Gauge Fields and Pairing in Double-Layer Composite Fermion Metals

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A symmetrically doped double-layer electron system with total filling fraction $\nu = 1/m$ decouples into two even-denominator composite fermion "metals" when the layer spacing is large. Statistical gauge fluctuations in this system mediate an attractive pairing interaction between composite fermions in different layers. A strong-coupling analysis shows that for any layer spacing d this pairing interaction leads to the formation of a paired quantum Hall state. [S0031-9007(96)01357-9]

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Composite fermions were introduced by Jain in order to understand the observed hierarchy of states in the fractional quantum Hall effect (FQHE) [1]. A composite fermion is an electron confined to move in two dimensions and tied to an even number of statistical flux quanta. Jain showed that the fractional quantum Hall effect for electrons at odd-denominator filling fractions can be viewed as an effective *integer* quantum Hall effect for composite fermions. Halperin, Lee, and Read (HLR) took Jain's suggestion further, arguing that at Landau level filling fraction $\nu = \frac{1}{2}$, or any even-denominator filling fraction $\nu = \frac{1}{2m}$, the statistical flux attached to composite fermions can, at the Hartree level, exactly cancel the physical flux of the applied magnetic field [2]. The composite fermions then form a new type of metal, and a growing number of experiments appear to support this description [3]. HLR also showed that fluctuations of the statistical gauge field in this metal give rise to singular inelastic scattering of sufficient strength to lead to a breakdown of Landau Fermi liquid theory. Though experimental proof of the non-Fermi-liquid nature of the composite fermion metal remains elusive, it has generated a great deal of excitement in the theoretical community [4].

Double-layer electron systems have been realized in both double quantum wells [5] and wide single quantum wells [6]. The (m, m, n) states at filling fraction $\nu = \frac{2}{m+n}$, proposed by Halperin [7], are double-layer generalizations of the Laughlin states. At even denominators, there are additional possibilities motivated by the composite fermion construction. As one of us has pointed out [8], in the limit where the layer spacing d is large, it should be possible to view a double-layer system at $\nu = \frac{2}{2m}$ as two decoupled $\nu = \frac{1}{2m}$ composite fermion metals. This description will be referred to as the double-layer composite fermion metal (DLCFM) description in what follows. The main result of this Letter is that an *ideal* DLCFM, by which we mean a DLCFM in which the carrier densities in the two layers are precisely equal, there is no interlayer tunneling, and there is no disorder, is *always* unstable to the formation of a paired quantum Hall state for any layer spacing d.

The Lagrangian density for an ideal DLCFM as defined above is given by ($\hbar = c = 1$)

$$\mathcal{L}(\mathbf{r}, \tau) = \mathcal{L}_1(\mathbf{r}, \tau) + \mathcal{L}_2(\mathbf{r}, \tau), \tag{1}$$

where

$$\mathcal{L}_1(\mathbf{r}, \tau) = \sum_s \Big(\psi^\dagger_{(s)}(\mathbf{r}, \tau)[\partial_\tau + i a_0^{(s)}(\mathbf{r}, \tau)]\psi_{(s)}(\mathbf{r}, \tau) + \frac{1}{2m_b}\psi^\dagger_{(s)}(\mathbf{r}, \tau)[-i\nabla - \mathbf{a}^{(s)}(\mathbf{r}, \tau) + e\mathbf{A}(\mathbf{r})]^2 \times \psi_{(s)}(\mathbf{r}, \tau)\Big), \tag{2}$$

$$\mathcal{L}_2(\mathbf{r}, \tau) = -\sum_{s,s'} \frac{i}{2\pi} K^{-1}_{ss'} a_0^{(s)}(\mathbf{r}, \tau)\epsilon_{ij}\partial_i a_j^{(s')}(\mathbf{r}, \tau) + \frac{1}{2}\sum_{s,s'}\int d^2r'\, \delta\rho_{(s)}(\mathbf{r}, \tau) V_{s,s'}(\mathbf{r} - \mathbf{r}') \times \delta\rho_{(s')}(\mathbf{r}', \tau). \tag{3}$$

