Spontaneous Emission from Tunneling Two-Level Atoms

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We put forward a theory of excitation decay in two-level atoms that tunnel through a square potential barrier while spontaneously emitting photons into an effectively one-dimensional mode continuum. The resulting decoherence can exponentially enhance the total tunneling probability. This enhancement is due to atoms whose final kinetic energy is raised above the barrier by the emission of photons detuned below resonance. [S0031-9007(96)01301-4]

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The vigorous development of the field of atom optics is centered on the interplay between the quantum dynamics of internal and translational atomic degrees of freedom [1]. A fundamental issue regarding this interplay is the loss of atomic wave-packet coherence via spontaneous photon emission, which has been extensively investigated in the context of atomic interferometry and diffraction [2]. By contrast, spontaneous emission in atomic tunneling has been virtually unexplored [3]. Yet, since tunneling is a distinct manifestation of wavelike properties, it is important to raise the basic questions: Can spontaneous decay of internal excitations in tunneling atoms be viewed as a decoherence process that is analogous to its counterpart in diffracted atoms? And if so, how would such decoherence manifest itself?

In this Letter we put forward a theory of spontaneous emission from a two-level atom as it tunnels through a square potential barrier. Our theory demonstrates that the emission process is describable as *loss of coherence between interfering classical trajectories in space-time*, which constitute the atom tunneling motion. The emitted photon at each frequency is correlated to particular atomic classical trajectories, in a way which makes them measurably distinguishable. This distinguishability destroys their interference [4], as does "which-way" ("Welcher-Weg") information, which is obtainable from spontaneous emission in diffracted atoms [2,5]. Several major findings follow from the present theory: (a) This loss of coherence can cause *exponentially large enhancement* of the barrier transmission by spontaneously emitting atoms. This result stands in contrast to WKB predictions of tunneling probability *suppression* by zero-temperature dissipation effects on structureless particles in double-well structures [6]. On the other hand, it bears a certain similarity to predictions of tunneling enhancement due to dissipative mixing among many potential-well levels of such particles [7], or to atomic reflection suppression by damped single-mode resonators [3]. The fundamental link between wave-packet coherence and transmission probability revealed here is akin to our findings for optical wave tunneling through dielectric structures: we have described such tunneling as *destructive* interference of propagating waves, so that decoherence should enhance the transmission [8]. (b) The barrier "filters through" almost entirely ground-state (decayed) atoms, which have emitted photons detuned below resonance.

Our model, which is in essence exactly solvable and experimentally feasible, involves an atom that is incident in the excited state $|e\rangle$ on a square potential barrier, which is the only region where spontaneous emission occurs from $|e\rangle$ to the ground state $|g\rangle$ into an effectively one-dimensional mode continuum. This model is realizable using excited cold atoms incident on an open high-Q cavity which is intersected by a nonresonant laser beam [Fig. 1(a)]. The laser beam creates a nearly square potential barrier by ac Stark shifts [1] of $|e\rangle$ and $|g\rangle$ relative to an upper (unpopulated) state $|u\rangle$, such that $V \approx \frac{1}{8}[\Omega_g^2/\delta_g + \Omega_e^2/\delta_e]$, $\Omega_{g(e)}$ and $\delta_{g(e)}$ being the laser Rabi frequency and detuning for the $|g\rangle \rightarrow |u\rangle$ ($|e\rangle \rightarrow |u\rangle$) transition. The barrier width L (which is comparable to the resonance wavelength c/ω_{eg}) should exceed the de Broglie wavelength of the incident atoms $\lambda_{\mathrm{dB}} = h/\sqrt{2mE_k} \sim h/\sqrt{2mV}$, if tunneling effects are to be appreciable. The cavity serves to strongly enhance the spontaneous emission at the transition from $|e\rangle$ to $|g\rangle$ [9], to the extent that the corresponding spontaneous emission outside the cavity (which coincides with the barrier) is insignificant.

