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Local Spin-Gauge Symmetry of the Bose-Einstein Condensates in Atomic Gases

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The Bose-Einstein condensates of alkali atomic gases are spinor fields with local "spin-gauge" symmetry. This symmetry is manifested by a superfluid velocity $\mathbf{u}_s$ (or gauge field) generated by the Berry phase of the spin field. In "static" traps, $\mathbf{u}_s$ splits the degeneracy of the harmonic energy levels, breaks the inversion symmetry of the vortex nucleation frequency $\mathbf{\Omega}_{c1}$, and can lead to *vortex ground states*. [S0031-9007(96)01231-8]

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The recent discoveries of Bose-Einstein condensation in atomic gases of ^{87}Rb [1], ^{7}Li [2], and ^{23}Na [3] have achieved a long sought goal in atomic physics. They have also provided condensed matter physicists opportunities to study interacting Bose systems at a wide range of densities. The realizations of these condensates are made possible by the invention of a number of special magnetic traps which trap alkali atoms in $F = 2$ (or $F = 1$) hyperfine spin states, with spins maximally aligned with the local magnetic field $\mathbf{B}$. These hyperfine states are referred to as "adiabatic" spin states.

An immediate question is whether these alkali condensates differ from the familiar ^{4}He condensate in any fundamental way. Unlike the spinless ^{4}He atoms, alkali atoms have nonzero hyperfine spins. The condensate of a (hyperfine) spin-F Bose gas is a spinor field

$$\langle \hat{\psi}_m(\mathbf{x}, t)\rangle = \zeta_m(\mathbf{x}, t)\Phi(\mathbf{x}, t)\,, \tag{1}$$

where $\hat{\psi}_m$ is the field operator, m is a label for F_z $(-F \le m \le F)$, Φ is a scalar, and ζ_μ is a normalized spinor. The fact that condensation takes place among the adiabatic spin states means that ζ is aligned with the magnetic field, i.e., $\hat{\mathbf{B}} \cdot \mathbf{F}\zeta = F\zeta$, where $\mathbf{F}$ is the hyperfine spin operator. With ζ specified by $\mathbf{B}$, the dynamics of $\langle\hat{\psi}_m\rangle$ is completely specified by that of the scalar field Φ, as in ^{4}He. One might then conclude that apart from extrinsic factors like density and external potential, there is no *intrinsic* symmetry difference between ^{4}He and alkali condensates. This is in fact the starting point of all current theories, which model the alkali systems as interacting dilute *spinless* Bose gases in harmonic potentials. Within these models, the effective Hamiltonian for the scalar Φ has a global U(1) gauge symmetry, as in ^{4}He.

The actual symmetry of the spinor field [Eq. (1)], however, is much larger than U(1). We call it *local* spin-gauge symmetry. It represents that a local gauge change $e^{i\chi(\mathbf{x},t)}$ of $\langle\hat{\psi}_m\rangle$ can be undone by a local spin rotation $e^{-i(\chi/F)\hat{\mathbf{B}}(\mathbf{x},t)\cdot\mathbf{F}}$. As we shall see, because of this symmetry, the effective Hamiltonian of the scalar Φ is not that of ^{4}He, but that of a neutral superfluid in a velocity field $\mathbf{u}_s$, or an electron in a vector potential $\mathbf{A}$. The velocity (or gauge field) $\mathbf{u}_s$ arises from the Berry phase of the spin field ζ and is a direct reflection of the underlying spin-gauge symmetry. The purpose of this paper is to discuss various forms of spin-gauge effects.