Here s is a layer index, $V_{s,s'}(\mathbf{r}) = e^2/\epsilon \times \sqrt{\vec{r}^2 + (1 - \delta_{s,s'})d^2}$ is the intralayer ($s = s'$) and interlayer ($s \neq s'$) Coulomb interaction, ψ_s is the fermion field in layer s,

$$\delta\rho_{(s)}(\mathbf{r}, \tau) = \psi^\dagger_{(s)}(\mathbf{r}, \tau)\psi_{(s)}(\mathbf{r}, \tau) - n \tag{4}$$

is the density fluctuation about the mean density in each layer, and $\nabla \times \mathbf{A} = B$ where $B = m\frac{2\pi n}{e}$ is the applied magnetic field. We work in the transverse gauge, $\nabla \cdot \mathbf{a}^{(s)}(\mathbf{r}, \tau) = 0$, and take $K_{11} = K_{22} = 2m$, $K_{12} = K_{21} = 0$, which is the natural choice in the limit of large layer spacing. We further specialize to $m = 1$, but all results may easily be generalized. Integrating out the $a_0^{(s)}$ fields enforces the constraint

$$\frac{1}{2\pi}\nabla \times (\mathbf{a}^{(s)} - e\mathbf{A}) = 2\delta\rho_{(s)}, \qquad (5)$$

which attaches two flux tubes of the appropriate statistical flux to each electron. In the following, we shall denote by $a^{(s)}$ the fluctuation in the transverse gauge field associated with layer s.

It is natural to describe the fluctuations of this system in terms of in-phase and out-of-phase modes. If the in-phase and out-of-phase gauge fields are defined as $a^{(\pm)} = a^{(1)} \pm a^{(2)}$, then within the random-phase approximation the relevant gauge field propagators at low frequency and long wavelengths are, in the limit $d \gg l_0$,

$$D^{(+)}(q, i\omega_n) \simeq (e^2 q/4\pi\epsilon + |\omega_n| k_f/4\pi q)^{-1} \qquad (6)$$

for the in-phase gauge fluctuations and

$$D^{(-)}(q, i\omega_n) \simeq \begin{cases} (e^2 d q^2/4\pi\epsilon \\ \quad + |\omega_n| k_f/4\pi q)^{-1} & \text{for } q \lesssim d^{-1}, \\ (e^2 q/4\pi\epsilon \\ \quad + |\omega_n| k_f/4\pi q)^{-1} & \text{for } q \gtrsim d^{-1} \end{cases} \qquad (7)$$

for the out-of-phase fluctuations [8]. The current-current interactions mediated by these gauge fields in the interlayer Cooper channel are then

$$V_{12}^{\pm}(k, k'; i\omega_n) = \pm\left(\frac{\mathbf{k} \times \hat{\mathbf{q}}}{m^*}\right)^2 D^{\pm}(q, i\omega_n), \qquad (8)$$

where $\hat{\mathbf{q}} = (\mathbf{k} - \mathbf{k}')/|\mathbf{k} - \mathbf{k}'|$ and m^* is the effective mass of the composite fermions. Fluctuations in $a^{(-)}$ are more singular at low frequencies than those in $a^{(+)}$ because the Coulomb interaction suppresses the in-phase density fluctuations but not the out-of-phase density fluctuations. As a consequence the effective interaction is dominated by the out-of-phase fluctuations. We will include both the in-phase and out-of-phase fluctuations in our calculations, while ignoring the less singular density-density and density-current interactions [8].

The dominant out-of-phase mode mediates an *attractive* pairing interaction between composite fermions in opposite layers. This attractive pairing interaction appears because $a^{(-)}$ couples to composite fermions in different layers as if they were oppositely charged. The fluctuating $a^{(-)}$ field strongly inhibits the coherent propagation of a single composite fermion, while a pair made up of composite fermions from different layers is neutral with respect to $a^{(-)}$. Such a composite fermion pair can then propagate coherently through the fluctuating $a^{(-)}$ field, much like a colorless meson propagating coherently through a strongly fluctuating gluon field.