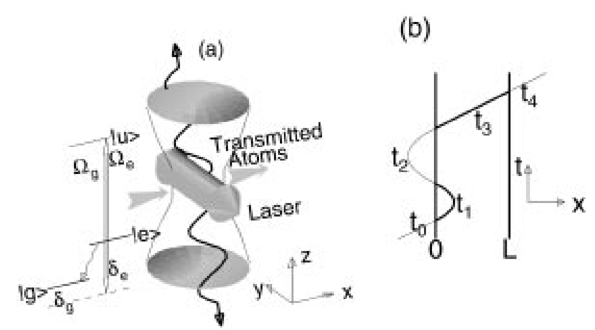

FIG. 1. (a) Two-level atoms tunneling through a laser-induced potential barrier while spontaneously decaying to the ground state by emission of cavity-mode photons. (b) Diagram of a Feynman path $x(t)$, nonclassically criss-crossing the brrier boundaries (within region $L = \lambda_{dB}$).

The ensuing analysis rests on two observations: (i) The overall duration of the decay process is much longer than the inverse transition frequency ω_{eg}^{-1} (see below). This allows us to resort to the rotating wave approximation (RWA), which is used in the Wigner-Weisskopf (WW) treatment of spontaneous emission [10]. (ii) Nearly all of the cavity-enhanced spontaneous emission is funneled into the continuum of nearly resonant modes with wave vectors $\mathbf{q} \approx (\omega/c)\hat{\mathbf{z}}$, which are aligned with the cavity axis z, perpendicular to the atomic incidence axis x. This allows us to use the dipole approximation, since $\mathbf{q} \cdot \mathbf{x} \approx 0$, and neglect off-axis photon recoil effects on the atomic wave packet. Hence, the RWA interaction Hamiltonian of the atom with the cavity-mode continuum becomes effectively one dimensional, $H_{\mathrm{int}} = -\zeta(x) \int d\omega\, \rho(\omega) [g_\omega a_\omega |e\rangle\langle g| + \text{H.c.}]$. Here $\zeta(x) = 1$ for $0 \le x \le L$ and 0 elsewhere; i.e., the interaction is confined to the cavity, whose x-axis extent coincides with that of the barrier; $\rho(\omega)$ is a Lorentzian mode-density distribution associated with the cavity-mode linewidth η [9]; g_ω is the coupling of the atom to the cavity mode at ω; and a_ω is the corresponding annihilation operator. The transition frequency ω_{eg} is shifted (renormalized) by the difference between the ac Stark shifts of $|e\rangle$ and $|g\rangle$, $\Delta_{\mathrm{ac}} = \frac{1}{4}(\Omega_e^2/\delta_e - \Omega_g^2/\delta_g)$.

In order to analyze the entanglement of emitted photon states with the translational degrees of freedom of the tunneling atom, we have developed a theoretical approach which combines the WW treatment [10], resulting in exponential decay of the excited state, with the Feynmann path-integral method, which yields a coherent sum over the atomic classical trajectories contributing to tunneling [11]. This approach is necessitated by the inadequacy of the plane-wave expansion for the translational degrees of freedom of tunneling atoms, which are often described by "imaginary wave vectors." The highly involved analysis of our model, implementing the outlined approach, is tractable using Refs. [12]. Its essential steps are as follows:

(1) Decomposition of each path into $n + 1$ intervals, $t_0, t_1, t_2, \ldots, t_n$, separated by events of crossing the barrier boundaries at $x = 0$ or L, so that at odd-numbered time intervals the atom is inside the barrier [Fig. 1(b)].

