Direct Observation of the Current-Phase Relation of an Adjustable Superconducting Point Contact

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By placing a mechanically controllable breakjunction in a superconducting loop the current-phase relation is measured over the complete phase range for atomic-size quantum point contacts with direct conductivity. The current-phase relations show at low temperatures a clear nonsinusoidal behavior with an extremum at phase differences between $\pi/2$ and π. When the self-inductance of the loop is reduced the measured current-phase relations approach the predictions of the theory for (quantum) point contacts in the ballistic limit. [S0031-9007(96)01205-7]

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When a phase difference φ exists between two superconductors, connected by a weak link, a supercurrent will flow through the weak link. The supercurrent I_s through this contact as a function of φ has been the subject of numerous theoretical surveys and a few experimental studies for various kinds of weak links. Originally Josephson [1] showed that for a tunnel junction the current-phase relation (CPR) is purely sinusoidal. Later on Kulik and Omelyanchouk [2,3] calculated CPRs, for dirty as well as for clean classical superconducting point contacts, which are at low temperatures explicitly nonsinusoidal showing a maximum between $\pi/2$ and π (in the phase range between 0 and π). This effect can be attributed to Andreev reflection processes [4] in the microconstriction.

Recently these theories have been extended into the quantum regime, where the width of the contact is comparable to the Fermi wavelength. Beenakker and van Houten [5] discussed the CPR for an adiabatic short and clean quantum point contact, Martin-Rodero *et al.* [6,7] calculated the maximum of the phase-dependent Josephson current that can be sustained by a junction of arbitrary length for different transmission coefficients, and Bagwell [8] derived the CPR of a one-dimensional quantum channel containing a single impurity.

Efforts to confirm part of these predictions by measuring the critical current-normal resistance product I_cR_n in the quantum regime have not (yet) succeeded. This may be due to fundamental physical or to environmental effects [9–11]. In this Letter we present current-phase measurements of the mechanically controllable break junctions (MCB) recently developed by Muller *et al.* [12]. We show the first experimental results on the full CPR of clean ballistic point contacts, measured in the quantum regime, that support the theories. For CPR measurements the MCB is short circuited by a bulk superconductor, thus forming a MCB-SQUID (MCBS) in which the phase difference can be influenced by applying an external magnetic flux Φ_e. Apart from being able to determine the CPR of a MCB a great advantage of the MCBS, over conventional SQUIDs, is the easy and reliable calibration procedure.

The MCBS's we designed are cut out of thin niobium or tantalum foils, using precision laser cutting techniques. The geometry is shown in Fig. 1(a); the enlargement [Fig. 1(b)] shows the future MCB. The enclosed area of the MCBS, indicated by A, determines the self-inductance of the MCBS and can be chosen at our convenience from sample to sample. The small squares in the drawing at either side of it are holes for anchoring the device by glue contacts and have no further physical meaning in this experiment. Once this device is glued onto a bending beam (Fig. 2) the constriction can be broken into a MCB in liquid helium by bending, guaranteeing a clean metallic point contact with a high mechanical stability and a variable contact size, as is standard practice now for MCB measurements [10,13]. A flux-detecting coil (Fig. 2) is placed directly on top of the enclosed area to ensure sufficient inductive coupling with the MCBS. Two gold wires are attached to the device enabling the application of an external current (I_a) to the ring for determination

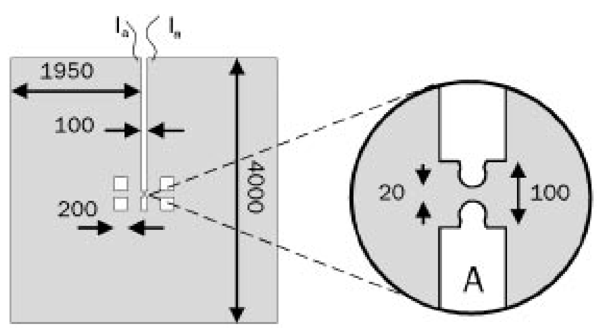

FIG. 1. The geometry of the MCBS as it is cut out of the 50 μm thick superconducting foil (the shaded area indicates the superconductor), the dimensions are given in μm. The two pairs of 200 × 200 μm holes are for anchoring the MCBS by glue contacts. The tiny hole in the middle of the device is the enclosed area. The enlargement shows the weakest spot of the MCBS which can be broken into a break junction and, below it, the top part of the enclosed area indicated by the letter A.

