New Type of Collective Motion for $N \sim Z$ Nuclei

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We study a new type of collective motion with α-particle type of correlations and show that it may be relevant for $N \sim Z$ nuclei. [S0031-9007(96)00505-4]

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New radioactive beam facilities will be studying proton rich nuclei with $N \sim Z$. The limited systematic information we have on such nuclei with $A \geq 60$ suggests different collective behavior than nuclei with $N \gg Z$. Although these nuclei have large deformations, the ratio of the excitation energy of the first angular momentum four state to the excitation energy of the first angular momentum two state, $E^*_{4_1^+}/E^*_{2_1^+}$, is closer to 2.5 than 3.3, suggesting gamma unstable collective motion. Furthermore, the energy spectrum for many such nuclei has $E^*_{4_1^+} \sim E^*_{2_2^+} \sim E^*_{0_2^+}$ resembling a *spherical* quadrupole vibrator. Within the interacting boson model with isospin and which includes neutron-proton correlations (IBM-3), we have found these features exhibited in a collective motion arising from a dynamical symmetry of IBM-3 not discussed heretofore. Neutron-proton correlations play a crucial role in this new collective motion; for nuclei with $N \sim Z$ we expect neutron-proton correlations to be larger than for nuclei with a large neutron excess because the valence neutrons and protons are filling the same major shell, and hence the radial wave functions of the neutrons will have a larger spatial overlap with the protons than neutrons filling different major shells.

The interacting boson model has been phenomenologically successful in describing the spectroscopy of heavy nuclei with a large neutron excess (IBM-2) [1]. However, when the valence neutrons and protons are filling the same major shell such as in nuclei with $N \sim Z$, then isospin must be introduced [2], which means a neutron-proton pair must be included in addition to the proton-proton and neutron-neutron pairs to complete the isospin triplet. These pairs are represented in the IBM-3 by three types of monopole ($s^\dagger_\tau$) and quadrupole ($d^\dagger_{m,\tau}$) bosons, where $m = (-2,-1,0,1,2)$ is the angular momentum projection and $\tau = (1,0,-1)$ is the isospin projection indicating neutron, proton-neutron, and proton bosons, respectively [2]. To the extent that IBM-3 is valid, we expect it to be valid for even-even nuclei; odd-odd nuclei will need additional isoscalar bosons [3].

We assume the IBM-3 nuclear Hamiltonian conserves angular momentum and isospin. The small isospin-breaking electromagnetic interactions can be treated in perturbation theory. Since there are eighteen bosons (six spatial-spin and three isospin degrees of freedom), the states of IBM-3 transform like the symmetric representations of U(18). One new dynamical symmetry, which includes the angular momentum subgroup, O(3), and the isospin subgroup, SU(2), is its orthogonal subgroup, O(18). The physical significance of O(18) is that the eigenstates have "α-particle-like" correlations because it leaves invariant the spin-isospin scalar two boson (aka four nucleon) system,

$$\mathcal{I}^\dagger = s^\dagger : s^\dagger - d^\dagger : d^\dagger, \tag{1}$$

where : means scalar product in both angular momentum and isospin space.

A basis for this dynamical symmetry can be determined by using the subchain,

$$\begin{aligned} &\mathrm{U}(18) \supset \mathrm{O}(18) \supset \mathrm{O}(15) \times \mathrm{SU}_s(2) \\ &\qquad \supset [\mathrm{O}(5) \supset \mathrm{O}(3)] \times \mathrm{SU}_d(2) \times \mathrm{SU}_s(2), \end{aligned} \tag{2}$$

where $\mathrm{SU}_s(2), \mathrm{SU}_d(2)$ are the isospin groups for the monopole and quadrupole bosons, respectively, and $\mathrm{SU}(2) = \mathrm{SU}_s(2) + \mathrm{SU}_d(2)$. We do not expect $\mathrm{O}(15) \times \mathrm{SU}_s(2)$ to be a conserved subgroup. [We shall not discuss the charge symmetric group chain, $\mathrm{U}(18) \supset \mathrm{U}(6) \times \mathrm{SU}_c(3)$, nor the other O(18) subchain, $\mathrm{U}(18) \supset \mathrm{O}(18) \supset \mathrm{O}(6) \times \mathrm{SU}(2)$, since U(6) and O(6) dynamical symmetry have been studied extensively [1,4,5].]

