Is There a Hot Electroweak Phase Transition at $m_H \gtrsim m_W$?

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We provide nonperturbative evidence for the fact that there is no hot first or second order electroweak phase transition at large Higgs masses, $m_H = 95$, 120, and 180 GeV. This means that the line of first order phase transitions separating the symmetric and broken phases at small m_H has an end point $m_{H,c}$. In the minimal standard electroweak theory $70 < m_{H,c} < 95$ GeV and most likely $m_{H,c} \approx 80$ GeV. If the electroweak theory is weakly coupled and the Higgs boson is found to be heavier than the critical value (which depends on the theory in question), cosmological remnants from the electroweak epoch are improbable. [S0031-9007(96)01335-X]

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The transition between the high temperature symmetric (or confinement) phase and the low T broken (or Higgs) phase in the standard electroweak theory (MSM) or its extensions is known to be of first order for small values of the Higgs mass m_H. This follows from perturbative studies of the effective potential [1] and nonperturbative lattice Monte Carlo simulations [2–4]. In the region of applicability of the perturbative expansion the strength of the electroweak phase transition decreases when m_H increases. However, the nature of the electroweak phase transition at "large" Higgs masses $m_H \gtrsim m_W$ remains unclear, since the perturbative expansion for the description of the phase transition is useless there. This Letter contains the results of the first nonperturbative lattice analysis of the problem for "large" Higgs masses, $m_H = 95$, 120, 180 GeV. We shall show that the system behaves very regularly there, much like water above the critical point. As there is no distinction between liquid water and vapor, there is no distinction between the symmetric and broken phases; there is no long-range order.

In Ref. [3] it has been shown that in a weakly coupled electroweak theory and in many of its extensions (supersymmetric or not) the hot electroweak (EW) phase transition can be described by an SU(2) $\times$ U(1) + Higgs model in three Euclidean dimensions. Dimensional reduction has its own limitations, described in detail in [3]. For example, for the MSM the 3D approximation is accurate to within a few percent for $30 \lesssim m_H \lesssim 250$ GeV. At the lower end of this inequality the high temperature expansion breaks down because the phase transition is very strongly first order and particle masses in the broken phase are $\sim T$ [5]. The upper end is the usual condition for the applicability of perturbation theory in the scalar sector of the MSM. In the minimal sypersymmetric standard model (MSSM) the latter condition is satisfied automatically. Hence, the 3D description is valid for a wide range of the phenomenologically interesting part of the parameter space of the MSM and MSSM.

Since the effects of the U(1) subgroup are perturbative deep in the Higgs phase and high in the symmetric phase, the presence of the U(1) factor cannot change the qualitative features of the phase diagram. Thus we shall take $\sin\theta_W = 0$. The effective Lagrangian is

$$L = \tfrac{1}{4}G_{ij}^a G_{ij}^a + (D_i\phi)^\dagger(D_i\phi) + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2, \quad (1)$$

where G_{ij}^a is the SU(2) field strength, ϕ is the scalar doublet, and D_i is the covariant derivative. The three parameters of the 3D theory (gauge coupling g_3^2, scalar self-coupling λ_3, and the scalar mass m_3^2) depend on temperature and on the underlying 4D parameters and can be computed perturbatively; the explicit relations for the MSM are worked out in [3] and for the MSSM in [6].

The phase structure of the theory (1) depends on one dimensionless ratio, $x = \lambda_3/g_3^2$. Indeed, the dimensionful coupling g_3^2 can be chosen to fix the scale, while the change of the second dimensionless ratio $y = m_3^2(g_3^2)/g_3^4$ corresponds to temperature variation. For $y \gg 1$ (large T) the system is in the strongly coupled symmetric phase, while at $y \ll -1$ (low T) the system is in the weakly coupled Higgs phase. Instead of x and y, we use a more physical set of variables m_H^* and T^* in presenting our results below. The parameter m_H^* is the tree-level Higgs mass in the 4D SU(2) + Higgs theory, and T^* is the temperature there. The explicit relationship between (x,y) and (m_H^*, T^*) is given by $x = -0.005\,50 + 0.126\,22h^2$, $y = 0.398\,18 + 0.155\,45h^2 - 0.001\,90h^4 - 2.580\,88(m_H^*/T^*)^2$, where $h \equiv m_H^*/80.6$ GeV. For large Higgs masses, m_H^* is close to the physical pole mass m_H in MSM [3].