In [8] it was proposed that this attractive interaction might lead to a "superconducting" instability of an ideal DLCFM. Such a superconducting state of composite fermions would be incompressible and thus exhibit the FQHE [9]. Here we investigate this possibility within the framework of Eliashberg theory. Related work, in different contexts, can be found in [10]. Using the Nambu formalism the matrix Green's function is

$$G(k, i\omega_n) = [i\omega_n Z_n - \epsilon_k\tau_3 - \phi_n\tau_1]^{-1}, \qquad (9)$$

where $\omega_n = (2n + 1)\pi T$ is a fermion Matsubara frequency, Z_n is the mass renormalization, ϕ_n is the anomalous self-energy, and $\Delta_n = \phi_n/Z_n$ is the gap function. The Eliashberg equations for l-wave pairing in this system are then given by

$$\begin{aligned} \omega_n(1 - Z_n) &= -\pi T \sum_m \frac{Z_m\omega_m}{(|Z_m\omega_m|^2 + \phi_m^2)^{1/2}} \times (\lambda_{m-n,0}^{(+)} + \lambda_{m-n,0}^{(-)}), \\ \phi_n &= -\pi T \sum_m \frac{\phi_m}{(|Z_m\omega_m|^2 + \phi_m^2)^{1/2}} \times (\lambda_{m-n,l}^{(+)} - \lambda_{m-n,l}^{(-)}), \end{aligned} \qquad (10)$$

where the coupling constants λ are obtained by averaging the effective interactions (8) over the Fermi surface:

$$\lambda_{m-n,l}^{(\pm)} = \frac{k_f}{2\pi m^*}\int_0^{2k_F} \cos\left(2l \sin^{-1}\frac{q}{2k_F}\right) \times D^{(\pm)}(q, |\omega_m - \omega_n|)\sqrt{1 - (q/2k_F)^2}\,dq. \qquad (11)$$

The pairing interaction mediated by $a^{(-)}$ is singular at small q and is thus attractive in all angular momentum channels. Here we consider the case of s-wave pairing and henceforth set $\lambda_{m-n}^{(\pm)} \equiv \lambda_{m-n,0}^{(\pm)}$.

Performing the integral (11) for $\lambda^{(-)}$ yields

$$\lambda_{m-n}^{(-)} \sim \frac{E_f}{(e^2/\epsilon l_0)}\left(\frac{l_0}{d}\right)^{2/3}\left(\frac{e^2/\epsilon l_0}{|\omega_m - \omega_n|}\right)^{1/3} + \text{less singular terms}, \qquad (12)$$

where $E_f = k_f^2/2m^*$ and $l_0 = 1/\sqrt{eB}$ is the magnetic length (for $\nu = 1/2$, $k_f = l_0^{-1}$). As discussed by HLR [2], the electron band mass must be renormalized so that $m^* \sim \epsilon/e^2 l_0$. For simplicity in what follows we will take $E_f \sim e^2/\epsilon l_0$ and

$$\lambda_{m-n}^{(-)} = \gamma\left(\frac{\omega_0}{|\omega_m - \omega_n|}\right)^{1/3}, \qquad (13)$$

where $\gamma = (l_0/d)^{2/3}$ is a dimensionless "coupling constant," and $\omega_0 = e^2/\epsilon l_0$. Performing the same integration for $\lambda^{(+)}$ we obtain

$$\lambda_{m-n}^{(+)} \sim \ln\left(\frac{\omega_0}{|\omega_n - \omega_m|}\right) + \text{less singular terms}. \qquad (14)$$

The two Eliashberg equations can be combined to obtain a single equation for Δ_n:

$$\Delta_n = \pi T \sum_m \frac{1}{(\omega_m^2 + \Delta_m^2)^{1/2}} \times \left[\left(\frac{\Delta_m \omega_n - \Delta_n \omega_m}{\omega_n} \right) \lambda_{m-n}^{(-)} - \left(\frac{\Delta_m \omega_n + \Delta_n \omega_m}{\omega_n} \right) \lambda_{m-n}^{(+)} \right]. \quad (15)$$

Note that there is a cancellation when $\omega_n = \omega_m$, which removes the divergence in the attractive interaction $\lambda_{m-n}^{(-)}$ when $m = n$. This cancellation can be understood as a consequence of Anderson's theorem [11]. The quasistatic ($\omega < T$) gauge fluctuations which are responsible for the destruction of Fermi liquid behavior in the "normal state," drop out of the gap equation because they can be viewed effectively as a random time-reversal invariant potential. Here by time reversal we mean combined time reversal and exchange of the two layers, under which $a^{(-)}$ is, indeed, invariant. $a^{(+)}$, on the other hand, is not time-reversal invariant, and hence is not governed by Anderson's theorem. As a result, there is a finite-temperature divergence at $m = n$. The origin of this divergence can be traced back to the fact that the composite fermion pairs are not "neutral" with respect to the $a^{(+)}$ field. As a result, the pairing equation is not gauge invariant and may contain unphysical divergences. These divergences are not present in gauge-invariant quantities such as the free energy, which was calculated by Ubbens and Lee [12] in a related problem arising in the gauge theory description of the spin gap in the cuprates.

