Spin Diffusion in 2D *XY* Ferromagnet with Dipolar Interaction

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In the ordered phase of 2D *XY* ferromagnet, the dipole force induces strong interaction between spin waves in the long-wave limit. This interaction leads to transformation of the spin-wave excitation into a new soft mode in an intermediate range of wave vectors, limited in magnitude and direction, and into an anomalous anisotropic diffusion mode at long wavelengths. The dissipation of spin waves at short wavelengths is found to be highly anisotropic. [S0031-9007(96)01230-6]

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The conventional condensed matter theory deals with elementary excitations and their interactions. The excitations such as electrons, phonons, spin waves, etc. have a propagating, wavelike nature. The momentum $\mathbf{p}$ and the energy ω of a single excitation are related by the dispersion relation (spectrum) $\omega = \epsilon(\mathbf{p})$. Weak interaction changes slightly the spectrum and leads to a finite lifetime of the excitations [1]. The effect of strong interaction is not so universal. Migdal [2] has shown that strong electron-phonon interaction renormalizes substantially electron velocities and, if it exceeds a critical value, reconstructs the ground state. Recently strong interaction of electrons in two dimensions with gauge fields has been shown to result in non-Fermi-liquid behavior [3].

In the long-wave (hydrodynamic) limit quasiexcitations turn into classical modes, such as sound or spin wave. Not only the propagating waves, but also particle, heat, and spin diffusion can be considered as hydrodynamic modes. The interaction between these modes has been shown to be substantial in the critical region [4], in the dynamics of liquid crystals [5,6], and in the dynamics of charge density wave interacting with impurities [7]. In all these systems interaction leads to a drastic reconstruction of the dispersion relation.

In this Letter we solve an experimentally feasible magnetic model, in which dipolar interaction between spin waves leads to the replacement of the propagating spin wave by a diffusion mode and to the appearance of a new soft mode in a range of momentum. This model is the two-dimensional *XY* ferromagnet with the dipolar interactions between spins.

The spin-diffusion mode appears naturally in the paramagnetic phase and in the vicinity of the Curie point [8]. We consider the low temperature ordered phase, in which no spin diffusion was expected so far, but rather a propagating, weakly dissipating spin-wave mode. Indeed, in 3D ferromagnets the exchange interaction between spin waves vanishes in the long-wave limit [9]. The dipole force generates three spin-wave processes [8,10] and violates the total spin conservation law. This interaction is dominant in the spin-wave dissipation via decay into the two spin waves or via merging with other spin waves. However, this dissipation vanishes in the long-wave limit.

It was shown in Ref. [11] that in the 2D *XY* ferromagnet at low temperatures the dipolar interaction is relevant in the long-wave limit, despite the low density of spin waves. The dipolar force induces an anomalous anisotropic scaling of spin-spin correlations in the ordered phase. In this Letter we find an analogous dynamical scaling for the diffusion mode.

First, we define the model. The dipolar force stabilizes the ferromagnetic long-range order [12], suppressing strong *XY* thermal fluctuation. Therefore, we represent the unit vector field of magnetization $\mathbf{S}$ by the two spin-wave fields—in-plane $\phi(\mathbf{x},t)$ and out-of-plane $\pi(\mathbf{x},t) = S^z$:

$$\mathbf{S} = (-\sqrt{1-\pi^2}\sin\phi;\sqrt{1-\pi^2}\cos\phi;\pi), \qquad (1)$$

where both π and ϕ are small. The field $\pi(t,\mathbf{x})$ is canonically conjugated to the field $\phi(t,\mathbf{x})$.

The quantum action of the 2D *XY* ferromagnet contains three terms: the exchange and the anisotropy energies,

$$A_{EA} = \sum_{\omega\mathbf{p}} (J\mathbf{p}^2|\phi_{\omega\mathbf{p}}|^2 + \lambda|\pi_{\omega\mathbf{p}}|^2)/2,$$

as well as the dipolar force term (see, e.g., [11]),

$$A = A_{EA} + g\sum_{\omega\mathbf{p}} |p_x\phi_{\omega\mathbf{p}} + p_y(\phi^2/2)_{\omega\mathbf{p}}|^2/2|\mathbf{p}|, \qquad (2)$$