An essential point in understanding the phase structure of the theory is the fact that the 3D SU(2) + Higgs theory (as well as the underlying electroweak theory) does not have a true gauge-invariant order parameter which can distinguish the high temperature symmetric phase and the low temperature Higgs phase [7]. There is no breaking or restoration of the gauge symmetry across the phase transition, just because physical observables are always gauge invariant.

In non-Abelian gauge Higgs theories on lattice, with matter in the fundamental representation and a fixed length of the scalar field, the Higgs (weakly coupled) and symmetric (strongly coupled) phases are analytically connected [7]; this was already seen in early lattice simulations [8]. This suggests the phase diagram on the (x, y) (Higgs mass, temperature) plane shown in Fig. 1. The knowledge of the phase diagram and the value of x_c is essential for cosmological applications. If $x_c = \infty$, the electroweak phase transition did occur in the early Universe at the electroweak scale independent of the parameters of the electroweak theory. This means that substantial deviations from thermal equilibrium took place at this scale, which might leave some observable remnants such as the baryon asymmetry of the Universe (for a review see [9] and references therein). In the opposite case of a finite x_c the EW phase transition never took place for a region of parameters of the underlying theory; any remnants from the electroweak epoch would then be unlikely.

There were up to now no solid results on the phase structure of the continuum 3D (and, therefore, high temperature 4D) SU(2) + Higgs theory. Various arguments in favor of and against finite x_c are listed below.

(1) $x_c = \infty$? The limit $x \longrightarrow \infty$ corresponds formally to $g_3^2 = 0$, i.e., to the pure scalar model with SU(2) global symmetry. The latter is known to have a second order phase transition, suggesting that $x_c = \infty$ in the SU(2) + Higgs theory.

The ϵ expansion also predicts a first order phase transition for any finite value of x, suggesting that $x_c = \infty$ [10]. However, it relies on the hope that $\epsilon = 1$ is small and, therefore, is not conclusive.

(2) $x_c = finite$? The absence of a true order parameter for the gauge Higgs system is certainly consistent with a finite x_c. Moreover, because there is no symmetry breaking, the existence of a line of second order phase transitions starting at x_c is very unlikely. However, the proof that the Higgs and symmetric phases are analytically connected [7] refers to a lattice system with a finite cutoff and is not applicable to the continuum system we are interested in.

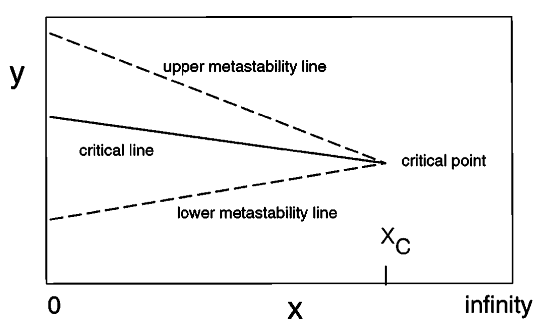

FIG. 1. A schematical phase diagram for the SU(2) + Higgs theory. Solid line is the phase transition and dashed lines indicate the metastability region.

A study of 1-loop gap equations for the SU(2) + Higgs theory argues in favor of a finite value of x_c [11]. However, the analysis relies on the applicability of perturbation theory near the phase transition point. This is known to break down at $m_H \sim m_W$.

In this Letter we present strong nonperturbative evidence for the fact that the line of first order phase transitions has a critical end point at a finite value of x, $0.09 < x_c < 0.17$, and most likely $x_c \approx \frac{1}{8}$. In terms of the physical Higgs mass in the MSM this means that the phase transition ends between $m_H = 70$ and 95 GeV, probably near $m_H = 80$ GeV.

When $m_H^* \leq 70$ GeV, both 3D and 4D [2,3] simulations have established the first order nature of the transition. The transition becomes weaker with increasing m_H^*, and at $m_H* \sim 80$ GeV the simulations have not been able to fully resolve the order of the transition [3,4]. Distinguishing a weak first order transition from a second order one is a very difficult task, often requiring prohibitively large lattice volumes.

To answer the question whether the transition ends near 80 GeV or continues as a weak transition we perform simulations at $m_H^* > 80$ GeV. If the transition is *absent* in this region, the system behaves completely regularly, which is relatively simple to resolve with lattice Monte Carlo methods.

