The Possibility of Flux Flow Spectroscopy

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(Received 7 June 1996)

A novel spectroscopic technique applicable to the study of spin excitations in magnetic materials is described. The probe frequency and wave vector are determined by the moving Abrikosov flux lattice of a type II superconductor in the mixed state. The feasibility of employing oxide superconductors and relevant material parameters are discussed. [S0031-9007(96)01211-2]

PACS numbers: 85.25.–j, 74.60.Ge, 76.20.+q, 76.50.+g

In this Letter we examine the feasibility of performing spectroscopic measurements of the excitations reflected as a resonance in the dynamic susceptibility $\chi(\vec{q},\omega)$ using flux flow devices. These devices can be fabricated from high temperature superconductors and used to probe the properties of another material (the specimen) placed in close proximity. A broad range of wave vector, $\vec{q}$, and frequency, ω, of the modes representing the specimen are potentially detectable with this technique. In fact, it should prove possible to examine a range of $\vec{q}$ amounting to a substantial fraction of the Brillouin zone (BZ). The proposed technique is complimentary to neutron diffraction but with the remarkable advantage that extremely small specimens (mass $\sim 10^{-14}$ g) may be investigated, making accessible submicroscopic systems.

The idea of using the ac Josephson effect to perform *in situ* spectroscopy, wherein the specimen under study is the junction barrier material itself, was proposed some time ago [1]. Electron-spin resonance (ESR) spectroscopy was envisioned and was later successfully demonstrated using conventional superconducting tunnel junctions [2,3]. Subsequently, Goldman *et al.* [4] and Barnes and Mehran [5] developed the theory further to include measurements of $\chi(q,\omega)$ in concentrated magnets. The present proposal is simpler in that the "device" is nothing more than a thin-film superconductor, and the sample need not form an integral part of the device—it need only be placed in close proximity.

The use of high-T_c materials for device fabrication has the obvious advantage that an expanded temperature range for studies is made available. Furthermore, most of the characteristic quantities are larger. The theoretical maximum Josephson frequency ($\nu_{\max}$) is limited by the zero-temperature superconducting energy gap to be $2\Delta(0)/h$; for the cuprates [$\Delta(0) \sim 30$ meV] this implies $\nu_{\max} \sim$ 15 THz. The maximum theoretical wave vector is, for simple flux flow, $\sim 1/\xi$, and for a weak link geometry, $\sim \lambda_L/\xi^2$, where ξ and λ_L are the superconducting coherence length and London penetration depth, respectively.

Each experimental element involved in the presently proposed experiment has already been demonstrated, although never together. The self-detection of electromagnetically excited resonance modes via the Josephson effect has been mentioned above. The detection of electromagnetic radiation using a sliding Abrikosov lattice in the cuprates has been demonstrated by Harris *et al.* [6], albeit at very low frequencies compared with those envisaged here. Doettinger *et al.* [7] have shown that it is possible to displace this lattice in high magnetic fields at velocities of more than 10^3 m/s. This value is comparable to or greater than those of typical spin waves and many other excitations. Finally, there is substantial recent experimental evidence [8] to show that a *moving* flux lattice is indeed crystalline and nearly perfect when driven at high velocities.

The basic principles are quite simple. Consider the flux flow device illustrated in Fig. 1. There is a potential difference V developed across the junction, and a magnetic field $B > B_{c_1}$ is applied in a direction perpendicular to the plane of the device. The Abrikosov lattice is in motion with some velocity v, and at a given point in space there will be a periodic radio frequency (rf) field, $\sim B_{c_1}$ inside and in proximity to the system. The Abrikosov lattice has a spatial period $\sim B^{-1/2}$, which therefore defines

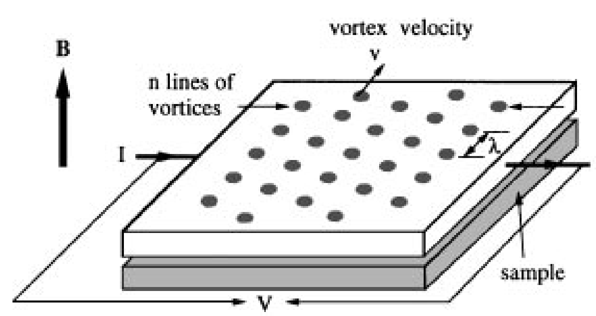

FIG. 1. The device, without shading, comprises a flux flow region with n lines of vortices in the direction of current flow. The wavelength λ is determined by the distance between vortices in the direction perpendicular to the current flow. The vortex lattice moves with a velocity v also in a direction perpendicular to the current flow. The sample, shown shaded below the device, must be in close proximity to but need not form a part of the device. Indicated is an applied magnetic field perpendicular to the system.