To begin, we first discuss the effective Hamiltonian. For brevity, we shall refer to hyperfine spins as simply "spins." We shall consider only hyperfine spin $F = 2$, as the treatment of the $F = 1$ case is exactly the same. The Hamiltonians of the alkali systems are of the form $H = H_s + V$, where $H_s = \int d\mathbf{x}\hat{\psi}^+_m(\mathbf{x})[-\frac{\hbar^2}{2M}\nabla^2 - \mu_a\mathbf{B}(\mathbf{x}, t)\cdot\mathbf{F}]_{mn}\hat{\psi}_n(\mathbf{x})$ is the single particle Hamiltonian, M and $\mu_a = -\mu_B/2$ are the mass and magnetic moment of the atom, μ_B is the Bohr magneton, and the factor $1/2$ is the g factor of the alkali atom. $\mathbf{B}$ is a sum of magnetic field configurations which can be static [2,3] or dynamic [1]. V is the two-particle interaction between the atoms. To form a trap, the Zeeman energy $-\mu_a B$ (or its time average) must behave like a potential well. If $\{\zeta^{(n)}\}$ are the spin eigenstates along $\hat{\mathbf{B}}$ ($\hat{\mathbf{B}} \cdot \mathbf{F}\zeta^{(n)} = n\zeta^{(n)}$, $-2 \leq n \leq 2$), then the Zeeman energy $-\mu_a \mathbf{B} \cdot \mathbf{F}$ reduces to $U^{(n)}(\mathbf{x}, t) = \frac{1}{2} n \mu_B B(\mathbf{x}, t)(n)$ for the states $\zeta^{(n)}$. If $\mu_B B$ is an attractive well, $U^{(n)}$ is confining (deconfining) for $n > 0$ ($n \leq 0$) [4]. This means that spin flips between $n > 0$ and $n \leq 0$ states can cause atoms to leave the trap. Since V generally causes spin flips unless both atoms are in the maximum spin state along the same quantization axis (in which case spin flips are prohibited by angular momentum conservation), it depletes all but the adiabatic spin states $\zeta^{(2)}$ in the trap. The resulting system is an interacting Bose gas with spins aligned with the local field $\mathbf{B}(\mathbf{x}, t)$ [4].

To construct a theory for the adiabatic spin states, we expand $\hat{\psi}_m$ in terms of the spin eigenstates $\zeta^{(n)}$, $\hat{\psi}_m(\mathbf{x}, t) = \sum_{n=-2}^{2} \zeta_m^{(n)}(\mathbf{x}, t) \hat{\psi}^{(n)}(\mathbf{x}, t)$. Expressing $\hat{\mathbf{B}} = \hat{\mathbf{z}} \cos\beta + \sin\beta(\hat{\mathbf{x}} \cos\alpha + \hat{\mathbf{y}} \sin\alpha)$, the explicit form is

$$\zeta_m^{(n)} = \langle m|\mathrm{U}|n\rangle, \qquad U = e^{-i\alpha F_z} e^{-i\beta F_y} e^{-i\chi F_z}, \quad (2)$$

where $F_z|n\rangle = n|n\rangle$. χ is arbitrary. It is the gauge degree of freedom of the system, and is usually chosen to make the spinor $\zeta^{(n)}$ single valued. The effective Hamiltonian $\mathcal{H}$ can be obtained by rewriting the equation of motion $i\hbar\partial_t \hat{\psi}_m = [\hat{\psi}_m, H]$ in the form $i\hbar\partial_t \hat{\phi}^{(n)} = [\hat{\phi}^{(n)}, \mathcal{H}]$. One then finds $\mathcal{H} = \mathcal{H}_{\mathrm{ad}} + \mathcal{H}_{\mathrm{nad}} + \mathcal{H}_{\mathrm{etc}}$. $\mathcal{H}_{\mathrm{ad}}$, referred to as the adiabatic Hamiltonian, contains $\hat{\phi}^{(2)}$ only. $\mathcal{H}_{\mathrm{nad}}$ is the spin-flip (or nonadiabatic) Hamiltonian which consists of cross terms between $\hat{\phi}^{(2)}$ and $\hat{\phi}^{(n \neq 2)}$. $\mathcal{H}_{\mathrm{etc}}$ describes the transitions between different $n \neq 2$ states and can be ignored. Denoting $\hat{\phi}^{(2)}$ and $\zeta^{(2)}$ as $\hat{\phi}$ and ζ, respectively, we have

$$\mathcal{H}_{\mathrm{ad}} = \int d\mathbf{x} \hat{\phi}^{+} \left[\frac{1}{2M} \left(\frac{\hbar\nabla}{i} + M\mathbf{u}_s \right)^2 + \mathcal{U} + \mathcal{W} \right] \hat{\phi} + \mathcal{V}, \quad (3)$$