In the present analysis we use previously published results at $m_H^* = 35-80$ GeV [3], and add new simulations at $m_H^* = 95$, 120, and 180 GeV. For $m_H^* = 120$ and 180 GeV we use two lattice spacings a corresponding to gauge couplings $\beta_G \equiv 4/(g_3^2 a) = 8$ and 12. For 120 GeV we use 6 volumes 12^3–64^3 and for 180 GeV 5 volumes 12^3–40^3 for both values of β_G. For $m_H^* =$ 95 GeV we have 6 volumes up to 56^3 at $\beta_G = 8$. The Monte Carlo program uses an optimized combination of heat bath and special overrelaxation updates [3]. The simulations were mainly performed on a 4 processor Cray C90. The number of new "runs"—combinations of lattice sizes and coupling constants—performed for this analysis is 251 (in addition to the 190 old runs at $m_H^* \leq 80$ GeV), with a total CPU time of ~1100 Cray CPU hours.

Among the many widely used tests of the order of the transition we shall use here (I) the finite-size scaling analysis of the order parameter $\phi^\dagger\phi$ susceptibility and (II) the analysis of the correlation lengths. We define the dimensionless $\phi^\dagger\phi$ susceptibility

$$\chi = g_3^2 V \langle(\phi^\dagger\phi - \langle\phi^\dagger\phi\rangle)^2\rangle, \qquad V(g_3^2)^3 = (4N/\beta_G)^3, \tag{2}$$

and measure it as a function of T^*. For each volume V we find the provisional "transition temperature" T_t^* where χ attains its maximum value $\chi_{\max}$. According to the standard finite-size scaling analysis [12], there are now 3 distinct possibilities: (a) In a first order phase transition the order parameter $\langle\phi^\dagger\phi\rangle$ has a discontinuous jump Δ_ϕ, and $\chi_{\max} \propto V\Delta_\phi^2$. (b) In a second order transition χ displays critical behavior, and $\chi_{\max} \propto V^\gamma$, where γ is a critical exponent. It is not excluded that $\gamma = 0$ for some choices of the order parameter. (c) If there is no transition, χ is regular and remains finite when $V \to \infty$ (on a system with periodic boundary conditions).

First we locate $\chi_{\max}$ approximately by investigating a wide range of temperatures with small lattices, and perform a series of simulations around the maximum with progressively larger lattices. Figure 2 shows $\chi(T^*)$ measured from a series of lattice sizes for $m_H^* = 60$ and 120 GeV, $\beta_G = 8$. In the $m_H^* = 60$ GeV case we observe [3] that the quantity $\chi_{\max}/V$ approaches a constant when $V \to \infty$ and the width of the peak of $\chi(T^*)/V$ decreases. This is a clear signal of a first order transition (asymmetric lattices were used for interface tension calculations).

The situation is markedly different when $m_H^* =$ 120 GeV: now the value of $\chi_{\max}$ grows very slowly when V increases and, within the statistical accuracy, approaches a constant. The $\chi(T^*)$ data still display an unambiguous peak at $T_t^* \sim 213$ GeV, signaling that the provisional transition has turned into a sharp—but regular—crossover. When $T^* > T_t^*$ the order parameter $\langle \phi^\dagger \phi \rangle$ remains small, and when T^* is decreased below T_t^*, $\langle \phi^\dagger \phi \rangle$ starts to increase rapidly.

The continuous lines with the error bands in Fig. 2 have been obtained with the Ferrenberg-Swendsen multihistogram technique. This allows us to combine several runs around the peak together; as an example, Fig. 2 shows the individual simulation points for 48^3, 64^3. The error analysis is done with the jackknife method.