with the corresponding couplings being the exchange constant J, the anisotropy λ, and the dipole constant g [13]. The action (2) is written in terms of the Fourier transformed fields $\phi_{\omega\mathbf{p}}$ and $\pi_{\omega\mathbf{p}}$; the abbreviation $(\phi^2/2)_{\mathbf{p}\omega}$ denotes the Fourier transform of $\phi^2(\mathbf{x},t)/2$. The anisotropy is assumed to be weak: $\lambda \ll J$. Therefore the spin $\mathbf{S}$ averaged over scales larger than $\sqrt{J/\lambda}$ turns into the plane. The bare spin-wave spectrum extracted from the quadratic part of the action (2) reads

$$\epsilon^2(\mathbf{p}) = c^2(p^2 + p_0 p_x^2/p), \qquad (3)$$

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where $p = |\mathbf{p}|$, the dipolar wave vector $p_0 = g/J$, and the spin-wave velocity $c = \sqrt{J\lambda}$. In the region $p_0 \ll p \ll \sqrt{\lambda/J}$ spin waves have the linear, acousticlike spectrum. We call this spherical shell of momentum the $\mathcal{A}$ shell.

The nonquadratic part of the action (2) A_{int} contains the three-leg and four-leg vertices. In particular, the three-leg vertex is

$$f(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \sum_{i=1}^{3} p_{ix} p_{iy} / |\mathbf{p}_i| \,. \tag{4}$$

Since the sum of momenta entering the vertex is zero, it depends on two momenta. Further we denote it as $f(\mathbf{p}_1, \mathbf{p}_2)$.

The dynamical equation, equivalently the Landau-Lifshitz spin precession equation, associated with the action (2) reads (we assume $\hbar = 1$)

$$\left(\frac{\omega^2}{\lambda} - \frac{\epsilon^2(\mathbf{p})}{\lambda} + i\,\frac{\omega}{\Gamma_0}\right)\phi_{\omega\mathbf{p}} = \frac{\delta A_{\text{int}}}{\delta\phi_{-\omega-\mathbf{p}}} + \eta_{\omega\mathbf{p}} \,. \tag{5}$$

The thermal noise $\eta_{\omega\mathbf{p}}$ and the bare dissipation coefficient Γ_0^{-1} generate the stochastic dynamics in (5). Their choice reflects a nonconserved nature of the in-plane spin. The noise-noise corelation function is related to the bare dissipation coefficient: $\langle|\eta_{\omega\mathbf{p}}|^2\rangle = 2T/\Gamma_0$. It vanishes at $T = 0$. We do not consider spin-lattice relaxation, and spin waves dissipate only due to the interaction generated by the dipole force. Hence, we set $\Gamma_0^{-1} = +0$, and look for the generated so-called dissipation function $b(\omega, \mathbf{p}) = \lambda\Gamma^{-1}(\omega, \mathbf{p})$, which has the dimension of energy (frequency).

The Green function $G(\omega, \mathbf{p})$ is defined as the linear response of the magnet to an external magnetic field with the frequency ω and the wave vector $\mathbf{p}$. The left hand side of Eq. (5) represents the inverse bare Green function $\lambda^{-1}G_0^{-1}(\omega, \mathbf{p})$.

The self-energy term $\Sigma(\omega, \mathbf{p})$ equals $G_0^{-1}(\omega, \mathbf{p}) - G^{-1}(\omega, \mathbf{p})$ by definition. We notify the real and the imaginary part of the self-energy term as $\Sigma = a^2(\omega, \mathbf{p}) - i\omega b(\omega, \mathbf{p})$. Thus, the Green function reads

$$G^{-1}(\omega, \mathbf{p}) = \omega^2 - \epsilon^2(\mathbf{p}) - a^2(\omega, \mathbf{p}) + i\omega b(\omega, \mathbf{p}) \,. \tag{6}$$

According to the fluctuation-dissipation theorem, the imaginary part of the Green function multiplied by the factor T/ω is the spin-spin correlation function $D(\omega, \mathbf{p})$:

$$D(\omega, \mathbf{p}) = \frac{b(\omega, \mathbf{p})}{[\omega^2 - \epsilon^2(\mathbf{p}) - a^2(\omega, \mathbf{p})]^2 + \omega^2 b^2(\omega, \mathbf{p})} \tag{7}$$

(we refer the factor T to vertices).