The normalized basis states for this subgroup chain for N bosons, representing N pairs of valence nucleons, are

$$|N, \alpha, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M\rangle = \mathcal{N}\, [\mathcal{I}^\dagger]^{(\mathcal{N}-\alpha)/2} |\alpha, \alpha, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M\rangle, \tag{3}$$

where $\mathcal{N} = \sqrt{(\alpha + 8)!/2^{N-\alpha}[(N-\alpha)/2]![(N+\alpha)/2+8]!}$,

$$\begin{aligned} |\alpha, \alpha, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M\rangle = &\sum_p A_p [s^\dagger : s^\dagger]^{[(\alpha-\delta-T_s)/2]-p} [d^\dagger : d^\dagger]^p \\ &\times |\delta + T_s, \delta + T_s, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M\rangle, \end{aligned} \tag{4}$$

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where

$$A_p = \frac{\sqrt{(\frac{\alpha-\delta-T_s}{2})!\,(\frac{\alpha-\delta+T_s+1}{2})!\,(\delta+\frac{13}{2})!\,(\frac{\alpha+\delta-T_s+13}{2})!\,(\frac{\alpha+\delta+T_s+14}{2})!\,(T_s+\frac{1}{2})!}}{2^{(\alpha-\delta-T_s)/2}p!\,(\frac{\alpha-\delta-T_s}{2}-p)!\,(\frac{\alpha-\delta+T_s+1}{2}-p)!\,(\delta+\frac{13}{2}+p)!\sqrt{(\alpha+7)!}}\,. \tag{5}$$

Thus these states have α-particle-like correlations. Although the number of monopole and quadrupole bosons is not a conserved quantum number, the monopole and quadrupole isospin are conserved in this group chain. Only the symmetric representations of O(18) occur and hence only one quantum number, α, is needed to label these representations. For N bosons the allowed values are $\alpha = N, N-2, \ldots, 0$, or 1. α counts the number of bosons *not* in the invariant in (1), and I will annihilate the states with maximum $\alpha = N$, $I|\alpha, \alpha, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M\rangle = 0$. The $\mathrm{O}(15) \times \mathrm{SU}_s(2)$ subgroup leaves both $s^\dagger : s^\dagger$ and $d^\dagger : d^\dagger$ separately invariant, and $\tilde{s} : \tilde{s}|\cdots\rangle = \tilde{d} : \tilde{d}|\cdots\rangle = 0$, where $|\cdots\rangle = |\delta + T_s, \delta + T_s, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M\rangle$. Likewise, only the symmetric representations for O(15) occur, with allowed eigenvalues $\delta = \alpha, \alpha - 1, \ldots, 0$, for each α. For a given value of α and δ, the allowed values of monopole isospin are $T_s = \alpha - \delta, \alpha - \delta - 2, \ldots, 0$, or 1. The allowed representations of the $\mathrm{O}(5) \times \mathrm{SU}_d(2)$ subgroup are given in Table I for the smallest values of δ [6], which are expected to lie lowest in energy. The allowed values of the quadrupole isospin are $T_d = \delta, \delta - 1, \ldots, 0$. This means that, for $\delta = 0$, $T = T_s$, and $\delta = 0$ occurs only for $N - T$ even.

The generators of the O(18) dynamical symmetry are given by the quadrupole operators, $P^{(2,t)} = [s^\dagger \tilde{d}]^{(2,t)} + (-1)^t [d^\dagger \tilde{s}]^{(2,t)}$, where $[\cdots]^{(\ell,t)}$ means coupled to angular momentum rank ℓ and isospin t and $\tilde{s}_\tau = (-1)^\tau \times s_{-\tau}$, $\tilde{d}_{m,\tau} = (-1)^{m+\tau} d_{-m,-\tau}$, the generators of the O(15) dynamical symmetry, $Q^{(\ell,t)}_{M,q} = [d^\dagger \tilde{d}]^{(\ell,t)}_{M,q}$, $\ell + t =$ odd, and the isospin generators for the monopole bosons, $\tilde{T}_{s,q} = -\sqrt{2}[s^\dagger \tilde{s}]^{(0,1)}_{0,q}$. The angular momentum generators are $\hat{J}_M = -\sqrt{30}\, Q^{(1,0)}_{M,0}$, and the total isospin is $\hat{T} = \hat{T}_s + \hat{T}_d$, where $\hat{T}_{d,q} = -\sqrt{10}\, Q^{(0,1)}_{0,q}$ are the quadrupole boson isospin generators. The Casimir operator is given by