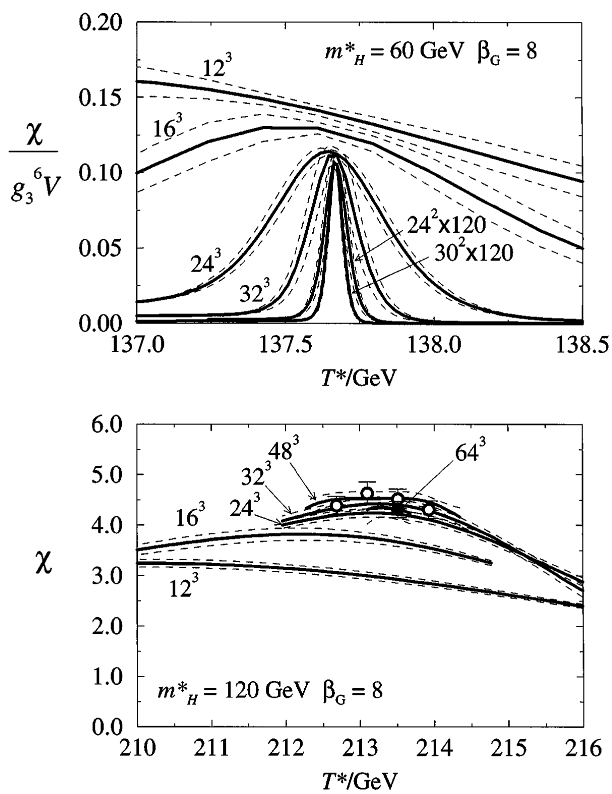

FIG. 2. The $\phi^\dagger\phi$ susceptibility χ at $m_H^* = 60$ and 120 GeV plotted as a function of T^* around the maximum for lattices of various sizes. Note that the 60 GeV plot shows χ divided by the volume. The continuous lines with error bands result from multihistogram reweighting; the individual Monte Carlo simulation points are shown for 48^3 and 60^3 lattices. The maximum values $\chi_{\max}$ are plotted in Fig. 3.

The $m_H^* = 120$ GeV, $\beta_G = 12$ case behaves quite similarly to the $\beta_G = 8$ data shown here, as do the $m_H^* =$ 95 and 180 GeV cases. The maximum values $\chi_{\max}$ for different m_H^* are shown as a function of V in Fig. 3. For $m_H^* = 35$, 60, and 70 GeV we use 3 different lattice spacings ($\beta_G = 8$, 12, 20); no systematic finite lattice spacing effect can be observed (the scatter in $m_H^* =$ 60 GeV is due to the large variation in lattice geometries: some volumes are long cylinders, some cubes).

The pattern of Fig. 3 very clearly suggests that the behavior of the system changes around $m_H^* = 80$ GeV from a first order transition to no transition or a second order phase transition with small or zero γ. At the critical point, we cannot yet discern the true value of γ; as an example we plot the mean field value $2/3$ in Fig. 3. A second order transition at $m_H^* > 80$ GeV can be ruled out by the study of the correlation lengths of the gauge-invariant composite operators, describing scalar (π) and vector (V) excitations, $\pi = \phi^\dagger \phi$, $V_j = i\phi^\dagger \overleftrightarrow{D}_j \phi$.

If the transition is of second order, the jump of the order parameter $\langle \phi^\dagger \phi \rangle$ vanishes together with the mass of the scalar excitation. At the same time, the vector correlation length may remain finite at the transition point, making the resolution of the nature of the transition to be numerically very difficult because of the hierarchy of the masses. A signature of this situation is a drastic increase of the scalar correlation length at some $y(x)$.

If, on the contrary, there is no transition at $x > x_c$, then all the correlation lengths of the system are finite, and

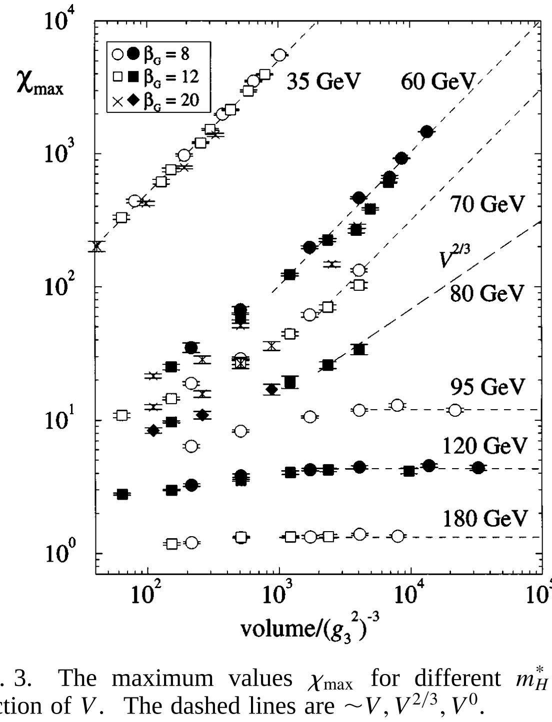

FIG. 3. The maximum values $\chi_{\max}$ for different m_H^* as a function of V. The dashed lines are $\sim V$, $V^{2/3}$, V^0.

expectation values of different gauge-invariant operators are continuous functions of y. After some minimum size, finite volume effects become negligible. In this case a reliable lattice Monte Carlo analysis, which is hardly possible to carry out near x_c, becomes comparatively quite simple.

