Off-Shell Supersymmetry versus Hermiticity in Superstrings

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We point out that off-shell four-dimensional spacetime supersymmetry implies strange Hermiticity properties for the $N = 1$ Ramond-Neveu-Schwarz superstring. However, these Hermiticity properties become natural when the $N = 1$ superstring is embedded into an $N = 2$ superstring. [S0031-9007(96)01339-7]

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Superstring theory is the only known renormalizable quantum theory of gravity, and spacetime supersymmetry plays a crucial role in removing the divergences. Spacetime supersymmetry also plays a crucial role in the recent conjectures [1] of strong-weak duality in superstring theory. However, spacetime supersymmetry is only an on-shell symmetry in the conventional $N = 1$ Ramond-Neveu-Schwarz (RNS) description of the superstring, which makes it difficult to observe its effects.

It will be shown in this Letter that after modifying the conventional Hermiticity properties, four-dimensional supersymmetry can be extended to an off-shell symmetry of the superstring. The modified Hermiticity properties are those of an $N = 2$ embedding of the $N = 1$ RNS superstring [2], where $N = 1$ and $N = 2$ refer to the number of worldsheet superconformal invariances. This suggests that the $N = 2$ description of the superstring [3] may be useful for illuminating the effects of spacetime supersymmetry. It is interesting to note that $N = 2$ superstrings have recently appeared in models [4] which try to explain the strong-weak duality symmetries.

In four-dimensional compactifications of the $N = 1$ RNS superstring, the spacetime-supersymmetry generators in the $-\frac{1}{2}$ picture are

$$q_a = \frac{1}{2\pi i} \oint dz\, e^{(1/2)(-\phi \pm i\sigma_0 \pm i\sigma_1 \pm iH_C)}, \tag{1}$$

where there are an even number of $+$ signs in the exponential ($a = 1$ to 4), ϕ comes from fermionizing the bosonic ghosts as $\beta = i\partial\xi e^{-\phi}$ and $\gamma = -i\eta e^{\phi}, \psi^3 = \pm\psi^0 = e^{\pm i\sigma_0}$ and $\psi^1 \pm i\psi^2 = e^{\pm i\sigma_1}$ where ψ^m is the fermionic vector, and $\partial H_C = J_C$ is the U(1) generator of the $c = 9, N = 2$ superconformal field theory representing the compactification manifold.

These spacetime-supersymmetry generators satisfy the anticommutation relations

$$\{q_a, q_b\} = \frac{1}{2\pi i} \oint dz\, e^{-\phi} \psi_m \gamma^m_{ab}\,, \tag{2}$$

which is not the usual supersymmetry algebra $\{q_a, q_b\} = \frac{1}{2\pi} \oint dz\, \partial x_m \gamma^m_{ab}$ where $\frac{1}{2\pi} \oint dz\, \partial x_m$ is the string momentum. However, after hitting the right-hand side of (2) with the picture-changing operator $Z = \{Q, \xi\}$, it becomes $\frac{1}{2\pi i} \oint dz\, Z e^{-\phi} \psi_m \gamma^m_{ab} = \frac{1}{2\pi} \oint dz\, \partial x_m \gamma^m_{ab}$. So up to picture changing, the q_a's form a supersymmetry algebra [5].

But off-shell supersymmetry requires that the q_a's form a supersymmetry algebra without applying picture-changing operations. This is because picture changing is only well defined when the states are on-shell. Off-shell, the states are not independent of the locations of the picture-changing operators.

So off-shell spacetime supersymmetry requires modification of the q_a's. Note that q_a has picture $-\frac{1}{2}$ and the momentum $\frac{1}{2\pi} \oint dz\, \partial x_m$ has picture 0, so we need generators with picture $+\frac{1}{2}$. The obvious solution [3] is to split q_a into a chiral part with picture $-\frac{1}{2}$ and an antichiral part with picture $+\frac{1}{2}$:

$$\begin{aligned} q_\alpha &= \frac{1}{2\pi i} \oint dz\, e^{(1/2)[-\phi \pm i(\sigma_0 + \sigma_1) + iH_C]}, \\ \overline{q}_{\dot\alpha} &= Z q_{\dot\alpha} = \frac{1}{2\pi i} \oint dz\, [b\eta e^{(1/2)[3\phi \pm i(\sigma_0 - \sigma_1) - iH_C]} \\ &\quad + i{:}(e^{\phi}\psi_m \partial x^m + e^{\phi} G^+_C + e^{\phi} G^-_C) \\ &\quad \times e^{(1/2)(-\phi \pm i(\sigma_0 - \sigma_1) - iH_C}{:}], \end{aligned} \tag{3}$$

where $G^{\pm}_C$ are the fermionic generators of the $c = 9$ $N = 2$ superconformal field theory. The $N = 1$ 4D supersymmetry algebra $\{q_\alpha, \bar q_{\dot\beta}\} = \frac{1}{2\pi} \oint dz\, \partial x_m \sigma^m_{\alpha\dot\beta}$ is now satisfied off-shell where we are using standard two-component Weyl notation.