 0031-9007/96/77(15)/3252(4)$10.00

a wave vector,

$$q = \frac{2\pi}{1.075}\left(\frac{B}{\phi_0}\right)^{1/2}. \tag{1}$$

The bias voltage implies some current density J which causes this lattice to have a velocity

$$v = \frac{\phi_0 J}{\eta c}, \qquad \eta = \frac{\phi_0 H_{c2}}{\rho_n c^2}. \tag{2}$$

This involves the Bardeen-Stephen [9] viscosity η which reflects the normal state resistance ρ_n. For the present purposes this expression for the velocity is misleading. If there are n lines of the vortex lattice in the direction of the current flow (see Fig. 1), then, using the above plus $\omega = vq$, gives

$$\hbar\omega = \frac{2eV}{n}, \tag{3}$$

which is just the usual Josephson relationship except for the factor of $1/n$ on the right-hand side. For a simple long Josephson junction, in a magnetic field, there is but a single line of vortices, and the phase difference across the junction advances by 2π each time a vortex passes by. When there are n such lines of vortices the phase advances, and hence the voltage across the junction is simply n times larger. The frequency of the rf field is thereby determined solely by geometrical factors and the applied fields; i.e., despite their appearance in Eqs. (2), material constants do not enter in the fundamental relationships when they are written in the form of Eqs. (1) and (3). Finally the magnitude of the rf field is $\sim B_{c_1}$ since this is the magnitude of the difference between the field at the center of the vortex and the minimum value between the vortices.

That the flux flow device is a self-detector is also easily appreciated. The motion of the Abrikosov lattice implies a dissipation as suggested by the theory of Bardeen-Stephen; i.e., there is a flux flow resistance. However, when the frequency of the rf field corresponds to a resonance in $\chi(\vec{q},\omega)$, the energy absorbed by the corresponding excitation must be supplied by the battery since there is no other source of energy in the system. For a given value of applied voltage, this implies a change in the current. The present linear response formalism is based upon this observation.

The principal theoretical problem is to determine the size of this current change. The change in current density ΔJ for a given electric field E is determined by

$$\Delta J E = \left.\frac{dU}{dt}\right|_{\text{sample}}. \tag{4}$$

The rate of absorption of energy can be written as

$$\left.\frac{dU}{dt}\right|_{\text{sample}} = \sum_{ij} E_{ij} T_{i\to j}, \tag{5}$$

where $T_{i\to j}$ is the probability for a transition of energy $E_{ij} = E_j - E_i$. By the golden rule,

$$T_{i\to j} = \frac{2\pi}{\hbar}|m_{ij}|^2 (e^{-\beta E_i} - e^{-\beta E_j})\delta(E_{ij} - \hbar\omega), \tag{6}$$

where $m_{ij} \sim b_{\text{rf}}$ are the relevant matrix elements of the radio frequency field b_{rf}, at the sample, generated by the moving vortex lattice. If the sample and vortex lattice are in close proximity and the sample is not too large $b_{\text{rf}} \sim B_{c1}$, which, as argued above, is the magnitude of the field in the device. If it is assumed that there is a narrow resonance due to some collective mode with a frequency ω_q, and that this mode dominates the imaginary part of the dynamic susceptibility, it is possible to replace $E_{ij} = \hbar\omega_q$ by $\hbar\omega$, and

$$\left.\frac{dU}{dt}\right|_{\text{sample}} = \hbar\omega \sum_{ij} T_{i\to j}, \tag{7}$$

where the $\sum T_{i\to j}$ is, to within constant factors, the definition of the imaginary part of the dynamic susceptibility times the field squared. It follows that the absorbed power

$$P = \Delta J E = \left(\frac{2}{\mu_0}\right) \text{Re}[-i\omega\chi(q,\omega)] b_{\text{rf}}^2. \tag{8}$$

This result simply amounts to taking the real part of the intuitive relationship $P = -i\omega\vec{B}\cdot\vec{M}$. To be specific consider the spin wave modes of a ferromagnet. The dynamic susceptibility is, near to a resonance, typically of the form

$$\chi = \frac{\mu_0\mu_B^2(S/V)}{\hbar(\omega_q - \omega) - i\Delta} = \frac{\mu_0\mu_B M_0}{\hbar(\omega_q - \omega) - i\Delta}, \tag{9}$$

where Δ is the width in energy units and $M_0 = \mu_B S/V$ is the net magnetization. The signal is therefore given by

$$\Delta J E = \left(\frac{2}{\mu_0}\right) \frac{\omega\mu_0 M_0(\Delta/\mu_B)}{(\hbar^2/\mu_B^2)(\omega_q - \omega)^2 + (\Delta/\mu_B)^2}\, b_{\text{rf}}^2. \tag{10}$$