In Ref. [3] we carried out the correlation length analysis for $m_H^* = 60$ and 80 GeV. For $m_H^* = 60$ GeV, a jump of the scalar and vector correlation lengths, typical of first order transitions, was clearly seen. For $m_H^* = 80$ GeV, a powerlike decrease of the mass of the scalar excitation with no change of the vector mass across the critical region has been observed within error bars [3,4].

In contrast, the scalar and vector masses behave smoothly for $m_H^* = 120$ and 180 GeV and are nonvanishing (Fig. 4). This signals the absence of first or second order phase transitions. Within the statistical accuracy, the masses and the susceptibility χ are independent of the lattice spacing, showing that the observed behavior is not a lattice artifact and persists in the continuum limit. The absence of (nearly) massless modes guarantees that the qualitative conclusions cannot be changed by the inclusion of higher order corrections coming from the procedure of dimensional reduction. The conclusions are hence valid for the 4D high temperature theory, as well.

A long distance effective field theory near the critical point $m_H^* \simeq 80$ GeV is likely to be a single component scalar model. This suggests that the critical point is of the 3D Ising type. The explicit mapping of the couplings of the 3D SU(2) + Higgs theory to the standard parameters of the Ising model (external field and temperature) is a complicated nonperturbative problem not attempted here.

To summarize, we demonstrated that the Higgs and confinement phases of 3D SU(2) + Higgs theory can be continuously connected. This means that the phase transition in weakly coupled electroweak theories is absent in a part of their parameter space. For the minimal standard model the critical value is near $m_H = 80$ GeV.

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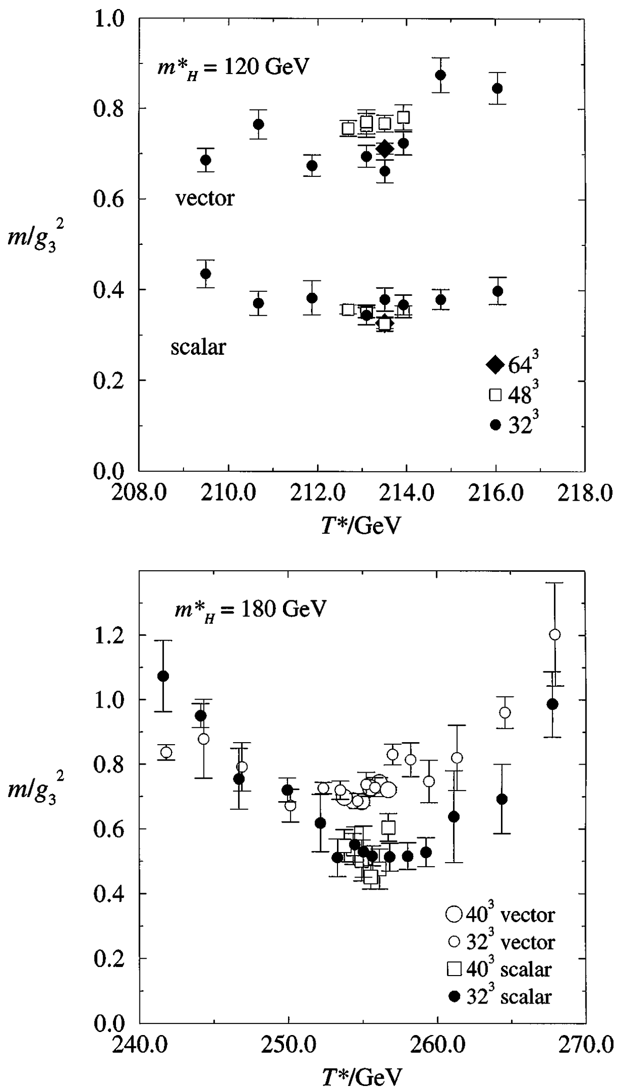

FIG. 4. The scalar and vector mass dependence on the temperature for "large" Higgs masses, $m_H^* = 120$ and 180 GeV.

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