Dynamical Phases of Driven Vortex Systems

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We study numerically the motion of vortices in dirty type II superconductors. In two dimensions at strong driving currents, vortices form highly correlated "static channels." The static structure factor exhibits convincing scaling behavior, demonstrating quasi-long-range translational order in the transverse direction. However, order in the longitudinal direction is only short range. We clearly establish the existence of a finite transverse critical current, suggesting strong barriers against transverse driving forces. We discuss these results in terms of recently proposed theories of the moving vortex systems. [S0031-9007(96)01255-0]

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Many condensed matter systems reach higher levels of organization by forming periodic media. Examples range from crystalline solids to Wigner crystals, charge density waves, and vortex lattices in type II superconductors. A central issue is the effect of disorder on the stability of such systems. Flux lattices, for weak disorder and short distances, are properly described by the standard elastic theory [1,2]. Topological excitations such as dislocation loops become relevant only for stronger disorder and longer length scales [3], possibly giving rise to a vortex glass phase. Concerning the behavior of weakly disordered systems at asymptotically long length scales, modifications of the elastic approaches were proposed. Following the early work of Nattermann [4], an extensive analysis was carried out in this regime by Giamarchi and Le Doussal, who concluded that density correlations decay according to a power law when the periodicity of the lattice is properly taken into account. They christened this phase a "Bragg glass" [5]. A recent analysis suggested that the elastic approach is self-consistent, as topological excitations were found irrelevant in three dimensions, and marginal in two dimensions [6]. However, a comprehensive picture is yet to be agreed upon. While numerous additional scenarios have been proposed [2], there is growing experimental evidence supporting the basic picture of two types of glasses as the disorder or the magnetic field is increased [7,8].

When these ordered media are exposed to an external force beyond a certain critical depinning strength, they become mobile. Early work developed perturbation studies at high velocities v in powers of $1/v$ [1,9]. A qualitatively new picture has been proposed recently by Koshelev and Vinokur, who argued that at large driving forces the effect of disorder can be adequately represented by a "shaking temperature" $T_{\mathrm{sh}} \sim 1/v$ [10]. Thus by increasing the velocity beyond some critical value a genuine *dynamic phase transition* may occur to a more ordered vortex state, characterized by a change from incoherent to coherent vortex motion. The nature of the ordered phase has been elucidated by Giamarchi and Le Doussal, who pointed out that some components of the disorder remain *static* [11]. These prevent the formation of a true solid order and stabilize a "moving glass" phase instead with quasi-long-range order (QLRO) only. In related charge density wave systems it is also found that the moving phase possesses only QLRO [12]. The physics of the moving glass phase is [11] that the vortices move along highly correlated *static channels*. This picture leads to a power law decay of the density correlations at large distances. Furthermore, it is characterized by diverging potential barriers against a transverse drive and, consequently, a finite transverse critical current.

In this Letter, we report a numerical study of the dynamical phases of driven vortex systems. We find that vortices indeed move along "static channels." We demonstrate the existence of translational QLRO in the transverse direction. However, we find only short-range order in the longitudinal direction, giving rise to a "moving transverse glass." We identify phase slips between neighboring elastic domains as a possible source of the breakdown of the elastic theory along the longitudinal direction. As a direct consequence the absence of a narrow band noise is predicted in driven 2D vortex systems. Finally, we measure the response to a transverse drive and observe a finite critical current.

We employ overdamped molecular dynamics (MD) simulations at zero temperature to study two dimensional interacting vortices in the presence of point disorder,

$$\gamma \frac{d\mathbf{r}_i}{dt} = \sum_{j\neq i} \mathbf{F}_v(\mathbf{r}_i - \mathbf{r}_j) + \sum_j \mathbf{F}_{\mathrm{pin}}(\mathbf{r}_i - \mathbf{R}_j) + \mathbf{F}_L\,. \tag{1}$$