(2) Calculation of the Feynmann propagator $K_e(x_t, x_0, t)$ which represents the probability amplitude that an incident atom initially excited at $x = x_0 < 0$ will remain excited at $x_t > L$ after a time t. This propagator is given by the integral $\int D[x(t)] \tilde{K}_e[x(t)] \exp\{iS[x(t)]\}$ over all paths $x(t)$ connecting x_t with x_0, where $S[x(t)]$ is the action along the path and $\tilde{K}_e[x(t)]$ is the amplitude for the atom to remain excited along the path $x(t)$. By our generalized WW ansatz this amplitude has the form

$$\tilde{K}_e[x(t)] = \exp(-\Gamma\tau_b), \quad (1)$$

where $\Gamma = \gamma + i\Delta_L$ is the sum of cavity-enhanced decay rate γ and Lamb shift Δ_L, while $\tau_b[x(t)] = \sum_{j=0}^{n/2-1} t_{2j+1}$ is the total time spent in the barrier (interaction region) by an atom following the path $x(t)$.

(3) Calculation of the probability amplitude $K_\omega(x_t, x_0, t)$ to decay to the ground state and emit a photon in mode ω between x_0 and x_t. This calculation involves a sum over path amplitudes similar to that of step 2 except that $\tilde{K}_\omega[x(t)]$, the amplitude to decay to the ground state and emit a photon of frequency ω along the path $x(t)$, is used instead of $\tilde{K}_e$. This amplitude is given in our generalized WW approach by

$$\tilde{K}_\omega[x(t)] = \frac{g_\omega^*}{\Delta + i\Gamma} \sum_{j=0}^{n/2-1} e^{i\Delta\tau_j} e^{-\Gamma\tau_{b,j}} [e^{(i\Delta-\Gamma)t_{2j+1}} - 1], \quad (2)$$

where $\Delta = \omega - \omega_{eg}$ is the detuning (which accounts for Δ_{ac}), $\tau_j = \sum_{i=0}^{2j} t_i$ is the total time spent by the atom before entering the barrier for the $(j+1)$th passage and $\tau_{b,j} = \sum_{i=0}^{j-1} t_{2i+1}$ is the time spent in the barrier during the first j passages. The sum in Eq. (2) is a result of the integration over the atom probability amplitudes to decay to the ground level during the odd-numbered intervals t_{2j+1}, when the atom is inside the barrier. The two terms with opposite signs in square brackets correspond to the upper and lower limits of integration over one such interval.

(4) Integration of $K_e(x_t, x_0, t)$ (introduced in step 2) over all paths. This integration is performed *exactly*, using Eq. (1) and the path decomposition method [12]. It is followed by the calculation of the Green function $G_e(E_k)$, which is the Laplace transform of the propagator $K_e(t)$, yielding

$$G_e(x_t, x_0, E_k) = G_0(x_t, x_0, E_k) e^{-ikL} \sigma(E_k, V - i\hbar\Gamma). \quad (3)$$

Here $G_0(E_k)$ is the Green function for free propagation and $\sigma(E_k, V)$ is the transmission amplitude for a structureless particle of kinetic energy E_k through a square potential barrier of height V and length L,

$$\sigma(E_k, V) = \left[\cos pL - i\,\frac{k^2 + p^2}{2kp}\,\sin pL\right]^{-1}, \quad (4)$$

$k = \sqrt{2mE_k}/\hbar$ and $p = \sqrt{2m(E_k - V)}/\hbar$ being the corresponding wave vectors outside and inside the barrier, respectively. The effect of spontaneous emission is to shift the effective potential V by $-i\hbar\Gamma$ (see below).

(5) Calculation of the Laplace transform of $K_\omega(x_t, x_0, t)$, which is somewhat more complicated than the former, but can also be performed exactly. This yields the following Green's function for an atom incident on the barrier with momentum $\hbar k = \sqrt{2mE_k}$ and exiting the barrier after having emitted a photon of frequency $\omega = \omega_{eg} + \Delta$, with momentum $\hbar\kappa_\omega = \sqrt{2m(E_k - \hbar\Delta)}$

$$G_\omega(E_k - \hbar\Delta) = \frac{g_\omega^*}{\Delta + i\Gamma} \frac{m}{ik} e^{i[-kx_0 + \kappa_\omega(x_t - L)]} \sigma_\omega(E_k, V). \quad (5)$$

