These values are shown in Fig. 5 together with theoretical predictions. The averages $\langle I_S \rangle$, which decrease with temperature, are nearly identical for both circuits. However, ΔI_S is about 1 order of magnitude higher for circuit #1 than for circuit #2. Furthermore, ΔI_S for circuit #1 decreases significantly when $\Theta > 0.2$. These effects are well explained by our calculation. At a given temperature, the exponent $\mathcal{B}$ vanishes when s reaches the maximum of the $S(u)$ curve. Thus, in the limit $\alpha \rightarrow \infty$, $\langle I_S \rangle / I_0 = \max[S(u)]$ [dashed line in Fig. 5(a)]. As damping is decreased, the dissipation barrier height decreases ($\mathcal{B} \propto \alpha$), and thermal fluctuations driving u above the dissipation barrier induce premature switching. The predicted curve $\langle I_S \rangle(\Theta)/I_0$ for circuit #1 [solid line in Fig. 5(a)] shows this effect and fits the experimental data. The corresponding curve for circuit #2 is indistinguishable from the $\max[S(u)]$ curve and agrees only qualitatively with the data. We attribute this discrepancy to the aforementioned smallness of α_0.

The large increase in the width of the histogram when going from circuit #2 to circuit #1 is a more direct manifestation of the effect of damping [see Fig. 5(b)]. As the damping α decreases, the relative change in the barrier height with s and, consequently, the slope of $\Gamma(s)$ decreases. Finally, the decrease of ΔI_S at high temperature is a consequence of the rounding of $S(u)$ with increasing Θ.

To summarize, a small unshunted current-biased junction connected to a RC impedance switches from a phase

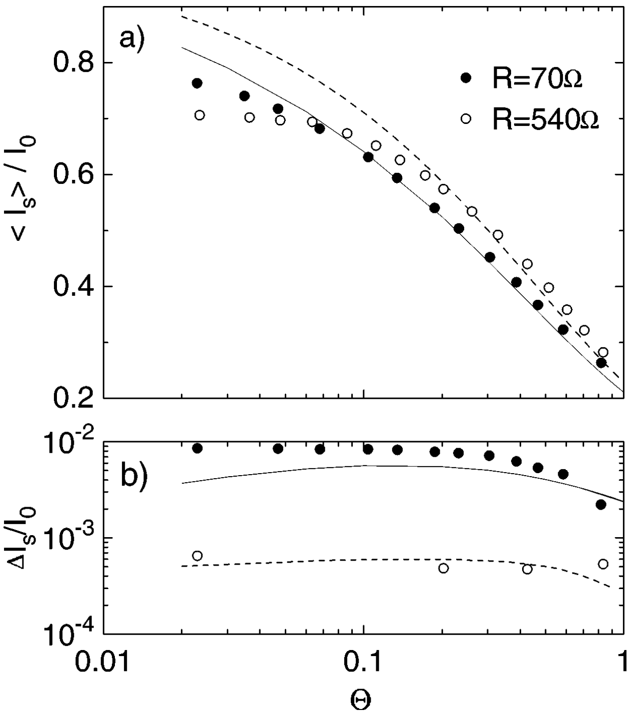

FIG. 5. Experimental (dots) and theoretical (lines) switching current average $\langle I_S \rangle$ (a) and mean square deviation ΔI_S (b) as a function of the dimensionless temperature $\Theta = k_B T/\varphi_0 I_0$, for circuits #1 and #2.

diffusion branch to a voltage branch by a process entirely different from the switching in large area junctions. This process is not dominated by thermal activation over the usual washboard potential barrier (or quantum tunneling through this barrier) but by thermal activation above a dissipation barrier for which an expression can be found in the large friction limit. The predictions based on this expression are well verified experimentally. When R increases, the width of switching histograms decreases, a direct consequence of the scaling of the dissipation barrier with the RC time constant of the impedance. The effect of temperature is twofold. It modifies the dependence of the dissipation barrier on bias current as well as producing the fluctuations driving the system above this barrier. This complexity must be taken into account if the average value of the switching current is to be used as a measurement of the critical current. Finally, the current dependence of the voltage in the diffusion state prior to switching is directly related to the shape of the dissipation barrier. Our results indicate that the dissipation barrier can be affected by quantum fluctuations of the phase difference when α_0 is small. Precise measurements of the voltage prior to switching as a function of α_0 in the large α regime would improve our knowledge of the quantum diffusion process in the tilted washboard.

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