The pole of the Green function (6) gives the dispersion of the quasiexcitation. Thus, we have to find the self-energy $\Sigma(\omega, \mathbf{p})$. We assume that the reduced temperature $t = T/4\pi J$ is small whereas $L = \ln(\sqrt{J\lambda}/g)$ is large (the latter means large $\mathcal{A}$ shell). Our theory is valid provided

$$t \ln(\sqrt{J\lambda}/g) = tL \ll 1 \,. \tag{8}$$

We employ the standard, so-called Janssen-De-Dominicis functional method [14] to account for the nonlinear terms in the stochastic Langevin equation (5). This method generates a diagrammatic expansion in powers of the bare vertices, associated with A_{int}. The main contribution to the self-energy is given by one-loop diagrams shown in Figs. 1(a) and 1(b). Under the condition (8), the two-loop correction represented by the diagram in Fig. 1(c) is small. Neglecting the two-loop diagrams (vertex correction) was a principal feature of the Migdal theory of electron-phonon interaction [2], and a major assumption in the so-called mode-coupling method in critical dynamics [8]. Later we prove this assumption for our model. Thus, we write the Dyson equation for our problem as follows:

$$\Sigma(\Omega, \mathbf{q}) = 2g^2\lambda^3 T \int \frac{d^2\mathbf{p}}{(2\pi)^2} \int \frac{d\omega}{2\pi} f^2(\mathbf{p}, \mathbf{q}) \times D(\omega, \mathbf{p}) G(\omega + \Omega, \mathbf{p} + \mathbf{q}) + \Sigma_b(\mathbf{q}) \,, \tag{9}$$

where Σ_b is the real self-energy [Fig. 1(b)]; it does not depend on the external frequency Ω. The functions $b(\omega, \mathbf{p})$ and $a(\omega, \mathbf{p})$ are even in both arguments [1]. The imaginary part of the self-energy is odd in frequency Ω. Hence, the equation for the dissipation function reads

$$b(\Omega, \mathbf{q}) = g^2\lambda^3 T \int \frac{d^2\mathbf{p}}{(2\pi)^2} \int \frac{d\omega}{2\pi} f^2(\mathbf{p}, \mathbf{q}) \times D(\omega, \mathbf{p}) D(\omega + \Omega, \mathbf{p} + \mathbf{q}) \,. \tag{10}$$

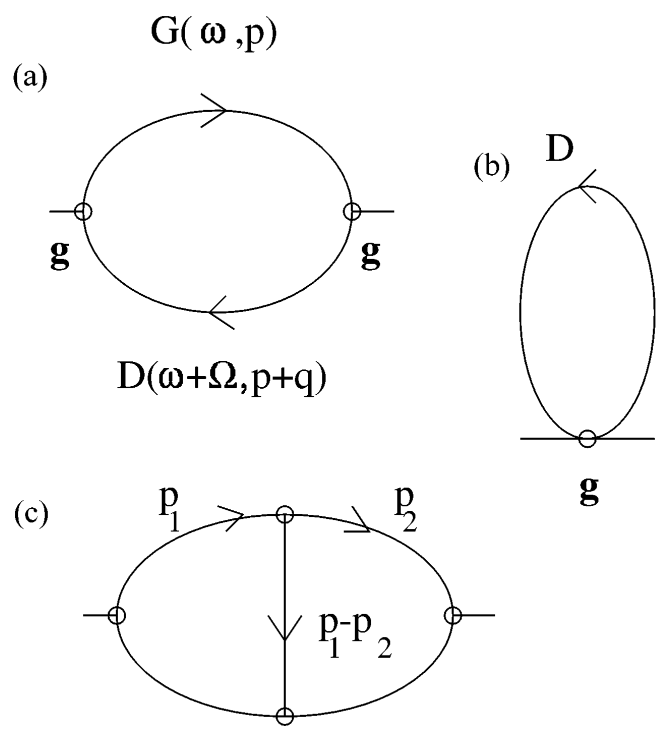

FIG. 1. (a) Main one-loop self-energy diagram. (b) One-loop four-leg vertex diagram. (c) Two-loop self-energy diagram. Momenta of internal lines are indicated.

The integrand in (10) is positive. Thus, the main contribution to $b(\Omega, \mathbf{q})$ comes from the region where poles of the two D functions coincide. The function $D(\omega, \mathbf{p})$ has poles at $\omega \approx \pm\epsilon(\mathbf{p})$, in the $\mathcal{A}$ shell [15]. Following the terminology of the field theory we call the surface $\omega^2 = \epsilon^2(\mathbf{p})$ the mass shell. The dissipation in the $\mathcal{A}$ shell is small and the D function can be represented as a sum of δ functions:

$$D(\omega, \mathbf{p}) \approx \sum_{\pm} \frac{\pi}{2\epsilon^2(\mathbf{p})} \, \delta(\Delta\omega_{\pm}), \tag{11}$$

where $\Delta\omega_{\pm} = \omega \pm \epsilon(\mathbf{p})$ measures the deviation from the mass shell. After integrating ω out from Eq. (10) with the D functions from (11), we recover the Fermi golden rule for the probability of the spin-wave decay and scattering processes.