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of the self-inductance [Fig. 1(a)]. An external magnetic field is applied by a second coil. This setup is enclosed by two lead-on-copper cans, shielding it from external electromagnetic noise [10]. The wiring is filtered by low-pass and copper-powder filters [14] as it enters the cans. The measurements of the CPR are performed inductively. We determine an averaged total flux $\langle\Phi_t\rangle$ embraced by the MCBS, as an external magnetic flux Φ_e is applied to it. The averaging is due to intrinsic (thermal) fluctuations. The difference $\langle\Phi_t\rangle - \Phi_e = \langle\Phi_s\rangle$ is the mean value of the self-induced flux of the MCBS, and the mean phase difference $\langle\varphi\rangle$ over the MCB equals $-2\pi\langle\Phi_t\rangle/\Phi_0$ where $\Phi_0 = h/2e$ is the flux quantum. The self-induced flux is generated by the phase-dependent supercurrent as it flows through the MCBS with self-inductance L,

$$\langle\Phi_s\rangle = L\langle I_s\rangle . \quad (1)$$

To determine $\langle\Phi_s\rangle$ a commercial SQUID magnetometer, providing a voltage proportional to the flux in the detecting coil, is used. With simple electronic means, called the compensator, this signal can be transformed into a compensated signal S_c proportional to the flux $\langle\Phi_s\rangle$ induced by the MCBS by subtracting the linear part of the signal. The compensator is calibrated by applying an external flux and comparing S_c, in the case where the MCBS is open and no current flows, to S_c when the MCBS is firmly closed to a bulk superconducting ring, where a persistent shielding current cancels the applied flux. S_c suddenly increases as the MCBS is opened, allowing us to monitor the breaking moment of the junction. An external current I_a (Fig. 1) entering the sample just at either side of the junction gives rise to an extra amount of flux in the MCBS, which offsets Φ_e by $(L - M)I_a = L^* I_a$, where M is the mutual inductance between the current leads and the MCBS. We expect M to be significantly smaller than L since L^* reproduces even when the I_a wiring geometry is altered drastically (for different samples with identical A), and approximate L by L^*. Having calibrated the system we can apply a known Φ_e and detect $\langle\Phi_t\rangle$, giving the self-induced flux $L\langle I_s\rangle = \langle\Phi_t\rangle - \Phi_e$ and the phase difference $\langle\varphi\rangle = 2\pi\langle\Phi_t\rangle/\Phi_0$. The averaged CPR can now be determined. By adjusting the contact size many different CPRs, with different amplitude for a fixed L, are recorded. The contact sizes, established where the normal resistance is in the kΩ range (i.e., the quantum regime), are the smallest possible ones; attempts to obtain still smaller contacts result in a jump to the vacuum tunneling regime where the amplitude of the CPR becomes too small to be detected. An example of the measured $\langle\Phi_t\rangle$ vs Φ_e relation is shown in reduced form in Fig. 2(a) for niobium at 1.3 K (which is much lower than the critical temperature of 9.2 K for niobium). Figure 2(b) shows the $\langle\Phi_s\rangle/\Phi_0$ vs $\langle\varphi\rangle$ relation. It will be clear from Fig. 2(a) that $\langle\Phi_s\rangle$, which equals $\langle LI_s\rangle$ should not exceed some critical value to prevent the basic Φ_t vs Φ_e relation from becoming multivalued, in order to be able to determine the CPR over the complete phase range. To fulfil this condition we choose the self-inductances of our samples small, and establish MCBs with a small conductance. Figure 2(b) also shows two other CPRs, for different contact sizes, established in the same MCBS with extremes in the supercurrent at approximately the same $\langle\varphi\rangle \equiv \langle\varphi\rangle_{\max}$, as is indicated by the arrow in the figure. The most striking feature in this figure is that $\langle\varphi\rangle_{\max}$ is between $\pi/2$ and π. Such behavior was predicted by Beenakker and van Houten [5] for a ballistic quantum point contact in the intrinsic fluctuation free case,

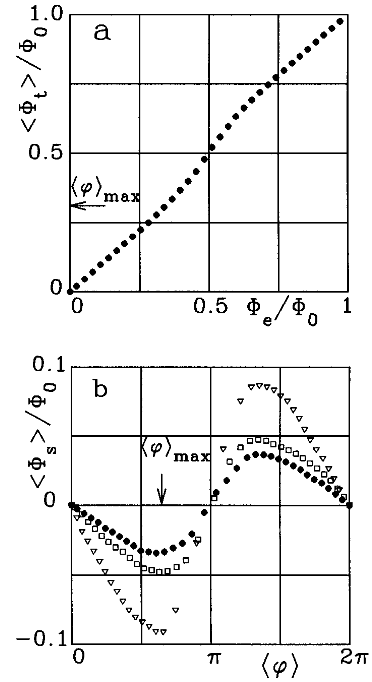

FIG. 2. From the relation between the total embraced flux $\langle\Phi_t\rangle$ and the applied flux Φ_e (a), the self-induced flux $\langle\Phi_s\rangle$ can be determined as a function of $\langle\varphi\rangle$, as is shown in (b) (filled circles) where also two other CPRs, measured on different contacts of the same niobium MCBS (at 1.3 K), are shown. The arrow indicates the $\langle\varphi\rangle_{\max}$ value, which is the mean value of the extremes in the self-induced flux for all the measured CPRs ($L^* = 0.16$ nH, $T = 1.3$ K, $T_c = 9.2$ K).