In order to obtain an expression useful for estimations, use is made of

$$\omega_q = vq, \tag{11}$$

where v is the velocity of the spin wave excitation. The flux lattice has its q matched with that of the excitation, implying an estimate,

$$q = \frac{2\pi}{\lambda_L}\frac{B}{B_{c_1}}, \tag{12}$$

where λ_L is the London penetration length. Useful is the relationship

$$v\mu_0 = \frac{v}{c}\, 377\ \Omega, \tag{13}$$

whence

$$\Delta J E = 2\,\frac{v}{c}\,(377\ \Omega)\,\frac{2\pi}{\lambda_L}\,\frac{B}{B_{c_1}}\,\frac{\mu_0 M_0(\Delta/\mu_B)}{(\hbar^2/\mu_B^2)(\omega_q - \omega)^2 + (\Delta/\mu_B)^2}\, h_{\text{rf}}^2. \tag{14}$$

At least for modest fields, i.e., somewhat greater than H_{c_1}, the radio frequency field, h_{rf}, is independent of v and the other parameters and is $\sim H_{c_1}$. So finally, in terms of physical quantities, the estimated response is

$$\Delta JE = \frac{4\pi(377\ \Omega)H_{c_1}^2}{\lambda_L}\frac{v}{c}\frac{B}{B_{c_1}}\frac{\mu_0 M_0(\Delta/\mu_B)}{(\hbar^2/\mu_B^2)(\omega_q - \omega)^2 + (\Delta/\mu_B)^2}. \tag{15}$$

In order to write an expression for $\Delta J/J$ it is noted that

$$JE = \frac{1}{\rho}E^2 = \frac{1}{\rho}v^2B^2 = \frac{1}{\rho}\left(\frac{v}{c}\right)^2 H_0^2, \tag{16}$$

where the relationship $E = vB$ has been used for the static fields. Combining the two relations and using $\rho \sim \rho_n(H/H_{c_2})$ for the flux flow resistance, yields

$$\frac{\Delta J}{J} = 2\pi\left(\frac{c}{v}\right)\left(\frac{(\rho_n/\lambda_L)}{(377\ \Omega)}\right)\frac{H_{c_1}}{H_{c_2}}\frac{\mu_0 M_0(\Delta/\mu_B)}{(\hbar^2/\mu_B^2)(\omega_q - \omega)^2 + (\Delta/\mu_B)^2}. \tag{17}$$

At resonance, and for the present assumption of maximum coupling, it follows:

$$\frac{v}{c}\frac{\Delta J}{J} = 2\pi\left(\frac{c}{v}\right)\left(\frac{(\rho_n/\lambda_L)}{(377\ \Omega)}\right)\frac{H_{c_1}}{H_{c_2}}\frac{\mu_0 M_0}{\Delta/\mu_B}. \tag{18}$$

Typical for the oxide superconductors are $\rho_n = 100\ \mu\Omega$ cm, $\lambda_L = 1500$ Å, $H_{c1} = 100$ G, and $H_{c2} = 10^6$ G. The sample is characterized by the velocity v (defined as ω/q) and the ratio, $\mu_0 M_0/\Delta/\mu_B$, of the internal field to the width. If $v = 10^4$ m/s then a typical value of 50 for the latter ratio gives $\Delta J/J \sim 1$.

The bridges of Doettinger *et al.* [7] are typically 100 nm thick, 20 μm wide, and 100 μm long (defined as the distance between the voltage contacts). For $B = 1.8$ T the wavelength $\lambda = 2\pi/q \sim 100$ Å and $v \sim 10^3$ m/s for a voltage of $\sim$0.2 V and a current of 0.05 A. These authors report that v is independent of the magnetic field B for fields of this magnitude. In order to reduce the steady state dissipation, pulsed currents are used and the film is deposited on a highly conductive MgO substrate. For spin waves in a typical ferromagnet, this velocity would be typical of or greater than the maximum values encountered in the entire BZ. Generating lower velocities presents no problem, and while the value of q would correspond to only a few percent of that at the zone edge, it corresponds to fields much smaller than H_{c2}, and there is no reason why this might not be increased by a factor of 10 without implying an extreme extrapolation of the parameter range already explored. This would permit more than 10% of the BZ to be investigated.