This Green's function is proportional to the atomic transmission function associated with ω-frequency emission, which is found to have the cumbersome form

$$\sigma_\omega(E_k, V) = \frac{k}{\kappa_\omega}\,\mu(\kappa_\omega/k, E_k, V - i\hbar\Gamma)\sigma(E_k - \hbar\Delta, V) - \mu(k/\kappa_\omega, E_k - \hbar\Delta, V)\sigma(E_k, V - i\hbar\Gamma), \tag{6}$$

where

$$\mu(\lambda, E, V) = 1 - \frac{1}{2}(1-\lambda) \times \left[1 - \frac{mV}{\hbar k}\frac{\sin pL}{\hbar p}\,\sigma(E, V)\right],$$

with λ standing for k/κ_ω or κ_ω/k.

The above analysis allows us to write the complete solution for the *entangled* state $|\psi\rangle_{\mathrm{AF}}^{\mathrm{tr}}$ associated with the transmitted atom at $x > L$ and the cavity field, taking the state of the excited incident plane-wave atom with kinetic energy E_k to be $(e^{ikx}/\sqrt{2k})|e\rangle$ and the cavity field to be initially in the vacuum state $|0\rangle$

$$|\psi\rangle_{\mathrm{AF}}^{\mathrm{tr}} = \sigma(E_k, V - i\hbar\Gamma)\,(e^{ik(x-L)}/\sqrt{2k})\,|e, 0\rangle + \int \frac{d\omega}{\sqrt{2\omega}}\frac{g_\omega^*}{\Delta + i\Gamma}\sqrt{\frac{\kappa_\omega}{k}}\,\sigma_\omega(E_k, V) \times \frac{e^{i\kappa_\omega(x-L)}}{\sqrt{2\kappa_\omega}}\,|g, \omega\rangle, \tag{7}$$

where the states $|g, \omega\rangle$, corresponding to an atom in state $|g\rangle$ and an emitted photon with frequency ω, are normalized such that $\langle g, \omega | g, \omega' \rangle = 2\omega\rho(\omega)\delta(\omega - \omega')$.

This solution yields the probability for an atom incident as a nearly monochromatic wave packet to be transmitted in the excited state

$$P_e^{\mathrm{tr}} = |\sigma(E_k, V - i\hbar\Gamma)|^2. \tag{8}$$

Plots of Eq. (8) (Fig. 3) reveal the overall diminishing of P_e^{tr} with γ in both the tunneling (below-barrier) and allowed (above-barrier) regimes of E_k. Also seen in Fig. 3 is the progressive suppression with γ of interference between multiply reflected excited-atom waves, resulting in smoothing out of P_e^{tr} oscillations as a function of E_k.

The corresponding probability P_g^{tr} of the transmitted ground-state wave packet is an incoherent sum (integral) of partial wave-packet transmission probabilities P_ω associated with photon emission at ω

$$P_g^{\mathrm{tr}} = \int_0^{E_k} d\omega\, P_\omega,$$

$$P_\omega = \mathcal{F}(\omega)\sqrt{1 - \frac{\hbar\Delta}{E_k}}\,|\sigma_\omega(E_k, V)|^2, \tag{9}$$

where $\mathcal{F}(\omega) = \rho(\omega)|g_\omega|^2/(\Delta^2 + \gamma^2)$. The most salient effect of spontaneous emission is seen to be (Figs. 2 and 3) the huge enhancement of P_g^{tr} as a function of γ for atoms initially in the deep tunneling regime $pL = \sqrt{2m(V - E_k)}\,L/\hbar > 1$.