where $\mathcal{U} = U^{(2)} = \mu_B B(\mathbf{x})$, $\mathcal{W} = (\hbar^2/2M)[|\nabla\zeta|^2 + (\zeta^{+}\nabla\zeta)^2] - i\hbar\zeta^{+}\partial_t\zeta$, and $\mathcal{V}$ is the projection of V onto the adiabatic spin states. It is of the form $\mathcal{V} = \int V(\mathbf{x} - \mathbf{y}) \hat{\phi}^{+}(\mathbf{x}) \hat{\phi}^{+}(\mathbf{y}) \hat{\phi}(\mathbf{y}) \hat{\phi}(\mathbf{x})$, where $V(\mathbf{x} - \mathbf{y})$ is a short range potential. The velocity $\mathbf{u}_s$ is defined as

$$M\mathbf{u}_s = (\hbar/i)\zeta^{+}\nabla\zeta \,. \quad (4)$$

Equation (3) describes a Bose fluid in a background velocity field $-\mathbf{u}_s$, or a charge e system in a vector potential $\mathbf{A}$ if $M\mathbf{u}_s \equiv e\mathbf{A}/c$. Under a local spin rotation $\exp[i\hat{\mathbf{B}} \cdot \mathbf{F}\chi(\mathbf{x})]$, $\mathbf{u}_s \rightarrow \mathbf{u}_s + (F\hbar/M)\nabla\chi(\mathbf{x})$, which is equivalent to a local gauge transformation $\hat{\phi} \rightarrow \exp[iF\chi(\mathbf{x})]\hat{\phi}$. This is a reflection of the underlying spin-gauge symmetry of ζ. The integral $\int_C \mathbf{u}_s \cdot d\mathbf{s}$ is the Berry's phase of ζ around a loop C. It can be easily calculated from the vorticity ($\mathbf{\Omega}_s$) of $\mathbf{u}_s$, which satisfies the Mermin-Ho relation [5]

$$\mathbf{\Omega}_s = \frac{1}{2} \nabla \times \mathbf{u}_s = \left(\frac{\hbar}{2M} \right) \epsilon_{\alpha\beta\gamma} \hat{B}_\alpha \nabla \hat{B}_\beta \times \nabla \hat{B}_\gamma \,. \quad (5)$$

Equation (5) shows that the spatial variations of $\mathbf{B}$ necessary to produce the trapping potential will inevitably generate to a nonvanishing superfluid velocity $\mathbf{u}_s$.

From the above derivation, it is clear that even in the absence of particle collisions, the adiabatic spin states are only *metastable*, for the nonadiabatic Hamiltonian $\mathcal{H}_{\mathrm{nad}}$ always induces transitions of adiabatic states to other less confined or completely deconfined spin states. The trapping of atoms therefore requires weak nonadiabatic effects. In the rest of this paper, we shall focus on the phenomena associated with the adiabatic spin fields (described by $\mathcal{H}_{\mathrm{ad}}$ only). We shall discuss nonadiabatic effects elsewhere, but point out here that they can be ignored if the Dirac centers of the magnetic trap are sufficiently far away from the atom cloud. A Dirac center is the point where $\mathbf{B} = 0$ and where the unit vectors $\hat{\mathbf{B}}$ surrounding D wrap around the unit sphere n times (n is a nonzero integer). If D resides in the cloud, the adiabatic spin field around D will develop a line singularity emerging from D (a Dirac string) which will cause a lot of spin flips. The field parameters discussed below are all within the range to keep the Dirac center sufficiently far away from the cloud.

To be concrete, we consider "static traps" of the form ($\nabla \cdot \mathbf{B} = \nabla \times \mathbf{B} = 0$)

$$\mathbf{B}(\mathbf{x}) = B_0\hat{\mathbf{z}} + G_1(x\hat{\mathbf{x}} - y\hat{\mathbf{y}}) + (G_2/2)[(z^2 - r^2/2)\hat{\mathbf{z}} - z\mathbf{r}], \quad (6)$$

where $\mathbf{r} \equiv (x, y)$ and G_1 and G_2 are the first and second order field gradients, respectively. The magnetic trap of the form Eq. (6) is similar to that used in the ^{7}Li experiment [2]. It is convenient to express the field gradients as $G_1 \equiv B_0(\gamma/L)$, $G_2 \equiv B_0/L^2$. The trapping potential $\mathcal{U}$ in Eq. (3) can then be expressed as