$$I_s(\varphi) = \frac{Ne\Delta_0(T)}{\hbar} \sin\left(\frac{\varphi}{2}\right) \tanh\left[\frac{\Delta_0(T)}{2kT} \cos\left(\frac{\varphi}{2}\right)\right], \quad (2)$$

where I_s is the supercurrent, φ the phase difference over the junction, $\Delta_0(T)$ the energy gap at both sides of the junction, and N the number of quantum channels. Equation (2) was originally obtained in the quasiclassical limit with a contact diameter much larger than the de Broglie wavelength of the conductance electrons by Kulik and Omelyanchouk [3]. In this description the same CPR takes place, in which $N2e^2/h$ is substituted for the Sharvin conductance $1/R_0$ of a normal ballistic point contact. Comparing the curves in Fig. 2 to Eq. (2) yields values of $1 \lesssim N \lesssim 5$. The absence of clear quantization can be explained in terms of the transmission parameter D, being (slightly) smaller than unity [15]. Low temperature experiments show that (in particular, for d metals) conductance of atomic-size point contacts is not quantized, not even for a single atom point contact [15,16]. For the conductance of atomic-size point contacts the Landauer expression [17] applies, $1/R_n = (2e^2/h)\sum_{n=1}^{N} D_n$, with D_n the transmission probability of the nth conductance channel, and N the total number of conductance channels available in the contact. Quantization of this conductance, and consequently of the supercurrent flowing through the contact, can only be observed for transmission parameters being either 1 (for all $n \leq N$) or 0 ($n > N$). When transmission parameters $0 < D_n < 1$ occur for separate conductance channels, the total conductance value of the point contact can take any value between quantized values.

In Fig. 3 the experimentally found $\langle\varphi\rangle_{\max}$ values for different contact sizes are shown for different fixed self-inductances at 1.3 and 4.2 K. The prediction for the noise free case, indicated by the arrow, exceeds the experimentally found values. Yet, one notices that with decreasing L^*, the $\langle\varphi\rangle_{\max}$ values increase. This was explained [18] by calculating the influence of intrinsic thermal fluctuations for the ballistic case [3]. The potential energy of the SQUID system is

$$U(\Phi_t,\Phi_e) = \frac{(\Phi_t - \Phi_e)^2}{2L} + U_J\left(\frac{2\pi\Phi_t}{\Phi_0}\right), \qquad (3)$$

where the Josephson coupling energy U_J actually corresponds to the CPR by means of $I_s = (2e/\hbar)\partial U_J(\varphi)/\partial\varphi$. If the critical current is adjusted to be sufficiently small, so that for all values of Φ_e the system is in thermodynamic equilibrium with the helium bath, the observed mean value $\langle\Phi_t\rangle$ as a function of Φ_e is determined by [18,19],

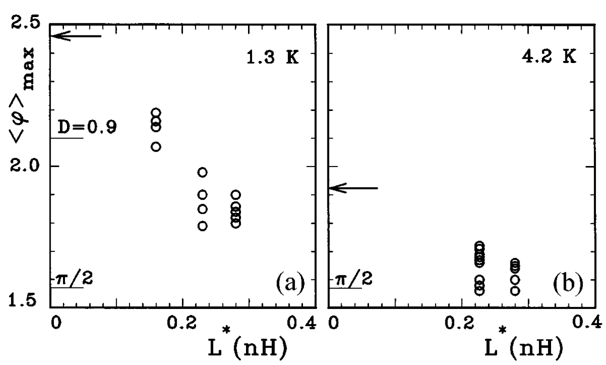

FIG. 3. In (a) and (b) the measured values of $\langle\varphi\rangle_{\max}$ are plotted versus L^* for 1.3 and 4.2 K, respectively. L^* is the experimentally determined value $L - M$ as it is defined in the text, and is 0.16, 0.23, 0.28 nH, respectively. Indicated are the predictions for a point contact with $D = 1$ (arrows) and $D = 0.9$, without fluctuations.

$$\langle\Phi_t\rangle = \frac{\int_{-\infty}^{\infty} \Phi_t \exp[-U(\Phi_t,\Phi_e)/kT]\,d\Phi_t}{\int_{-\infty}^{\infty} \exp[-U(\Phi_t,\Phi_e)/kT]\,d\Phi_t}. \qquad (4)$$