Although we have solved the problem of finding off-shell supersymmetry generators, we now have a new problem. Using the standard RNS definition of Hermiticity where all fundamental fields are Hermitian or anti-Hermitian (the anti-Hermitian field is σ_0), the Hermitian conjugate of q_α is no longer $\overline{q}_{\dot\alpha}$. Fortunately, this new problem can be solved by modifying the definition of Hermiticity. However, this new Hermiticity definition will be natural only if one embeds the $N = 1$ superstring into an $N = 2$ superstring.

To find the appropriate Hermiticity definition, one first writes $\overline{q}_{\dot\alpha}$ in the form

$$\overline{q}_{\dot\alpha} = e^{R}\left(\frac{1}{2\pi i} \oint dz\, b\eta e^{(1/2)[3\phi \pm i(\sigma_0 - \sigma_1) - iH_C]}\right) e^{-R},$$

where

$$R = \frac{1}{2\pi} \oint dz\, c\xi e^{-\phi}(\psi^m \partial x_m + e^{\phi} G_C^+ + G_C^-) \tag{4}$$

and $e^R F e^{-R} = F + [R, F] + \frac{1}{2}[R, [R, F]] + \cdots$ (the expansion usually stops after two terms).

One then defines Hermiticity as

$$\begin{aligned} (x_m)^\dagger &= e^R x_m e^{-R}, \qquad (\psi_m)^\dagger = e^R \psi_m e^{-R}, \qquad (F_C)^\dagger = e^R \overline{F}_C e^{-R}, \\ (e^{\phi/2})^\dagger &= e^R (c\xi e^{-(3/2)\phi}) e^{-R}, \qquad (e^{-\phi/2})^\dagger = e^R (b\eta e^{(3/2)\phi}) e^{-R}, \\ (b)^\dagger &= e^R (ib\eta\partial\eta e^{2\phi}) e^{-R}, \qquad (c)^\dagger = e^R (-ic\xi\partial\xi e^{-2\phi}) e^{-R}, \\ (\eta)^\dagger &= e^R (i\eta b\partial b e^{2\phi}) e^{-R}, \qquad (\xi)^\dagger = e^R (-i\xi c\partial c e^{-2\phi}) e^{-R}, \end{aligned} \tag{5}$$

where F_C are the worldsheet fields in the $c = 9, N = 2$ superconformal field theory. It is straightforward to check that the new Hermiticity definition satisfies $(F^\dagger)^\dagger = F$ for all F, preserves OPE's, and implies that $(q_\alpha)^\dagger = \overline{q}_{\dot\alpha}$.

One strange feature of the Hermiticity definition of (5) is that a field may have a different conformal weight from its Hermitian conjugate since $(T)^\dagger = T + i\partial(bc + \xi\eta)$ where T is the RNS Virasoro generator. Another strange feature is that the BRST operator is not Hermitian since $Q^\dagger = \frac{1}{2\pi}\oint dz\, b$. [This easily follows from writing $Q = e^R(\frac{i}{2\pi} \oint dz\, b\eta\partial\eta e^{2\phi})e^{-R}$.]

Although these features are strange in the $N = 1$ RNS description of the superstring, they are natural if the $N = 1$ superstring is embedded into an $N = 2$ superstring. As discussed in Ref. [2], any critical $N = 1$ superstring can be embedded into a critical $N = 2$ superstring where the $c = 6, N = 2$ superconformal generators are [2]

$$\begin{gathered} T_{N=2} = T_{N=1} + \frac{i}{2}\partial(bc + \xi\eta), \\ G^+_{N=2} = j_{\mathrm{BRST}}, \quad G^-_{N=2} = b, \quad J_{N=2} = bc + \xi\eta, \end{gathered} \tag{6}$$

and $j_{\mathrm{BRST}} = e^R(ib\eta\partial\eta e^{2\phi})e^{-R}$.

Using the Hermiticity definition of (5), $(T_{N=2})^\dagger = T_{N=2}$, $(G^+_{N=2})^\dagger = G^-_{N=2}$, and $(J_{N=2})^\dagger = J_{N=2}$, which are the standard Hermiticity properties of an $N = 2$ string. [This Hermiticity can be made manifest by writing the $N = 2$ generators of (6) in terms of spacetime-supersymmetric variables [3].] So the Hermiticity properties implied by off-shell four-dimensional supersymmetry are natural only if the $N = 1$ superstring is embedded into an $N = 2$ superstring.

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