$$\mathcal{U} = \hbar\Omega_{\mathrm{Zee}} + \tfrac{1}{2}M(\omega_\perp^2 r^2 + \omega_z^2 z^2) + O|\mathbf{x}/L|^4, \quad (7)$$

where $\hbar\Omega_{\mathrm{Zee}} = \mu_B B_0$, $\omega_z^2 = \mu_B B_0/(ML^2)$, $\omega_z/\omega_\perp = (\gamma^2 - 1/2)^{-1/2} \equiv \lambda$, $\gamma^2 > 1/2$. For later use, we denote the longitudinal and transverse widths of the ground state Gaussian of the harmonic well [Eq. (7)] as a_z and $a_\perp$, where $a_z = (\hbar/M\omega_z)^{1/2}$, $a_\perp = (\hbar/M\omega_\perp)^{1/2}$. Typically $a_z, a_\perp \ll L$. It is straightforward to show that $\mathcal{W} = (\hbar^2/2M)([\sin\beta\nabla\alpha]^2 + [\nabla\beta]^2)$, where α, β are polar angles of $\mathbf{B}$ as defined earlier. This term is smaller than the harmonic potential in $\mathcal{U}$ by a factor $(\gamma a_\perp/L)^4$ and can be ignored in general.

From Eq. (5) it is straightforward to show that [with $\chi = -\alpha$ in Eq. (2)] $\mathbf{u}_s = \frac{2\hbar}{M}(1 - B_z/B) \times \nabla[\tan^{-1}(B_y/B_x)]$, and

$$\mathbf{u}_s = -\frac{\hbar}{M}\left(\frac{\gamma}{L}\right)^2 \hat{\mathbf{z}} \times \mathbf{r} + O(\mathbf{x}^2/L^3),$$
$$\boldsymbol{\Omega}_s = -\hat{\mathbf{z}}\frac{\hbar}{M}\left(\frac{\gamma}{L}\right)^2 + O(|\mathbf{x}|/L^3). \tag{8}$$

Thus for $|\mathbf{x}| < L$, the spin-gauge effect generates a constant effective "rotation" $-\boldsymbol{\Omega}_s$ along $\hat{\mathbf{z}}$.

An immediate consequence of $\boldsymbol{\Omega}_s$ is that it generates a Coriolis force on the alkali system. This force can be detected by applying an ac magnetic field along $\hat{\mathbf{x}}$, $\mathbf{b} = be^{-i\omega t}\hat{\mathbf{x}}$. This field will generate a term $(\mu_B b\gamma/2L)xe^{-i\omega t}$ in the effective Hamiltonian $\mathcal{H}_{\rm ad}$, as if a time dependent force $\mathbf{f} = (\mu_B b\gamma/2L)e^{-i\omega t}\hat{\mathbf{x}}$ is present. It is easy to see that the equation of motion of the center of mass in the x-y plane, $\mathbf{R} = \int \hat{\phi}^+ \mathbf{r}\hat{\phi}$, $\mathbf{r} = (x, y)$, assumes the form

$$M\frac{d^2\mathbf{R}}{dt^2} = -M\omega_\perp^2\mathbf{R} + 2M\frac{d\mathbf{R}}{dt} \times \boldsymbol{\Omega}_s + \mathbf{f}, \tag{9}$$

which has resonances at $\omega = \omega_\perp \pm \Omega_s$ (for $\omega_\perp \gg \Omega_s$). The degenerate clockwise and counterclockwise harmonic modes $\omega_\perp$ are split by the Coriolis force. This splitting exists in both normal and superfluid phases and can be easily shown to be independent of particle interactions.