The thermal averaged CPR $\langle I_s\rangle$ vs $\langle\varphi\rangle$ obtained from this expression is changed in comparison with the CPR of Eq. (2). In the calculated thermally averaged CPR curve the maximum in the current, as well as $\langle\varphi\rangle_{\max}$ is reduced. The latter effect is very pronounced since $U(\Phi_t,\Phi_e)$ is strongly asymmetric in the ballistic case. Because of fluctuations, in general, $\langle\Phi_t\rangle$ does not equal the flux value corresponding to the minimum in $U(\Phi_t)$. If L is very small the parabolic term in Eq. (3) dominates and $\langle\Phi_t\rangle$ nearly coincides with the Φ_t value corresponding with the minimum in $U(\Phi_t)$, because the thermal averaging takes place in a nearly symmetric potential. Numerical calculations of $\langle\varphi\rangle_{\max}$ using Eqs. (4) and (2) still exceed the experimental $\langle\varphi\rangle_{\max}$. We believe that this is because the intrinsic noise in our system may be larger than just the thermal noise (e.g., zero point fluctuations [20]) and the transmission D of the junctions is not (always) equal to 1. In case of arbitrary transmission Haberkorn *et al.* [21] obtained the following expression:

$$I_s(\varphi) = \frac{\pi\Delta_0(T)}{2eR_n}\frac{\Delta_0(T)}{\epsilon}\sin(\varphi)\tanh\left(\frac{\epsilon}{2kT}\right)$$
$$\epsilon \equiv \Delta_0\sqrt{1 - D\sin^2(\varphi/2)}, \qquad (5)$$

where the normal resistance $R_n = R_0/D$. This result was extended to the quantum regime by Bagwell [8], who found that the same CPR takes place in which $1/R_0$ becomes $N2e^2/h$. This equation not only describes the intermediate case between tunneling and direct conductivity but also summarizes the well-known results for tunneling ($D \ll 1, \epsilon \approx \Delta_0$) of Josephson [1] and Ambegaokar and Baratoff [22], predicting a purely sinusoidal CPR, and for clean ballistic conductivity [$D = 1, \epsilon \approx \Delta_0\cos(\varphi/2)$] of Kulik and Omelyanchouk [3]. Close to the critical temperature T_c it reduces to the results of Aslamazov and Larkin [23] obtained from the Ginzburg Landau theory, the CPR is sinusoidal irrespective of the transmission. The CPR measurements shown in Fig. 4 were performed on a tantalum sample at 4.2 K, which has a T_c of 4.5 K. These CPRs coincide with the theory when thermal fluctuations are taken into account [19].

Because there is no clear correlation between the amplitude and the $\langle\varphi\rangle_{\max}$ of the different CPRs in Fig. 3, the spread of the points at the fixed values of L^* is attributed to changes in the contact geometry as the contact is adjusted causing small variations in D. It is this transmission that determines the shape of the CPR according to Eq. (5). Also variations in $U_J(\Phi_t)$, as the maximal (critical) current is changed, affect the averaging in $\langle\Phi_t\rangle$ by intrinsic fluctuations [Eq. (4)], but they do not dominate. Despite the spread it is clear from Fig. 3 that at 1.3 K, for the smallest self-inductance, we measured $\langle\varphi\rangle_{\max}$ values exceeding the noise free (!) $\langle\varphi\rangle_{\max}$ for a point contact with $D = 0.9$ as indicated in

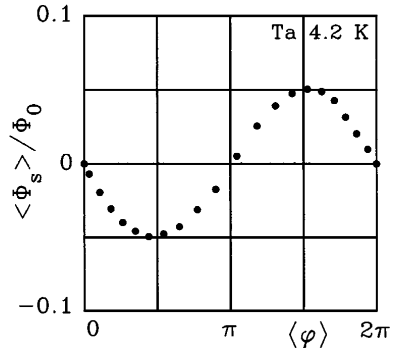

FIG. 4. The sinusoidal CPR of a tantalum point contact (T_c = 4.5 K) measured at 4.2 K. The L^* of this tantalum MCBS is 0.16 nH.

the figure. Since the effect of an enhanced effective noise temperature and a reduced transmission reduce $\langle\varphi\rangle_{\max}$, this sets a lower limit for the transmission of these points at $D = 0.9$. D being smaller than 1 can be attributed to the influence of interface roughness and pointlike defects in the break junction [9].

In conclusion, we present in this Letter a novel experiment which constitutes the first experimental proof for the nonsinusoidal CPR in atomic-size (quantum) point contacts with direct conductivity. Comparison of the measured CPRs to the theories for ballistic quantum point contacts shows that the MCB is a (nearly) ballistic point contact and establishes a lower limit for the transmission coefficient D at 0.9. Furthermore, we have shown how, by reducing the self-inductance of the SQUID, the intrinsic (thermal) noise effect on the supercurrent can be reduced.

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