The work of Ref. [7] *does* illustrate a problem associated with the extrapolation to higher velocities. Their system is current fed, and, following a theory of Larkin and Ovchinnikov [10], there exists a negative resistance region beyond a certain temperature-dependent critical velocity $v^* \sim 10^3$ m/s for $YBa_2Cu_3O_{7-\delta}$. The theory predicts that the resistive losses actually *decrease* with increasing voltage beyond that which corresponds to v^*. The existence of this negative resistance region leads to an instability when the flux flow device is driven by a current source (as also occurs, e.g., for Shapiro steps). This value of v^* is likely to be adequate for most purposes; however, for high velocity excitations, it is probable in some temperature ranges that this instability will represent an important limitation. A typical resistance is $\sim 1\ \Omega$, and there would seem to be no insurmountable obstacle to the use of a voltage source. It might be noted in connection with these experiments that the current density $\sim 10^{10}$ A/m^2 is several orders of magnitude larger than the conventionally defined critical current density.

That the flux lattice tends to be deformed [8] for currents of the order of the critical current implies the existence of a *minimum* useful velocity v_m. Using $v = (E/B)$ and $\rho = \rho_n(B/B_{c_2})$ for the flux flow resistance implies $v_m \sim \rho_n J_c/B_{c_2} \sim 10^{-2}$ m/s for the values given above and a typical $J_c \sim 10^6$ A/m^2. This is not a serious restriction but serves to illustrate that in the present context a *good* superconductor is one with small pinning and hence small critical currents.

Assuming that the experimental extrapolation to higher fields is without serious problems, it remains the case that the upper bound of q is limited by the coherence length which in the *a-b* plane of cuprates is $\sim$30 Å. It is possible to imagine the use of the *a-c* plane which, by virtue of the short coherence length in the c direction, would lead to q values comparable to those at the BZ edge. However, the field corresponding to these limiting values are beyond current laboratory practice.

The magnitude of the required field, for a given q, might be reduced by increasing the area occupied by the flux quantum. Perhaps the optimal fashion by which to achieve the maximum usable q is with a long Josephson junction. Such high quality junctions exist naturally at grain boundaries in high temperature superconductors. For the present objective the advantage of a junction is its large magnetic width $d = 2\lambda_L + a \sim 2\lambda_L$ since the London penetration depth λ_L is invariably much greater than a, the barrier width. As usual, there must be a single flux quantum per wavelength and hence the wave vector is determined, in a junction, by the condition that $B\lambda d = \phi_0$ or that

$$q = \frac{\xi}{\lambda_L}q_0, \tag{19}$$

where q_0 is the value given by Eq. (1). This might also be written as $B \sim B_{c_1} \lambda_L / \lambda$ and with $B_{c_1} \sim 10^{-2}$ T, and $\lambda_L \sim 1500$ Å implies $B \sim 10$ T for $\lambda \sim 1.5$ Å. The resulting *electromagnetic field* is a reasonably well defined plane wave which extends a distance $2\lambda_L \sim 3000$ Å on either side of the barrier. Clearly this solution is sensible for large enough q such that the uncertainty $\Delta q \sim (2\lambda_L)^{-1}$ is not important. (A more complicated possibility would have a series of parallel junctions, or simply channels of weakened superconductivity, separated by $\sim 2\lambda_L$. This would solve the Δq problem and enhance the active area of the device.)

For both junctions *and* flux flow devices there are important issues relevant to the coupling of the device to the sample that must be addressed. It should be observed that, in the absence of the sample, the rf field outside the bridge or junction falls off exponentially with a characteristic length which is simply the wavelength λ. This is a trivial property of the solution of Maxwell's equations for the region outside the device. For maximum coupling the sample must be in very close proximity on an atomic scale for the highest wave vectors envisaged. Also, the sample should not exceed the region occupied by the field. In general, the coupling to the measuring device will be reduced by an antifilling factor f which represents the fraction of the sample occupied by the field. For a flux flow device two dimensions of the region filled by the field are defined by the area of the flux flow region [envisaged $\sim (1\ \mu\text{m})^2$] while the active area for a junction corresponds to the rectangle which extends λ_L on either side of the tunneling barrier. The other dimension of this active region is either the wavelength for an insulator or the microwave penetration depth for a metal. The latter, in the extreme anomalous region, is typically of the order of several hundreds Å, and is actually greater than the wavelength for such high frequencies. This can be thought of as arising from the better impedance match of the device to a metal (or another superconductor).

The relative orientation of the static and rf fields also needs discussing. For the situation illustrated in Fig. 1, it would appear that the rf and static fields are parallel within the sample. This would present a problem since a strong direct coupling to, e.g., spin waves requires the field to be perpendicular to the static applied field. In fact, the decaying solutions of Maxwell's equations always contain a component of the rf field which is in the required plane. More directly, a component in this plane might be generated by tilting the applied field by a small angle. The vortices, and with them the rf field at the surface of the device, will remain almost perpendicular to the plane while the internal static field within the sample will subtend a finite angle to this direction, as desired.

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