More pronounced effects can be found in the superfluid phase of alkali atoms with positive scattering length $a > 0$. Because of spin-gauge symmetry, the ground state energy functional becomes

$$\mathcal{E}(\Phi) = \frac{1}{2M}\left|\left(\frac{\hbar\nabla}{i} + M\mathbf{u}_s\right)\Phi\right|^2 + (\mathcal{U} + \mathcal{W})|\Phi|^2 + (2\pi\hbar^2 a/M)|\Phi|^4. \tag{10}$$

When $\mathbf{u}_s$ is small, Eq. (10) can be written as $\mathcal{E}(\Phi, \mathbf{u}_s) = \mathcal{E}(\Phi, \mathbf{0}) - \Omega_s\mathcal{L}_z$ ($\mathcal{L}_z = -i\hbar\Phi^*\hat{\mathbf{z}}\cdot \mathbf{r} \times \nabla\Phi$), which is the Hamiltonian density of a scalar superfluid in a container rotating with frequency $\Omega_s\hat{\mathbf{z}}$. Let Ω_{c1}^0 denote the vortex nucleation frequency in the absence of spin-gauge effect (i.e., $\Omega_s = 0$). Because of the background rotation Ω_s, the actual vortex nucleation frequencies $\Omega_{c1}^\pm$ for vortices with 2π circulation around $\pm\hat{\mathbf{z}}$ will be $\Omega_{c1}^\pm = \Omega_{c1}^0 \mp \Omega_s$. In particular, when $\Omega_s \geq \Omega_{c1}^0$, hence $\Omega_{c1}^+ \leq 0$, vortex ground state will emerge in the absence of external rotation.

The value of Ω_{c1}^0 has been studied for harmonic traps by a number of authors [6,7]. Using the Thomas-Fermi approximation (TFA) [6,8], which is good at large N, Baym and Pethick have shown that Ω_{c1}^0 is reduced by particle interactions from its noninteracting value $\omega_\perp$ as

$$\Omega_{c1}^0/\omega_\perp = Q^{-2}\ln Q^2,$$
$$Q = R_\perp/a_\perp = (15\lambda \mathrm{Na}/a_\perp)^{1/5}, \tag{11}$$

where $R_\perp$ is the transverse width of the condensate. Since $\Omega_{c1}^0 \propto N^{-2/5}$, $\Omega_s \propto N^0$, the inversion asymmetry of the nucleation frequencies, $(\Omega_{c1}^+ - \Omega_{c1}^-)/(\Omega_{c1}^+ + \Omega_{c1}^-) \approx \Omega_s/\Omega_{c1}^0$, increases as $N^{2/5}$. Thus for sufficiently large N the condition of vortex ground state $\Omega_s/\Omega_{c1}^0 \geq 1$ can always be met. From Eqs. (8) and (11), one finds that the ratio Ω_s/Ω_{c1}^0 increases as the externally controllable parameters $N, G_1, B_0^{-1}, \lambda$ increase.

Figure 1 shows the ratio Ω_s/Ω_{c1}^0 [calculated from Eqs. (8) and (11)] as a function of field gradient G_1 for different oblate traps ($\lambda = \omega_z/\omega_\perp > 1$) for a Na condensate with $N = 5 \times 10^6$ particles. The scattering length a of ^{23}Na is +4.9 nm. The field at the center of the trap is set at $B_0 = 1$ G. The reason that we consider oblate traps instead of prolate ones is because spin-gauge effect turns out to be much more prominent in oblate clouds. For the field gradients shown, and for $N = 5 \times 10^6$, TFA [6,8] is valid and Eq. (11) applies. Moreover, it is easily shown that the Dirac centers (X_D, Y_D, Z_D) of the static trap Eq. (6) are located at $(\pm L\sqrt{8\gamma^2 + 2}, 0, 2\gamma L)$ and $(0, \pm L\sqrt{8\gamma^2 + 2}, -2\gamma L)$. For the cases we considered, these centers lie sufficiently far above the oblate cloud that nonadiabatic effects are unimportant. (If R_z denoted the width of the condensate along z, which is related to $R_\perp$ as $R_z = R_\perp/\lambda$ [6], we have $R_\perp = 1.08 \times 10^{-3}$ cm, 2.13×10^{-3} cm, 5.3×10^{-3} cm, and $Z_D/R_z = 2.2, 4.51, 10.2$ for $\lambda = 5, 10, 25$.)