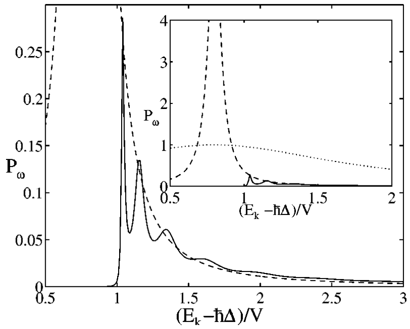

FIG. 2. The energy spectrum of transmitted ground-state atoms: Solid curve: transmission probability P_ω [Eq. (9)] (in units of $\hbar/V$) for $E_k/V = 0.8$, $L = 2.5\lambda_{\mathrm{dB}}(E_k/V = 1)$, $\gamma = 0.05V/\hbar$, $\omega_{eg} = 100V/\hbar$ as a function of kinetic energy following emission. Dashed curve: spontaneous line shape. Inset: Idem, on a small scale. Dotted curve: cavity line shape.

In order to gain more insight into the above general results, we shall henceforth assume that the cavity linewidth η and E_k satisfy the following inequalities:

$$|E_k - V| \ll \hbar\eta < E_k < \hbar\omega_{eg}, \qquad \gamma \ll \eta\,. \tag{10}$$

The spectrum of spontaneous emission is then limited to $|\Delta| \ll E_k$ and becomes Lorentzian in this range, $\mathcal{F}(\omega) \approx \mathcal{L}_\gamma(\Delta)$, since the spectral variation of $\rho(\omega)$ and $|g_\omega|^2$ is slow, $\rho(\omega)|g_\omega|^2 \approx 2\pi\gamma$, in accordance with the WW approximation. Equation (6) can now be simplified, since Eq. (10) implies that $\kappa_\omega/k \approx 1$ and $\mu \approx 1$, yielding

$$\sigma_\omega(E_k, V) \approx \sigma(E_k - \hbar\Delta, V) - \sigma(E_k, V - i\hbar\Gamma). \tag{11}$$

It is seen from Eqs. (9) and (11) that the dramatic enhancement effects in the tunneling regime are due to the first term in (11), corresponding to atoms that have decayed to the ground state shortly after entering the barrier and are subsequently transmitted through the barrier as unexcited atoms with kinetic energy $E_k - \hbar\Delta$, *which can be above the barrier* if $\Delta < 0$. By

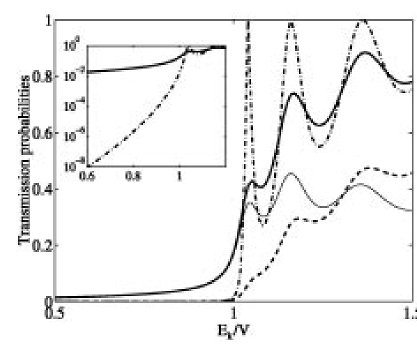

FIG. 3. Transmission probabilities for $\gamma = 0.025V/\hbar$, $\omega_{eg} = 100V/\hbar$, and $L = 2.5\lambda_{\mathrm{dB}}(E_k = V)$ as a function of initial kinetic energy: [Eqs. (8)–(12)]. Dashed curve: P_e^{tr}. Thin solid curve: P_g^{tr}. Thick solid curve: $P_{\mathrm{tot}}^{\mathrm{tr}}$. Dash-dotted curve: corresponding transmission probability of a structureless particle. Inset: The last probability and $P_{\mathrm{tot}}^{\mathrm{tr}}$ on a logarithmic scale in the tunneling regime.

contrast, the second term in (11) corresponds to atoms that have decayed shortly before exiting the barrier after having effectively been transmitted as excited atoms with the *initial* kinetic energy E_k, whence this term is exponentially small in the tunneling regime. The use of Eq. (11) in Eq. (9) therefore leads to the enhancement of P_ω (Fig. 2) and $P_g^{\rm tr}$ (Fig. 3) due to the possibility to gain kinetic energy from the broad vacuum field reservoir by emitting a photon detuned below the resonance $\hbar\omega_{eg}$. In the deep tunneling regime, assuming that $\gamma \ll (V - E_k)$, Eqs. (9)–(11) allow us to roughly estimate that the atoms have probability of order

$$P_g^{\rm tr} \sim \int_V^{E_k + \hbar\omega_{eg}} dE\, \mathcal{L}_\gamma[(E - E_k)/\hbar] \approx \frac{\gamma}{V - E_k} \tag{12}$$

to jump over the barrier into the allowed energy regime by emitting a photon with $\Delta < E_k - V < 0$ (Fig. 2).