$$C_{\mathrm{O}(18)} = \sum_{t=0,1,2} (-1)^t P^{(2,t)} : P^{(2,t)} + C_{\mathrm{O}(15)} + \hat{T}_s \cdot \hat{T}_s\,, \tag{6}$$

with eigenvalues $\alpha(\alpha + 16)$. The O(15) Casimir operator is given by

$$C_{\mathrm{O}(15)} = \sum_{\ell,t,\ell+t\,\mathrm{odd}} Q^{(\ell,t)} : Q^{(\ell,t)}, \tag{7}$$

with eigenvalues $\delta(\delta + 13)$.

Hence an attractive interaction quadrupole interaction will have the eigenvalue

$$-\kappa\langle N, \alpha, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M| \sum_{t=0,1,2} (-1)^t P^{(2,t)} : P^{(2,t)} |N, \alpha, \delta, [T_s, T_d]^{(T)}_{T_z}, (\tau_1, \tau_2), J, M\rangle = -\kappa[\alpha(\alpha + 16) - T_s(T_s + 1) - \delta(\delta + 13)]\,. \tag{8}$$

Thus, the most symmetric irreducible representation (IR) of O(18), $\alpha = N$, will be the lowest in energy, whereas the smallest IR's of O(15), $\delta = 0, 1$, etc., will be the lowest in energy. The O(15) quantum number, δ, is similar to a phonon quantum number. However, because of the additional monopole and quadrupole isospin quantum numbers, in general, there are more eigenstates than for the usual quadrupole vibrator which does not distinguish between neutrons and protons. For example, for $\delta = 1$, and $T = N$ and $T = 0$, there is one 2^+ excited state and it has $T_s = N - 1$ and 1, respectively, but, for all other isospins there are two 2^+ excited states for each total isospin T corresponding to two different monopole isospins, $T_s = T \pm 1$. Furthermore, for $\delta = 2$, the 0^+ state can only have $T_d = 2$, whereas the $2^+, 4^+$ states can have both $T_d = 0, 2$. This means that there are a different number of states for $\delta = 2$ depending on the angular momentum. For example, although for $\delta = 2$, and $T = N$, there is only one $0^+, 2^+, 4^+$ excited state and it has $T_s = N - 2$, for $T = 0$, there is one 0^+ state which has $T_s = 2$, but two $2^+, 4^+$ states corresponding to

TABLE I. The allowed values of (τ_1, τ_2), T_d, and J for the lowest IR's of O(15) labeled by δ.

δ	(τ_1, τ_2)	T_d	J
0	(0,0)	0	0
1	(1,0)	1	2
2	(2,0)	0,2	2,4
	(1,1)	1	1,3
	(0,0)	2	0
3	(3,0)	1,3	0,3,4,6
	(2,1)	1,2	1,2,3,4,5
	(1,1)	0	1,3
	(1,0)	1,2,3	2

$T_s = 0$ and 2, and for $T = 1$ there are two 0^+ states corresponding to $T_s = 1$ and 3, but three $2^+, 4^+$ states corresponding to $T_s = 1$, 3 and $T_d = 2$, and $T_s = 1$, $T_d = 0$. On the other hand, for all other isospins, there are three 0^+ states ($T_s = T - 2, T, T + 2$), but four $2^+, 4^+$ states corresponding to $T_s = T - 2, T, T + 2, T_d = 2$, and $T_s = T$, $T_d = 0$. Most likely these states will mix in general, separating them in energy; a detailed analysis of nuclear spectra with a Hamiltonian with O(18) dynamical symmetry is underway using an IBM-3 diagonalization [7]. Nevertheless, we anticipate that there will be changes in the phonon multiplet going from nuclei with $T \neq 0$ to nuclei with $T = 0$ because for $T \neq 0$ there are only 25% more $2^+, 4^+$ two phonon states than 0^+ two phonon states, whereas for $T = 0$ there are 100% more $2^+, 4^+$ two phonon states than 0^+ two phonon states.

