Interacting Self-Guided Beams viewed as Particles: Lorentz Force Derivation

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The trajectories of interacting self-guided beams in a homogeneous nonlinear medium are derived directly from the Lorentz force of classical electromagnetic theory, treating the beams as particles with mass. This, to our knowledge, is the first self-consistent particle description of beam interaction, and, in particular, one that does not rely upon interpretation of the wave equation. The method is applicable to any stable beams, in both two and three dimensions. [S0031-9007(96)00609-6]

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This Letter addresses a fundamental question: Can interacting self-guiding beams be described directly from the Lorentz force of classical electromagnetic theory by treating the beams as particles with mass? Of course it is well known that solitons are *particlelike*, but this conclusion has been deduced previously after solving and interpreting the *wave* equation [1]. The scalar wave equation for monochromatic waves in the slowly varying approximation is identical to Schrödinger's equation. Now, adopting an entirely different tact, we describe here soliton interaction directly from first principles following from the classical Lorentz law governing the electromagnetic force. Furthermore, our derivation applies to all self-guided beams of one and two transverse dimensions that are stable to perturbations. Unlike the inverse scattering technique, our description is not restricted to the singular case of one-dimensional beams of a cubic nonlinearity. This underscores the fact that beams which are more general than solitons can also be particlelike. Nevertheless, we are not necessarily advocating the Lorentz force approach for actually performing calculations. However, the Lorentz force approach is appealing on fundamental physical grounds by providing a self-consistent demonstration that stable, self-guided beams can be treated as particles with mass, without regard to their wavelike properties. Finally, interacting self-guided beams have potential use for all optical switching [2].

Self-guidance occurs when beam diffraction is balanced by the containment effects of a nonlinearly induced refractive index [3]. The most familiar self-guided beams are spatial solitons. These are a very special class of one-dimensional self-guided beams of a cubic medium which are revealed by the inverse scattering method [4] and are highly stable even to collisions with each other. However, our analysis requires only that the beams be stable to infinitesimal perturbations. This is a condition far less restrictive than that necessary for solitons. The analysis is carried out rigorously for the interaction of two such beams when they are well separated. We hasten to emphasize that solitons are not the only example of stable self-guided beams. Stable beams also exist with circular symmetry as well as planar symmetry. While beams with circular symmetry are not stable in a Kerr law medium, they are in a saturating material. This can be shown [5] directly by an elementary extension of the proof for stability first given by Kolokolov [6] in a Kerr material.

There is a force between any electromagnetic field and a polarized medium. This is the Lorentz force of classical electromagnetic theory. Thus, parallel linear waveguides exert a force on each other which is *mediated* by the presence of electromagnetic fields. This force would cause them to attract or repel if they were free to move. Analogously, two parallel beams of light should exert a force on each other which is *mediated* by the presence of a polarizable medium. Thus, the beams should attract or repel, and the force should follow directly from the Lorentz force. This motivates the conceptual approach adopted here.

If a self-guided beam is stable to small perturbations then it can be thought of as an elastic body which has an internal restoring force under perturbations which tends to maintain the beam shape. Thus, the forces acting on such a beam can be replaced by a single net force which acts on the center of mass of the beam and the beam will move without changing shape. In particular, we determine the trajectory of two stable self-guided beams treating them as rigid bodies or particles and using the classical Lorentz force.

We begin by calculating the electromagnetic force between an electromagnetic wave and a dielectric material. The material can be considered to be made up of individual dipoles which are induced by the presence of the electromagnetic wave. The classical Lorentz force can be used to calculate the force on each dipole. The force $\mathbf{f}$ acting on a stationary particle or molecule with dipole moment $\mathbf{p}$ is then given by [7]

$$\mathbf{f} = (\mathbf{p} \cdot \nabla)\mathbf{E} + \frac{\partial \mathbf{p}}{\partial t} \times \mathbf{B}. \tag{1}$$

For a monochromatic wave propagating in an isotropic, dispersionless medium, the polarization (or dipole moment per unit volume) is given by $\mathbf{P} = \varepsilon_0(n^2 - 1)\mathbf{E}$. If we replace the dipole moment $\mathbf{p}$ in Eq. (1) with the dipole moment per unit volume $\mathbf{P}$ and then integrate over all space, the net force on the medium is obtained. Assuming an $e^{i\omega t}$ time dependence for the fields, using $\nabla \times \mathbf{E} = i\omega \mathbf{B}$, and also the vector identity $(\mathbf{E} \cdot \nabla)\mathbf{E} + \mathbf{E} \times (\nabla \times \mathbf{E}) = \frac{1}{2}\nabla(\mathbf{E} \cdot \mathbf{E})$ the time average force $\mathbf{F}$ exerted on the medium is given by

$$\mathbf{F} = \frac{1}{2} \int \varepsilon_0 (n^2 - 1) \nabla |\mathbf{E}|^2 \, dA \,. \tag{2}$$

From Newton's third law, the medium must exert an equal but opposite force on the beam. If $\mathbf{r}_0$ is the center of the beam and m is its effective mass, then the force is found by integrating Eq. (2) by parts and reversing the sign giving

$$m \frac{d^2 \mathbf{r}_0}{dt^2} = \frac{1}{2} \int \varepsilon_0 |\mathbf{E}|^2 \nabla n^2 \, dA \,. \tag{3}$$