We first ignore the $a^{(+)}$ fluctuations and calculate what T_c would be in their absence. Linearizing (15) and setting $\lambda_{m-n}^{(+)}$ to zero we obtain an equation which, because of the scaling behavior of $\lambda_{m-n}^{(-)}$, allows the dependence on the temperature, ω_0, and γ all to be factored out. The resulting equation is

$$\Delta_n = \gamma \left(\frac{\omega_0}{2\pi T} \right)^{1/3} \sum_m |m - n|^{-1/3} (1 - \delta_{m,n}) \times \left(\frac{\Delta_m}{2m + 1} - \frac{\Delta_n}{2n + 1} \right) \mathrm{sgn}(2m + 1). \quad (16)$$

Unlike conventional BCS theory there is no need for a frequency cutoff in the gap equation. Because the effective interaction falls off as $\omega^{-1/3}$, it is possible to take the Matsubara sum to infinity. The resulting expression for T_c in this limit is

$$T_c \simeq 4.3 \omega_0 \gamma^3 \propto \frac{1}{d^2}, \quad (17)$$

where only the proportionality constant needs to be determined numerically.

We find a similar result for the zero-temperature gap if we continue to neglect $a^{(+)}$. The zero-temperature Eliashberg equation on the imaginary frequency axis can be written as

$$\Delta(i\omega) = \frac{1}{2} \int_{-\infty}^{\infty} d\omega' \frac{1}{(\omega'^2 + \Delta^2)^{1/2}} \times \left(\frac{\omega \Delta(i\omega') - \omega' \Delta(i\omega)}{\omega} \right) \lambda^{(-)}(i\omega - i\omega'). \quad (18)$$

Within the approximation $\Delta(i\omega) = \text{const}$ the equation for $\Delta(0)$ obtained by taking the $\omega \to 0$ limit of the right-hand side of (18) is

$$1 = \frac{\gamma \omega_0^{1/3}}{3} \int_{-\infty}^{\infty} d\omega' \frac{1}{(\omega'^2 + \Delta^2)^{1/2}} \frac{1}{\omega'^{1/3}} \propto \gamma \left(\frac{\omega_0}{\Delta} \right)^{1/3}. \quad (19)$$

It follows that $\Delta(0) \propto \gamma^3 \omega_0$. A fully self-consistent solution of (18) yields

$$\Delta(0) \simeq 8.4 \gamma^3 \omega_0 \propto \frac{1}{d^2}. \quad (20)$$

Thus, in the absence of $a^{(+)}$ fluctuations, the superconducting energy gap at zero temperature falls off as $1/d^2$. We emphasize that the gap $\Delta \sim \gamma^3 \theta(\gamma)$ is not analytic at $\gamma = 0$ and is not a perturbative effect.

At zero temperature, the $a^{(+)}$ fluctuations do not lead to any divergences, so the Eliashberg equations may be solved without special precaution. Again, we consider the approximation $\Delta(i\omega) = \text{const}$. The equation for $\Delta(0)$ is then, in the limit $\gamma \ll 1$,

$$1 = \frac{\gamma \omega_0^{1/3}}{3} \int_{-\infty}^{\infty} d\omega' \frac{1}{(\omega'^2 + \Delta^2)^{1/2}} \frac{1}{\omega'^{1/3}} - \int_{-\Lambda}^{\Lambda} d\omega' \frac{1}{(\omega'^2 + \Delta^2)^{1/2}} \ln \frac{\omega_0}{\omega'} \simeq A\gamma \left(\frac{\omega_0}{\Delta} \right)^{1/3} - B \left(\ln \frac{\omega_0}{\Delta} \right)^2 + \text{less singular terms}. \quad (21)$$