Recently, the spectrum of ${}^{64}_{32}\mathrm{Ge}_{32}$ has been measured [8], and the first few excited states are shown in Table II with O(18) and unperturbed O(15) quantum numbers. The spectrum indicates a phonon structure. The average measured excitation energy of the two phonon states 4_1^+ and 2_2^+, $\bar{E}_2^* = \sum_{J_i = 2_2^+, 4_1^+} (2J + 1) E_{J_i} / \sum_{J_i = 2_2^+, 4_1^+} (2J + 1)$, is 1.88 MeV and the ratio of two phonon energy to one phonon energy is then $\bar{E}_2^* / E_{2_1^+}^* = 2.09$, in between a spherical (2.0) and deformed quadrupole O(6) vibrator (2.5) [1]. For $T = 0$ nuclei, the 2_1^+ has $\delta = 1$ and $T_s = 1$ and is unique (Table I). The 4_1^+ and 2_2^+ have $\delta = 2$, but there are two states each, one with $T_s = 0$ and one with $T_s = 2$, which will probably mix. Using the quadrupole interaction (8) to set the scale of the phonon energy, the O(18) excitation energy is $E_{\delta, T_s}^* = \kappa[\delta(\delta + 13) + T_s(T_s + 1)]$ and hence $E_{2, T_s}^* / E_{1,1}^*$ ranges between 1.88 and 2.25 for $T_s = 0$ or 2, respectively. An even admixture of $T_s = 0$ and 2 for the $\delta = 2$ state gives the observed ratio. Of course, a ratio of 2.09 is also consistent with a spherical vibrator. We can distinquish between a spherical vibrator and the O(18) limit by the variation of the excitation energy of the first excited state with isospin. For maximal isospin, $T = N$, the O(18) model is equivalent to the spherical vibrator; as can be seen from (4) there are no correlations since $\delta + T_s = \alpha = N$, and the number of quadrupole bosons, N_d, is a conserved quantum number and $N_d = \delta$. As the isospin decreases for a given N, N_d remains a conserved quantum number for a spherical vibrator, and hence the energy spectrum of the first excited state remains a constant. However, for O(18), N_d is not conserved for $T < N$, and the expectation value $\langle \cdots | N_d | \cdots \rangle$ where $| \cdots \rangle = | N, N, \delta, [T_s, T_d]_{T_z}^{(T)}, (\tau_1, \tau_2), J, M \rangle$ is

$$\langle \cdots | N_d | \cdots \rangle = \delta + \frac{(N - \delta - T_s)(N + T_s - \delta + 1)}{2(N + 7)} . \tag{9}$$

We clearly see then for maximal isosopin, $T = N = \delta + T_s$, $\langle \cdots | N_d | \cdots \rangle = \delta$, the same as a spherical vibrator. However, for T, T_s small and N large, $\langle \cdots | N_d | \cdots \rangle \approx \frac{1}{2}(N + 1) + O(1/N)$; that is, the same for all states, and hence the effect of the quadrupole boson energy goes to zero in the *spectrum,* and thus the excitation energy of the first excited state decreases.

A measurement of the $B(E2)$ will also distinguish between the two types of collective motion. For nuclei with $T = 0, N$ even, only the isoscalar quadrupole operator contributes to $B(E2)$; hence, the quadrupole operator is $Q = Q_{\mathrm{sp}} \sqrt{3} (1 + 2\Delta) P^{(2,0)}$, where Q_{sp} is the single-particle quadrupole moment and Δ is the effective charge. $B(E2)$ from the ground state for O(18) becomes

$$B(E2 : 0_1^+, T = 0 \rightarrow 2_1^+, T = 0)_{\mathrm{O}(18)} = \frac{(1 + 2\Delta)^2}{3} N(N + 16) B(E2)_{\mathrm{sp}} . \tag{10}$$