Here γ is the damping parameter, $\mathbf{R}_j$ specifies the pinning center positions, $\mathbf{r}_i$ denotes the location of the ith vortex, and $\mathbf{F}_L$ is the Lorentz force, exerted by the external driving current. The force between vortices is given by

$$\mathbf{F}_v(\mathbf{r}) = F_0(1 - \tilde{r}^2)^2 \frac{\tilde{\mathbf{r}}}{\tilde{r}^2}\,, \tag{2}$$

where $\tilde{r} = r/R_{\mathrm{cut}}, F_0 = V_0/R_{\mathrm{cut}}$, and we choose $R_{\mathrm{cut}} = 3.6a_0$, where a_0 is the mean vortex spacing. Here a_0, V_0 $[\approx 2s\Phi_0^2/(4\pi\lambda)^2]$, and $\gamma a_0^2/V_0$ define the units of

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length, energy, and time, respectively, and s is the sample thickness. The pinning force is taken as

$$\mathbf{F}_{\text{pin}}(\bar{r}) = -4F_p(1 - \bar{r}^2)\bar{\mathbf{r}}. \quad (3)$$

Here $\bar{r} = r/R_{\text{pin}}$ and $R_{\text{pin}} = 0.25a_0$. We worked with a fixed pin density of $\rho_{\text{pin}} = 5\rho_{\text{vortex}}$.

Now we construct the phase diagram in the driving force-pinning strength plane, at zero temperature. With increasing driving currents three phases emerge: a pinned glass, a plastic flow regime, and some kind of an ordered phase. At low drive the vortices remain pinned, forming a glassy phase. As the Lorentz force is increased beyond a critical value F_d, the vortices depin. This transition, and in particular the value of F_d, can be well captured by studying the current-voltage (*IV*) characteristics. The resulting values of F_d were used to construct the lower phase boundary in Fig. 1. Above F_d the vortices form a pattern of pinned and unpinned regions, often described as an incoherent, or "plastic flow" [2]. For strong disorder, F_d scales linearly with the pinning strength, whereas for weak disorder the relation is quadratic. The near-linearity of the phase boundary in Fig. 1 indicates that we concentrated on the regime of strong disorder.

Upon further increase of the driving force, Koshelev and Vinokur proposed [10] that at some $F_L = F_g$ a dynamic phase transition occurs from the incoherent plastic flow to a regime with coherent vortex motion. Concerning the nature of this phase, Giamarchi and Le Doussal suggested [11] that the vortex system forms a moving glass. We now explore these propositions. The phase boundary F_g can be established by measuring the static structure factor $S(\mathbf{k})$. In the plastic flow regime the absence of ordering manifests itself in a central peak and a structureless ring (lower panel in Fig. 2). In the high velocity phase $F_L > F_g$, one expects to see sixfold coordinated Bragg peaks representing some sort of solid ordering. We do indeed observe a sharp transition into a phase with well developed peaks (upper panel in Fig. 2).

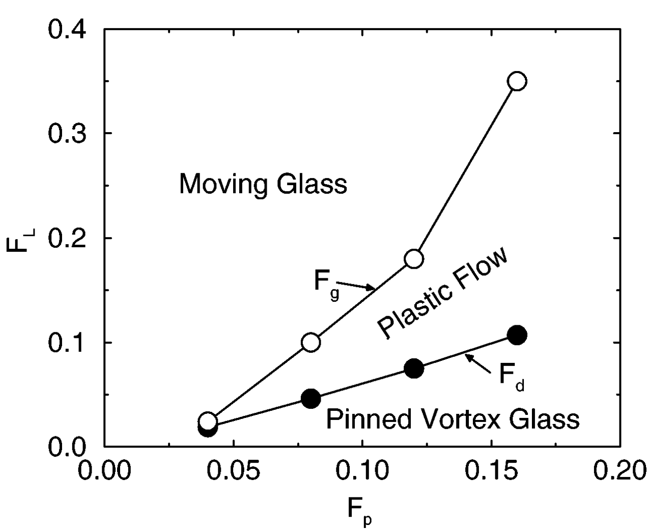

FIG. 1. The dynamic phase diagram. F_p is the pinning strength, F_L is the Lorentz force. Open circles represent F_g, solid circles represent F_d.