Here A and B are numbers of order 1 and Λ is a high-energy cutoff $\Lambda \sim \omega_0$. The presence of the $a^{(+)}$ fluctuations leads to a substantial suppression of the gap. In the limit $\gamma \ll 1$ we find that

$$\Delta \sim \omega_0 \frac{\gamma^3}{(\ln \gamma)^6} \propto \frac{1}{d^2 (\ln d)^6}. \quad (22)$$

Although the gap is suppressed, the $a^{(+)}$ fluctuations do not eliminate the zero-temperature pairing instability. This is the central result of this paper—an ideal DLCFM, as defined above, is *always* unstable to the formation of a paired state no matter how large the layer spacing is.

We now comment on the solution of the finite-temperature Eliashberg equations including the $a^{(+)}$ fluctuations. As stated above, the problem is that the logarithmic singularity in $\lambda_{m-n}^{(+)}$ is not canceled by

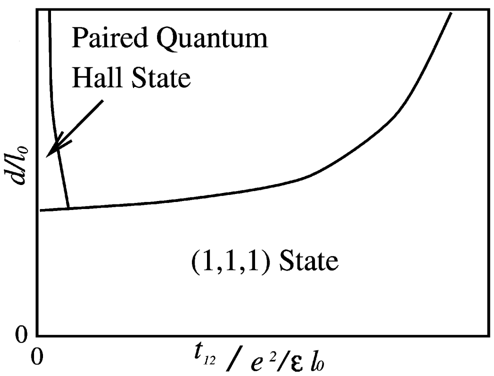

FIG. 1. Proposed phase diagram for a double-layer $\nu = 1$ system including the paired quantum Hall state discussed in this paper. Here t_{12} is the interlayer tunneling amplitude.

Anderson's theorem. However, in the presence of a superconducting gap the $a^{(+)}$ fluctuations also acquire a gap which in turn cuts off the logarithmic divergence in $\lambda^{(+)}$ when $m = n$ so that $\lambda_0^{(+)} \sim \ln(\omega_0/\Delta)$. Therefore, when treated self-consistently, the $a^{(+)}$ fluctuations do not lead to any divergence in the finite-temperature Eliashberg equations. We believe that this treatment is equivalent to the free energy analysis of Ubbens and Lee [12] and, like them, we find that within Eliashberg theory the singular $a^{(+)}$ fluctuations drive the finite-temperature pairing transition to first order. However, we do not expect this result to be physically relevant in our case. Fluctuations about the mean-field Eliashberg treatment presented here will drive the transition temperature to zero because, as in the Chern-Simons Landau-Ginzburg theory of the FQHE [9], the gauge fields screen vortices, rendering their energy finite rather than logarithmically divergent. Thus there is no Kosterlitz-Thouless transition.

While our results should be valid for any ideal DLCFM with total filling fraction $\nu = 1/m$, the case $\nu = 1$ is particularly interesting. For this case a transition from a FQHE state to a compressible state as d/l_0 is increased has been observed experimentally [5]. In the absence of interlayer tunneling the FQHE state at $\nu = 1$ for small d/l_0 is expected to have long-range phase coherence associated with an XY-like order parameter [13]. The observed $T = 0$ incompressible-compressible transition with increasing d/l_0 can then be viewed as an unbinding transition of vortex configuration (merons) of this order parameter [13]. Understanding the relation between the "unbound meron liquid" and the DLCFM state is an open problem deserving further study. One possible scenario is shown in Fig. 1. While the paired DLCFM state has not yet been observed experimentally for $\nu = 1$ this may be due to disorder or unbalancing of the wells.

To conclude, we have shown that in the absence of disorder and interlayer tunneling a perfectly balanced DLCFM is, at the level of the Eliashberg equations, *always* unstable to the formation of a paired state at zero temperature, regardless of how large the layer spacing is. Such a paired state will be incompressible and thus exhibit the FQHE [9]. Motivated by this result we propose the qualitative phase diagram shown in Fig. 1 for the $\nu = 1$ double-layer system. The experimental observation of the paired quantum Hall state discussed in this paper would provide strong evidence for the existence of gauge fluctuations in composite fermion metals.

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