From Fig. 1, one sees that vortex ground states can be reached (i.e., when $\Omega_s/\Omega_{c1}^0 = 1$) by increasing the field gradient G_1 or increasing λ. For $\lambda = 10$ and 25 the "critical" field gradient G_1^* that stabilizes the vortex ground state is 463 and 1151 G/cm, respectively. The corresponding trap frequencies $(\omega_\perp, \omega_z)/(2\pi)$ are (60.6, 1212) and (399, 7980) Hz. Note that even if the field gradient is below the critical value G_1^*, there is a large range of field gradients which has sizable asymmetry in the vortex

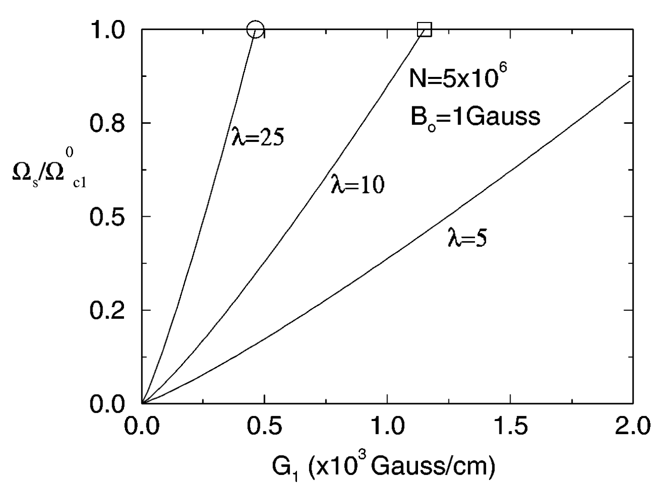

FIG. 1. The increasing spin-gauge effect for increasing field gradients and increasing oblateness of the trap. The square and circle indicate the appearance of vortex ground states, i.e., when $\Omega_s/\Omega_{c1}^0 = 1$.

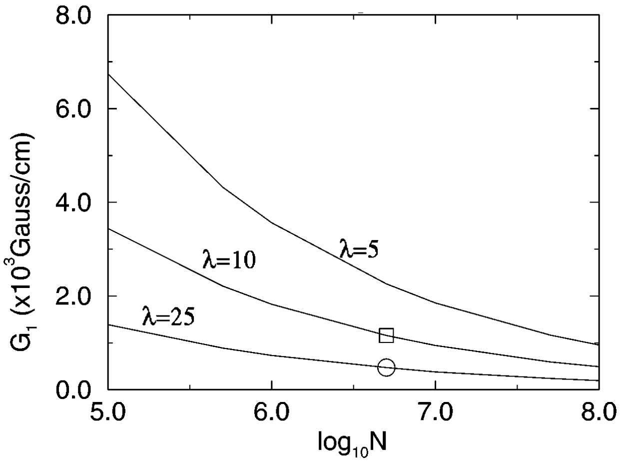

FIG. 2. The phase boundary of separating vortex and nonvortex ground states for different ratios of $\omega_z/\omega_\perp\lambda$. The vortex (and nonvortex) states lie above (below) the phase boundary. The states labeled by circle and square correspond to those of Fig. 1.

nucleation frequencies, manifesting spin-gauge effects. For example, for $\lambda = 10$, the asymmetry $(\Omega_{c1}^+ - \Omega_{c1}^-)/(\Omega_{c1}^+ + \Omega_{c1}^-) \approx \Omega_s/\Omega_{c1}^0$ increases rapidly beyond 20% for $G_1 \geq 400$ G/cm. In Fig. 2 we show the phase boundaries in the G_1-N plane (for different λs) separating the vortex and nonvortex ground states. Regions above and below the boundary (labeled by λ) are vortex and nonvortex ground states, respectively. The negative slopes of these boundaries show that the vortex ground state is easier to achieve for larger condensates. We also note that the ratio $\Omega_s/\omega_\perp$, which describes the amount of energy level splitting in Eq. (9), is of the order of 10^{-2} to 10^{-1} for almost the entire range of field gradients considered in Fig. 1 except for the small range $G_1 < 50$ G/cm. This splitting, though small, is within the limit of detectability.