The upper phase boundary in Fig. 1 was determined by mapping out F_g for several disorder strengths. One can see that for strong disorder indeed all three expected phases are observed, whereas for weak disorder there is no evidence for an intervening plastic flow regime. Either that phase occupies a very slim region in the parameter space or there is a direct pinned Bragg glass-to-moving glass transition. This transition is much harder to identify because both phases exhibit quasi-long-range order, and thus the structure factors are very similar.

The central goal of our paper is to elucidate the nature of the high velocity phase. To address this issue we first analyze $S(\mathbf{k})$,

$$S(\mathbf{k}) = \frac{1}{L^d} \sum_{i,j} e^{i\mathbf{k}\cdot[\mathbf{r}_i(t)-\mathbf{r}_j(t)]}. \quad (4)$$

The pinning strength F_p is fixed to be 0.16 and the applied force $F_L = 0.6$ is well above the corresponding critical force $F_g \approx 0.35$. We simulate five different system sizes with fixed vortex density and number of vortices ranging from 240 to 1500. The initial configurations are chosen randomly. We let the MD simulations evolve with time, make sure that the system reaches its steady state, then freeze the vortex configuration and measure $S(\mathbf{k})$. In the steady state the vortices form an orderly array. Its

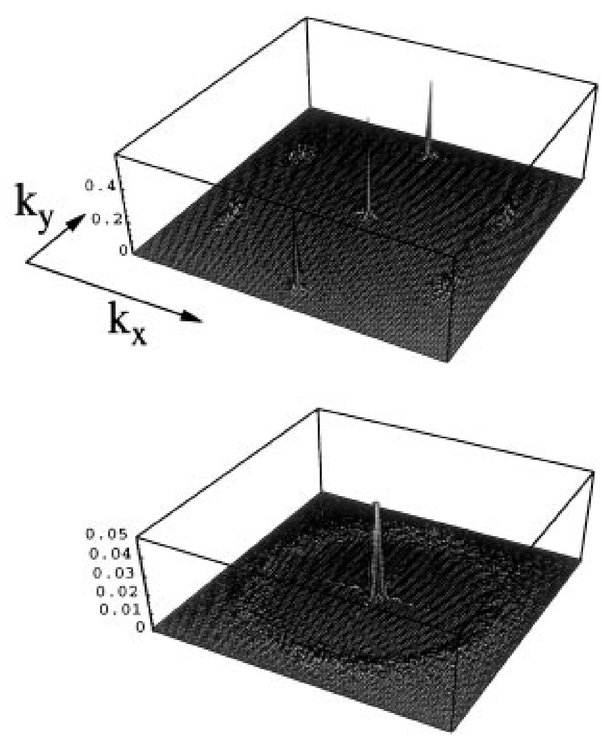

FIG. 2. The static structure factor $S(\mathbf{k})$. $F_L = 0.6$ in the upper panel and 0.2 in the lower panel. The number of vortices is 960 and the disorder strength $F_p = 0.16$. The critical force is $F_g \approx 0.35$.

principal lattice vector in most cases is aligned with the direction of motion. It was argued that the system chooses such an orientation to minimize the power-dissipation [9]. However, the details of this alignment have yet to be understood.

To study the peaks of $S(\mathbf{k})$ at the reciprocal lattice vector $\mathbf{G}_0 = (0, \pm 4\pi/\sqrt{3})$, we write down the following finite size scaling form for $\mathbf{k} \parallel \mathbf{G}_0$:

$$S(\delta k, L) = L^{d-\nu_s} G_L(\delta k\, L)\,, \tag{5}$$

with $\delta k = |\mathbf{k} - \mathbf{G}_0|$. The peaks of $S(\mathbf{k})$ are anisotropic and the half-width in the direction $\mathbf{k} \parallel \mathbf{G}_0$ is considerably smaller than $\mathbf{k} \perp \mathbf{G}_0$. In Fig. 3 the scaling function $G_L(x)$ is plotted with respect to the dimensionless scaling variable $x = \delta k L$ for the five system sizes. The peak amplitudes scale with the system size as $L^{2-\nu_s}$ with $\nu_s = 0.53 \pm 0.1$, as shown in the inset of Fig. 3. Using this value of ν_s the normalized structure factors collapse onto a single curve, confirming the scaling behavior $S(\mathbf{k}) \cong |\mathbf{k} - \mathbf{G}_0|^{-1.47}$ around the peaks. We find that the exponent ν_s is not independent of F_L and F_p [13]. The moving glass picture predicts a power-law decay of density correlations with a universal exponent in 3D but possibly a nonuniversal one in 2D, thus providing a natural description of our results in the transverse directions. In contrast, the peak heights at momenta with nonzero longitudinal components decay rapidly with system size. To understand the underlying physical mechanism we studied a large number of snapshots of vortices. We identified all of the lattice defects and determined the corresponding Burgers' vectors. The overwhelming majority points in the direction of the velocity, suggesting that *phase slips* between longitudinal boundaries of elastic domains are present in a finite density and should be incorporated in a full theory. This observation is not consistent with an elastic approach for the longitudinal direction. Clearly, a more detailed understanding is needed on this issue, especially in two dimensions [14]. A measurable consequence of this absence of translational order in the longitudinal direction should be the corresponding absence of narrow band noise. It is noteworthy that the experimental search for narrow band noise in moving vortex systems so far has been fruitless [15]. The same conclusion was reached for the analogous CDW models in 2D [12].

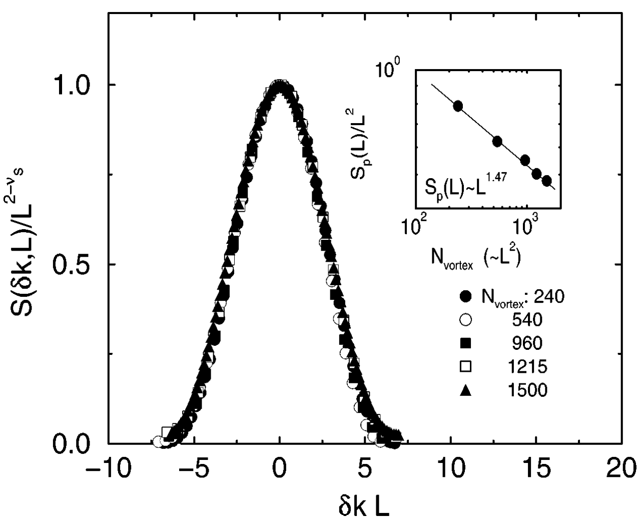

FIG. 3. Finite size scaling of the static structure factor $S(\mathbf{k})$. The driving force $F_L = 0.6$ is well above the critical force $F_g \sim 0.35$. The inset shows power law dependence of the peak heights with varying system size; $S_p(L) \sim L^{2-\nu_s}$ with $\nu_s \sim 0.53 \pm 0.1$.

The proposition of the moving glass phase rests on the argument that certain components of the disorder *do not* average out, but present a *static* perturbation. If so, the moving vortices should form static channels, which do not change their shape with time. To study this we map out the trajectories of the vortices. Making sure that the flow reached its steady state, we take a large number of consecutive snap shots, which are then displayed on top of each other. We display only a small portion of the result in Fig. 4 to show how vortices retrace each other's path to a remarkable degree, clearly demonstrating the formation of static channels. One observes that the transverse wandering of most channels is at best comparable to the lattice spacing. The fact that we nevertheless see power law decay of the density correlations implies either that the crossover to the glassy asymptotics occurs at anomalously short distances, or that additional physics, such as phase slips, may play a role.