The center of the beam is defined by

$$\mathbf{r}_0 = \int |\mathbf{E}|^2 \mathbf{r} \, dA \Big/ \int |\mathbf{E}|^2 \, dA \,. \tag{4}$$

The effective mass of the beam is related to the total stored energy in the electromagnetic field by

$$mc^2 = \frac{1}{2} \int (\varepsilon |\mathbf{E}|^2 + \mu |\mathbf{H}|^2) \, dA = \int \varepsilon |\mathbf{E}|^2 \, dA$$
$$\cong \varepsilon_0 n_0^2 \int |\mathbf{E}|^2 \, dA \,. \tag{5}$$

The final form is valid when the minimum and maximum values of the refractive index are approximately equal and given by n_0. It can then be shown that the field is predominantly transverse, and obeys the scalar wave equation [8,9]. The reaction force acting on the beam can be considered to be exerted on the individual photons as they trasverse the empty spaces between the atoms and thus they are traveling at the speed of light. Combining Eqs. (3) and (5) and using the velocity of the photons to convert from time derivatives to z derivatives gives

$$\frac{d^2 \mathbf{r}_0}{dz^2} = \frac{1}{2n_0^2} \int |\mathbf{E}|^2 \nabla n^2 \, dA \Big/ \int |\mathbf{E}|^2 \, dA \,. \tag{6}$$

This is the fundamental equation for the trajectory of the center of the beam in both two and three dimensions and is equivalent to the eikonal equation for rays propagating in a graded index (axial uniform) fiber:

$$\frac{d^2 \mathbf{r}_0}{dz^2} = \frac{1}{2n_0^2} \nabla n_{\mathrm{eff}}^2(\mathbf{r}_0) \,, \tag{7}$$

provided we identify the effective graded index profile in Eq. (7) with the expression appearing in Eq. (6). Because n_{eff} is axially uniform, the gradient operator can be replaced by its transverse part ∇_t.

The equation of motion (6) was derived by finding the force on the beam treating the beam as a particle. Alternatively, the above expression can also be obtained from the scalar wave equation in the slowly varying amplitude approximation by differentiating Eq. (4) twice, substituting from the wave equation, and performing integrations by parts.

Our fundamental result, Eq. (6), can be applied directly to a single beam in a smoothly varying refractive index gradient and gives the motion of the center of the beam. If we apply Eq. (6) to the situation of two beams in a uniform nonlinear medium then we find that the center of the two beam system does not move. What we are interested in is the relative motion of the two beams. The result is obtained by regarding one of the beams as moving in the refractive index profile induced by the presence of the second beam. A simple expression for the interaction of two self-guided beams located at $\mathbf{r}_0$ and $-\mathbf{r}_0$ can be calculated from Eq. (6) by making a suitable well separated beam approximation for the composite two beam field such as

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_s(\mathbf{r} - \mathbf{r}_0) + \mathbf{E}_s(\mathbf{r} + \mathbf{r}_0) \,, \tag{8}$$

where $E_s(\mathbf{r})$ is the field amplitude for a single self-guided beam. The field will induce a waveguide $n_{\mathrm{ind}}^2(\mathbf{r})$ via the nonlinearity of the medium [3,9]. In addition, we also calculate the waveguide $n_{\mathrm{mode}}^2(\mathbf{r})$ for which the field in Eq. (8) is the *exact* mode. Since a waveguide does not exert any forces on its own modes, the beams "feel" only the difference between the induced and modal waveguides $n_{\mathrm{ind}}^2(\mathbf{r}) - n_{\mathrm{mode}}^2(\mathbf{r})$. The motion of the center of one of the beams can be determined from Eq. (6) using this difference waveguide. After some algebra, we obtain the following expression for $n_{\mathrm{eff}}(\mathbf{r}_0)$ appearing in Eq. (7):

$$n_{\mathrm{eff}}(\mathbf{r}_0) \cong n_0 + \int [n_s(\mathbf{r}_0) - n_0] \mathbf{E}_s(r) \mathbf{E}_s(\mathbf{r} - 2\mathbf{r}_0)^* \, dA \,, \tag{9}$$

where $n_s(\mathbf{r})$ is the refractive index induced by the single soliton $E_s(\mathbf{r})$. Using this procedure, we note that if the two beams are in phase then the force is attractive, and if they are π out of phase then the force is repulsive. In addition, a simple analysis reveals that the variation of the force with separation depends only on the tail of the self-guided beam. In planar geometry, this is an exponential decay, and in three dimensions it is an exponential divided by the square root of the distance. In planar geometry the possible trajectories are beams which periodically cross each other or which continually diverge. With circular beams, in addition to cross and diverging beams, it is also possible to obtain beams which spiral around each other [10]. We previously derived Eq. (9) from the invariants of the slowly varying wave equation [11] and also from a purely linear perspective [9].

We have derived the trajectories of two self-guided beams of arbitrary cross section by treating the beams as particles with mass and calculating the classical electromagnetic force between them. To our knowledge, this is the first self-consistent particle description of beam interaction, and, in particular, one that does not require solving and interpreting the wave equation as an intermediate step. The beams must be stable only to small perturbations as discussed in the introduction. They need not be solitons. The special case of interacting spatial solitons of planar symmetry in a cubic (Kerr) material can be treated by the inverse scattering method [4]. Finally, the physical consequences of beam interaction have been discussed elsewhere [10,11].

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