Under the assumptions leading to Eq. (11), along with the approximation $\sqrt{1 - \hbar\Delta/E_k} \approx 1$, we can obtain a simplified expression for the total transmission probability $P_{\rm tot}^{\rm tr} = P_g^{\rm tr} + P_e^{\rm tr}$ by extending the integration over P_ω in Eq. (9) to $E_k = \infty$. This yields

$$\begin{aligned} P_{\rm tot}^{\rm tr}(E_k, V) &\approx \int_{-\infty}^{\infty} d\Delta\, \mathcal{L}_\gamma(\Delta) |\sigma(E_k - \hbar\Delta, V)|^2 \\ &= \int_0^\infty d\tau \int_0^\infty d\tau'\, e^{-\gamma|\tau - \tau'|} \\ &\quad \times \hat{\sigma}^*(\tau, V)\hat{\sigma}(\tau', V), \end{aligned} \tag{13}$$

where $\hat{\sigma}(t, V)$, the Fourier transform of $\sigma(E, V)$, is the impulse response (to a temporal δ function) for transmission of a structureless particle, and the simple relation $e^{-\gamma|\tau|} = \int d\Delta e^{-i\Delta\tau} \mathcal{L}_\gamma(\Delta)$ has been used. We thus obtain the following important result: the total transmission probability $P_{\rm tot}^{\rm tr}$ coincides, in the limit of narrow spontaneous linewidth γ [Eq. (10)], with the transmission probability of a partially incoherent wave packet of a structureless particle with coherence time γ^{-1} (see Ref. [8]).

The following conclusions can be inferred from the above analysis: (a) The probability distribution of the transmitted atoms is approximately Lorentzian for final kinetic energies $E_k - \hbar\Delta$ above the barrier, whereas their counterparts below the barrier only contribute an exponentially small tail to this distribution. (b) The fact that fast atoms emerging from the barrier are almost always unexcited means that the barrier acts as a "filter" that transmits almost only atoms that have already decayed.

The results of this paper open a new vista into the transition from quantum dynamics to classicality via decoherence by focusing on the effects of excitation decay on atomic tunneling. In the limit of negligible decay $\gamma \to 0$, which is realizable by detuning the cavity off resonance with ω_{eg}, the excited atomic wave packet with $E_k < V$ exhibits tunneling, which is a result of interference between many classical trajectories, and is characterized by exponentially low transmission $P_e^{\rm tr}$ [Eq. (8)]. When γ is appreciable, the wave packet is dominated by the portion that has decohered by decay into the field-mode continuum and has thereby lost its tunneling properties: its energy spread becomes classical (statistical), giving rise to a Lorentzian tail into the above-barrier energy range, thereby allowing for enhancement of the transmission [Eqs. (9) and (13)]. The effects of this decoherence on barrier traversal times will be discussed elsewhere.

The results predicted here can be experimentally realized by a variety of cold atoms. In accord with Eq. (10), the lifetime of the $|e\rangle \to |g\rangle$ transition should preferably be long, above 10^{-6} sec. A confocal cavity whose finesse is $\sim 10^5$ and subtends a solid angle of ~ 0.1 sr can enhance spontaneous emission rate γ by a factor of ~ 30. The cavity linewidth η should be much larger than γ, i.e., preferably above 10 MHz. Correspondingly, the potential energy V and the kinetic energy E_k must be above 0.1 GHz, which requires the laser Rabi frequency $\Omega_{e(g)}$ and detuning $\delta_{e,(g)}$ to be well within the GHz range. This implies that the transition frequency ω_{eg} can lie anywhere between the GHz and the optical ranges.

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