Looking for the long wavelength quasiexcitations, we need the self-energy at very small momenta $q \ll p_0$ which we denote as Σ_0. We anticipate quasiexcitations to be soft, $\Omega \ll cq$, and restrict the quasiexcitation wave vector $\mathbf{q}$ to be directed along the magnetization: $|q_x| \ll q$. An essential contribution to the integral in Eq. (9) comes from internal momenta p being in the $\mathcal{A}$ shell and the internal frequency $\omega \approx \epsilon(\mathbf{p})$. Integrating over ω with the D function from Eq. (11), we find [15]

$$\Sigma_0 = \frac{c^2 p_0^2 t}{4\pi} \int \frac{c^4 p^3 dp}{\epsilon^4(\mathbf{p})} \frac{\Omega \sin^2(2\psi) \, d\psi}{\Omega - cq\cos\psi + ib_1(\psi)}, \tag{12}$$

where we have omitted an Ω-independent term which contributes a negligible change to the spectrum Eq. (3) [16]. $b_1(\psi)$ is the dissipation function $b(\omega, \mathbf{p})$ of a spin wave inside the $\mathcal{A}$ shell and ψ is the angle between the direction of magnetization and the internal spin-wave wave vector $\mathbf{p}$: $\sin\psi = p_x/p$. Later we prove that b_1 depends on ψ only.

If $cq \gg b_1$, we make the integral over ψ in Eq. (12) to find

$$\Sigma_0(\chi) = c^2 p_0^2 tL \cos^2\chi \exp(-2i\chi), \tag{13}$$

where χ is defined by $\cos\chi = \Omega/cq$. It measures the deviation from the mass shell. If q is so small that $cq \ll b_1$, Eq. (12) implies a q-independent constant dissipation function:

$$b_0 = c^2 p_0^2 tL \int \frac{d\psi}{4\pi} \frac{\sin^2(2\psi)}{b_1(\psi)}. \tag{14}$$

To find b_0, we need $b_1(\psi)$. An unusual feature of our theory is that the dissipation process in the $\mathcal{A}$ shell is mediated by an off mass shell, virtual spin wave. Indeed, Eq. (3) does not allow for decay or merging processes because the function $\epsilon(\mathbf{p})$ is convex upwards in the momentum space. Hence, the dissipation of a spin wave in the $\mathcal{A}$ shell ($q \gg p_0$) propagating in the direction specified with the angle ψ ($\sin\psi = q_x/q$) is mediated by an internal virtual spin wave in (10), with a momentum $p \ll p_0$ and a frequency $\omega < cp$. The integration over ω, with one of the D functions in (10) taken in the form (11), leads to following equation:

$$b_1 = \frac{c^2 p_0^2 t f^2(\mathbf{0}, \mathbf{q})}{8\pi q^2} \int d^2\mathbf{p} D\big(\epsilon(\mathbf{p} + \mathbf{q}) - \epsilon(\mathbf{q}), \mathbf{p}\big). \tag{15}$$

Since $\omega = \epsilon(\mathbf{p} + \mathbf{q}) - \epsilon(\mathbf{q})$, we conclude that $\omega = cp\cos\Phi$, where $\Phi = \psi - \varphi$ is the angle between the vectors $\mathbf{q}$ and $\mathbf{p}$. Invoking the definition of the "mass-shell" angle χ for virtual spin wave, we find that $\chi = \Phi$. Now we look at Eq. (15) in more detail:

$$b_1(\psi) = \frac{c^2 p_0^2 t}{2\pi} \operatorname{Im} \int \frac{\sin^2\psi \cos\psi \, dp \, d\varphi}{p^2 \sin^2\psi + p_0 p \varphi^2 + \Sigma_0(\psi)/c^2}, \tag{16}$$

where $\varphi^2 \sim p/p_0 \ll 1$ and, thus, we have substituted $\chi = \psi$. In other words, the dissipation of a short wavelength spin wave propagating in the direction ψ is determined by scattering on a long virtual spin wave which lies on a specific distance off the mass shell: $\omega/cp = \cos\psi$. The integration over p in (16) is confined towards the crossover region: $p \sim p_c = p_0\sqrt{tL}$.