The heaviest even-even nuclei with $T = 0$ for which the $B(E2)$ are measured are ^{44}Ti and ^{48}Cr. Using no effective charge ($\Delta = 0$), we find that (10) gives 0.055 and 0.138 $(e\,\mathrm{b})^2$, respectively, in very good agreement with the measured values 0.061 ± 0.015 and 0.133 ± 0.020 $(e\,\mathrm{b})^2$, respectively [9]. For spherical nuclei, $B(E2 : 0_1^+, T = 0 \rightarrow 2_1^+, T = 0)_{\mathrm{spherical}} = 5(1 + 2\Delta)^2 N B(E2)_{\mathrm{sp}}$ [1], producing a smaller $B(E2)$ with a different mass dependence. In Fig. 1 we compare the two limits as a function of N.

In summary, the O(18) dynamical symmetry developed in this paper shows promise that it may have the correct neutron-proton correlations to describe heavy nuclei with $N \sim Z$. Only additional measurements on nuclei with $N \sim Z$ will be able to decide.

TABLE II. The excitation energy $E_{J_i^+}^*$ in MeV of the lowest energy levels in ^{64}Ge [8] and their O(18) and unperturbed O(15) quantum numbers.

J_i^+	$E_{J_i^+}^*$	α	δ	T_s	(τ_1, τ_2)	T_d
$0_1^\dagger$	0.0	4	0	0	(0,0)	0
$2_1^\dagger$	0.902	4	1	1	(1,0)	1
$2_2^\dagger$	1.579	4	2	0 or 2	(2,0)	0 or 2
$4_1^\dagger$	2.052	4	2	0 or 2	(2,0)	0 or 2

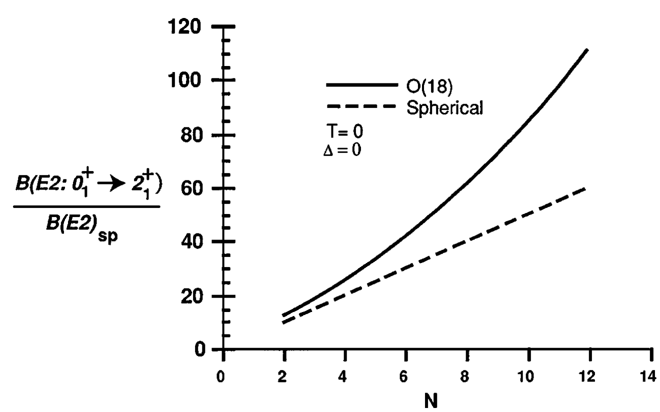

FIG. 1. $B(E2)$ from the ground state to the first excited state, relative to the single-particle value, $B(E2)_{\mathrm{sp}}$, versus N for $T = 0$ and $\Delta = 0$. The solid line is for the O(18) limit; the dashed line is for the spherical limit.

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[1] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).
[2] J. P. Elliott and A. P. White, Phys. Lett. **97B**, 169 (1980).
[3] J. P. Elliott and J. A. Evans, Phys. Lett. **101B**, 216 (1981).
[4] J. N. Ginocchio and A. Leviatan, Phys. Rev. Lett. **73**, 1903 (1994); J. N. Ginocchio and A. Leviatan (to be published).
[5] G. Long, Chin. J. Nucl. Phys. **16**, 331 (1994).
[6] W. G. McKay and J. Patera, *Tables of Dimensions, Indices, and Branching Rules for Representations of Simple Lie Algebras,* Lecture Notes In Pure And Applied Mathematics, Vol. 69 (M. Dekker, New York, 1981); W. G. McKay, J. Patera, and D. W. Rand, SIMPLIE, Macintosh software for representations of Lie algebras.
[7] M. Sugita, in *Perspectives for the Interacting Boson Model,* edited by R. Casten *et al.* (World Scientific, Singapore, 1994); M. Sugita (private communication).
[8] P. J. Ennis *et al.*, Nucl. Phys. **A535**, 392 (1991).
[9] S. Raman *et al.*, At. Data Nucl. Data Tables **36**, 1 (1987).