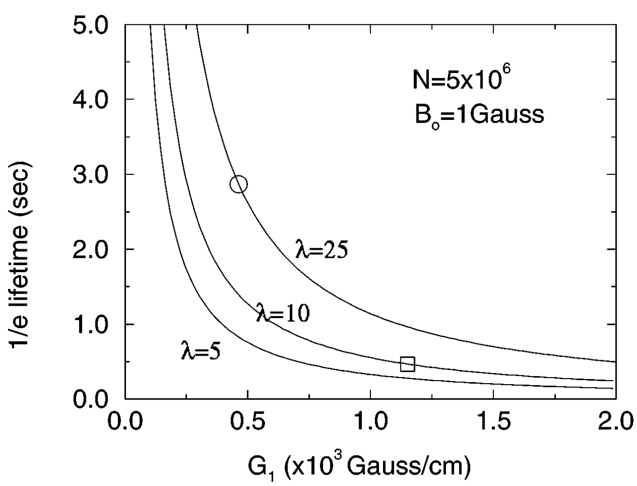

FIG. 3. The increasing $1/e$ lifetime as a function of field gradient for nonvortex ground states for different aspect ratios λ. The actual ground states to the left of the circle and square are in fact vortex ground states, which will have lifetimes longer than the nonvortex states because of the reduction of particle density near the vortex core.

The field gradients considered in Figs. 1 and 2 are typically 10 times those in current experiments. These traps are more confining than the current ones [1–3]. Since tighter traps means higher densities at the center of the atom cloud, it implies more frequent two- and three-body collisions and hence a faster decay of the condensate. The decay rate due to two- and three-body collisions has recently been studied by Edwards *et al.* [9]. It is found to be $dN/dt = \alpha \int |\Phi|^4 + \beta \int |\Phi|^6$, where α and β correspond to two- and three-body collisions. The value of α for both Rb and Na is about 10^{-16} to 10^{-15} cm^3 sec^{-1} [9,10], whereas β is typically of the order of 10^{-30} cm^3 sec^{-1} [9,10]. Before estimating the lifetimes of the vortex ground states, we note that they are bounded below by those of the nonvortex ground states. This is because vortex ground states have a lot fewer particles at the center of the trap than the nonvortex states because of their vortex cores, which reduces the frequency of two-body collisions. Calculating nonvortex ground states Φ in TFA from Eq. (3), it is straightforward to calculate the integrals $\int |\Phi|^4$ and $\int |\Phi|^6$, which are functions of N, G_1, and λ. From these functions one can integrate the equation of dN/dt to obtain the $1/e$ decay time τ. In all cases considered the effect of three-particle collisions is much smaller than that of two-particle collisions and is therefore ignored. Figure 3 shows the lifetime τ of nonvortex ground states as a function of field gradient G_1 for different aspect ratios λ, with $N = 5 \times 10^6$ and $B_0 = 1$ G. It shows that τ decreases with increasing G_1, which implies increasing central density. Figure 3 also shows τ increases as λ increases. This is because increasing λ will flatten the condensate towards a two dimensional disk, which reduces the central density and hence two-body collision effect. Figure 4 shows the lifetime of nonvortex ground

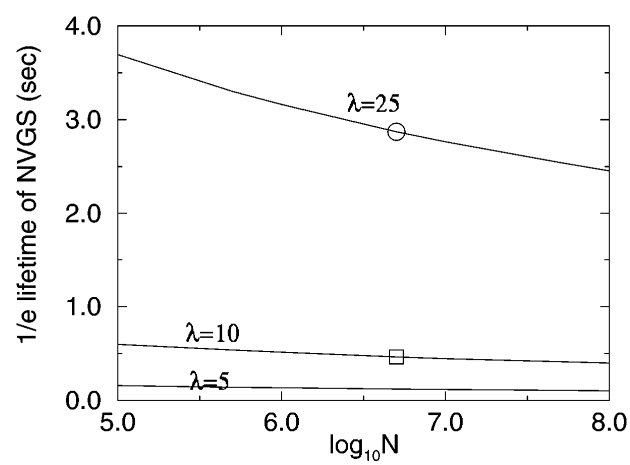

FIG. 4. The $1/e$ lifetime along (but slightly below) the phase boundary of Fig. 2 (so that one remains in the nonvortex regime). The states labeled by circle and square correspond to those of Fig. 1.

states (NVGS) just below the phase boundaries in Fig. 2. For $\lambda = 25$, the lifetime is about 3 to 4 sec. It reduces to about 0.5 sec for $\lambda = 10$. Lifetimes of the vortex ground states, which are longer than the NVGS's shown in Fig. 4, are sufficiently long for measurements to be performed.

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