Substituting $\Sigma_0(\psi)$ from Eq. (13) into Eq. (16), we find the anisotropic dissipation of a spin-wave mode in the $\mathcal{A}$ shell:

$$b_1(\psi) = \beta_1 t^{3/4} c p_0 \frac{\sin^{3/2}(2\psi)\sin(\psi/2)}{L^{1/4}\cos\psi}, \tag{17}$$

where the direction of the spin wave is limited to the fundamental quadrant: $0 < \psi < \pi/2$. We found $\beta_1 = \Gamma^2(1/4)/4\sqrt{2\pi} \approx 1.31$.

Let us return to very low quasiexcitation momenta $q \ll b_1/c$. Plugging Eq. (17) into Eq. (14), one finds

$$b_0 = \beta_0 c p_0 t^{1/4} L^{5/4}, \tag{18}$$

where $\beta_0 \approx 1.24$ was found numerically. The condition $cp_{DM} \sim b_1$ defines the crossover wave vector $p_{DM} \sim p_0 t^{3/4}/L^{1/4}$, separating ranges of validity for Eqs. (13) and (14). The self-energy (13) and the dissipation functions (17) and (18) conclude the self-consistent solution of the Dyson equation (9) and (10).

Next we verify that the two-loop correction [see Fig. 1(c)] is negligible. Note that the main contribution to the diagram [Fig. 1(c)] comes from the two internal momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, restricted to the $\mathcal{A}$ shell. Inside the $\mathcal{A}$ shell the Green and the D functions live on the mass shell. However, the prohibition of the three spin-wave processes confines the arguments of the Green function $G(\omega_1 - \omega_2, \mathbf{p}_1 - \mathbf{p}_2)$ off the mass shell. The Green function off the mass shell is small in temperature. A simple counting shows that the two-loop correction to the dissipation function is $b_0' \sim b_0 t^{1/4}$. The two-loop correction to b_1 is small in $t^{1/4}$, and is also small in the ratio $p_0/q \ll 1$.

Having the explicit expression for the self-energy, we can analyze the dispersion relation $\omega^2 = \epsilon^2(\mathbf{p}) + \Sigma(\omega, \mathbf{p})$ in the range of small ω and p. New results are expected for the region $p < p_c = p_0\sqrt{tL}$ in which Σ_0 becomes comparable with $\epsilon^2(\mathbf{p})$. In a range of momentum $p_{DM} \ll p \ll p_c$ and angles $\psi \ll \sqrt{p_0 tL/p}$ we find a new propagating soft mode with the dispersion

$$\omega = cp(p^2 + p_0 p\psi^2)^{1/2}/p_0\sqrt{tL}\,. \qquad (19)$$

The dissipation of the soft mode grows to the boundary of the region and becomes of the order of its energy at $\psi \sim \sqrt{p_0 tL/p}$ or $p \sim p_{DM}$. There is no soft mode beyond the indicated range. The spin-wave mode persists at $p > p_0\sqrt{tL}$. In the momentum range $p \ll p_{DM}$ we find a new diffusion mode with the dispersion

$$\omega = -it^{-1/4}L^{5/4}\epsilon^2(\mathbf{p})/\beta_0 c p_0\,. \qquad (20)$$

The angular range of the diffusion mode increases with decreasing p and captures the entire circle at $p < p_0 tL$.

At even smaller wavelengths $p < p_A \ll p_{DM}$ the interaction between diffusion modes should be taken into account ($p_A \sim p_0 t^{11/4}$ is the anomalous diffusion setup wave vector [17]). The problem can be solved by the renormalization group method [17]. The growing interaction, although leaving invariant the diffusive nature of the spin propagation, changes the dispersion. For the propagation along the spontaneous magnetization it is $\omega \propto -ip^{47/27}$, whereas for the propagation in the perpendicular direction it is $\omega \propto -ip^{47/36}$. This is a dynamic analog of the non-Gaussian fixed point found in [11].

In conclusion, we discuss the experimental feasibility of the above considered effects in the epitaxial magnetic films. The main difficulty is that all of them are confined to rather long waves. Therefore even weak in-plane anisotropy can suppress them. One needs to use substrates with the sixfold symmetry axis. The sixfold anisotropy is much weaker than the tetragonal one. Moreover, it was predicted in [18] that there exists a temperature interval in which the hexagonal anisotropy vanishes at large distances. A proper substrate is, for example, a (111) face of Cu, Au, etc. In a recent work [19] a growth of an ultrathin Ru/C(1000) film with in-plane magnetization has been reported.

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