The moving glass picture also implies the existence of diverging barriers against small transverse currents, leading to a *finite transverse critical current*. This critical current is the largest when one of the principal lattice vectors is parallel to the motion [11]. We select 50 disorder realizations which lead to a steady state with one of its primitive lattice vectors parallel to the direction of the velocity. After the system reaches the steady state, a small additional transverse force is applied and the transverse velocity v_y is measured. Figure 5

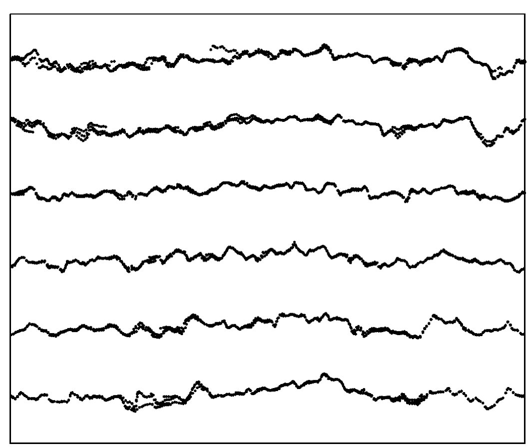

FIG. 4. The static channels in the steady state.

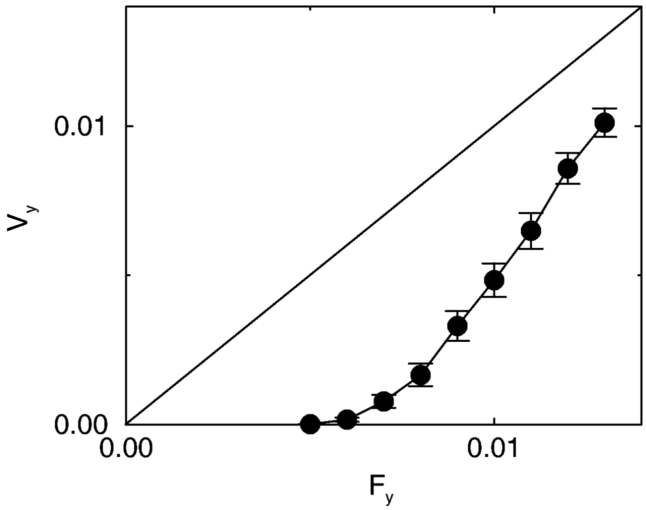

FIG. 5. *I-V* characteristics showing transverse velocity v_y as a function of transverse force F_y. Here $F_L = 0.6$. The straight line represents a free flux flow response.

clearly exhibits a finite critical transverse force $F_d^y \approx 0.006 \pm 0.001$. Simulations for several different system sizes result in identical critical currents, indicating that this threshold behavior is not a finite size effect [13]. The absence of a threshold current for $F_p = 0$ verifies that this phenomenon *is not a result* of the periodic boundary conditions or the alignment of the driving force with the sides of the simulation box.

Our simulations are consistent with the following aspects of the moving glass scenario: power law decay of density correlations; existence of static channels; and a finite transverse critical current. However, the manifestly short-range longitudinal correlations clearly call for further studies. Our real space snapshots point towards the possible importance of phase slips between the longitudinal boundaries of elastic domains. Finally, the truly glassy nature of the transverse fluctuations remains to be verified, since the divergent nature of the barrier heights would be proven only by identifying *nonlinear I-V* characteristics at *finite* temperatures [3,16] or by analyzing the statistics of the barrier heights. Both of these tests are numerically very demanding.

In summary, we explored the dynamic phases of driven vortex systems. We measured the *I-V* characteristics to determine the critical depinning force F_d, establishing the phase boundary between the pinned glass and the plastic flow regime. Next we studied the structure factor $S(\mathbf{k})$, which exhibited a sharp transition from its ring shape in the plastic flow regime into a phase with an anisotropic peak pattern. The Bragg peaks establish the existence of power law order in the transverse directions, but only short-range longitudinal order. Correspondingly, we also expect the absence of a narrow band noise. We demonstrated the formation of static channels and found a finite critical transverse current. We suggested that longitudinal phase slips might be responsible for the discrepancies between the moving glass picture and our numerical results. Thus this new phase might be more accurately called a moving transverse glass.

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