Ion Transport in Turbulent Edge Plasmas

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Edge plasmas, such as the tokamak scrape-off layer, exist as a consequence of a balance between cross-field diffusion and parallel losses. The former is usually anomalous, and is widely thought to be driven by strong electrostatic turbulence. It is shown that the anomalous diffusion affects the parallel ion transport by giving rise to a new type of thermal force between different ion species. This force is parallel to the magnetic field, but arises entirely because of perpendicular gradients, and could be important for impurity retention in the tokamak divertor. [S0031-9007(96)01236-7]

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Cross-field transport in tokamak edge plasmas is observed to be anomalous, and is generally thought to be driven by electrostatic turbulence. Indeed, direct probe measurements indicate strong fluctuations in the density and the electrostatic potential [1]. The theoretical understanding of the mechanisms driving the turbulence is, however, incomplete. In the modeling of the tokamak edge, especially in numerical computations, *ad hoc* anomalous diffusion coefficients across the magnetic field are therefore invoked, usually in such a way as to match experimentally observed density and temperature profiles. On the other hand, the transport is generally taken to be classical along the field. It is the purpose of the present Letter to reconsider the kinetic theory underlying ion transport in the edge, and to point out the strong coupling that exists between the transport along and across the field. It is found that if the radial transport is anomalous, parallel ion transport cannot in general be entirely classical. This basic conclusion is independent of the details of the anomalous transport; it is a simple consequence of the fundamental property of edge plasmas that parallel fluxes can be driven by radial gradients.

We consider an impure edge plasma with electrostatic turbulence present. For simplicity, we assume that the perpendicular wavelength of the turbulence $2\pi/k_\perp$ is large compared with the ion Larmor radius, $k_\perp \rho_i \ll 1$. Writing the $\mathbf{E}\times\mathbf{B}$ drift velocity $\mathbf{V}_E = \overline{\mathbf{V}}_E + \tilde{\mathbf{V}}_E$ and the distribution function $f_i = \overline{f}_i + \tilde{f}_i$ for each ion species i as sums of average and fluctuating parts, and taking the average $\langle\cdots\rangle$ of the drift kinetic equation over fluctuations gives

$$(\mathbf{v}_\parallel + \mathbf{V}_d + \overline{\mathbf{V}}_E)\cdot\nabla\overline{f}_i + \langle\tilde{\mathbf{V}}_E\cdot\nabla\tilde{f}_i\rangle + eE_\parallel v_\parallel \frac{\partial\overline{f}_i}{\partial\epsilon} = \overline{C}_i\,, \tag{1}$$

where $v_\parallel$ is the parallel velocity, $\epsilon \equiv m_i v^2/2$ the kinetic energy, and $E_\parallel$ the parallel electric field. $\overline{C}_i$ is the collision operator with all other species averaged over fluctuations. The magnetic drift velocity $\mathbf{V}_d$ may be neglected since it is usually smaller than the parallel streaming term, as follows from the estimate

$$\frac{\mathbf{V}_d\cdot\nabla\overline{f}_i}{v_\parallel\nabla_\parallel\overline{f}_i} \sim \frac{\rho_i v_{Ti}/WR}{v_{Ti}/L_\parallel} \sim \frac{\rho_i}{W} \ll 1\,,$$

where R is the major radius, $L_\parallel$ the connection length, $v_{Ti} \equiv (2T_i/m_i)^{1/2}$ a thermal ion velocity, and W the scape-off layer (SOL) width.

The anomalous diffusion generally invoked to account for radial transport comes from the term $\langle\tilde{\mathbf{V}}_E\cdot\nabla\tilde{f}_i\rangle$ in (1). A number of assumptions must be made to justify the diffusive nature of the transport. In a Fokker-Planck expansion of the change in f_i due to the random $\mathbf{E}\times\mathbf{B}$ drift in the radial (r) direction

$$\begin{aligned}\left\langle\frac{\partial f_i}{\partial t}\right\rangle_{\tilde{\mathbf{E}}\times\mathbf{B}} &= \langle\tilde{\mathbf{V}}_E\cdot\nabla\tilde{f}_i\rangle \\ &= -\frac{\partial}{\partial r}\left(\frac{\langle\Delta r\rangle}{\Delta t}f_i\right) \\ &\quad + \frac{1}{2}\frac{\partial^2}{\partial r^2}\left(\frac{\langle(\Delta r)^2\rangle}{\Delta t}f_i\right),\end{aligned} \tag{2}$$

it is first of all assumed that higher-order terms are negligible. This is true if the random displacement of a particle on times longer than the correlation time τ_c is distributed as a Gaussian, which is indeed generally observed in numerical simulations of test particles in strong turbulence [2]. Furthermore, the random step size Δr taken in the time Δt must be small in comparison with the SOL width W. This assumption can be justified in a standard manner for weak turbulence since the correlation time is then short enough. In strong turbulence, particles convected by the $\mathbf{E}\times\mathbf{B}$ drift tend to move many times around convective cells in a correlation time. The step size therefore becomes of the order of the cell width, i.e., $\Delta r \sim k_\perp^{-1}$, and we must require $k_\perp W \gg 1$. Under these assumptions, the influence of the turbulence on the distribution becomes diffusive, and we can write (2) as

$$\langle\tilde{\mathbf{V}}_E\cdot\nabla\tilde{f}_i\rangle = -\frac{\partial}{\partial r}\left(D\frac{\partial\overline{f}_i}{\partial r}\right), \tag{3}$$

where D is the usual diffusion coefficient

$$D \equiv \frac{1}{2} \int_0^{\Delta t} \langle \tilde{V}_{Er}(t) \tilde{V}_{Er}(0) \rangle \, dt \,. \tag{4}$$

This integral is taken along particle orbits, with $\Delta t > \tau_c$. An additional pinch term is easily included by incorporating it in $\overline{\mathbf{V}}_E$ in (2).

Although $\tilde{\mathbf{V}}_E$ is independent of the particle velocity $\mathbf{v}$, the diffusion coefficient D does in general depend on the perpendicular and parallel velocities of the particle since the decorrelation mechanism may be affected by $\mathbf{v}$. However, as seen by the ions, typical edge plasma turbulence is effectively two dimensional since $k_\parallel v_{Ti} \ll \omega$, where ω is the frequency and $k_\parallel \sim L_\parallel^{-1}$ the parallel wave number. This ordering follows either from simple estimates such as $\omega \sim \omega_* = k_\perp v_{Ti} \rho_i / W$ or from complete stability calculations of the dominant modes in the plasma edge [3]. The correlation time may be estimated by $\tau_c \sim 1/\gamma$ since the growth rate γ for edge instabilities is of the order of ω. Thus, we expect that $k_\parallel v_{Ti} \tau_c \ll 1$, implying that decorrelation does not occur as a result of parallel motion. The diffusion coefficient is then independent of $v_\parallel$. Collisions are capable of achieving decorrelation if particles are able to diffuse by classical diffusion from one convective cell to the next in less than a correlation time, i.e., if $\nu \tau_c > (k_\perp \rho_i)^{-2}$. Since this would require a very high collision frequency ν, we conclude that the diffusion coefficient is largely independent of the particle velocity in the small-Larmor-radius limit we are considering. In other words, the decorrelation in (4) is likely to occur as a consequence of the randomness of the velocity field $\tilde{\mathbf{V}}_E$ rather than by other small details in the particle motion.

Thus, although it is not crucial for the general argument, we shall take D to be constant. More importantly, we expect the diffusive term to be of the same order as parallel streaming, i.e.,

$$v_{Ti}/L_\parallel \sim D/W^2, \tag{5}$$

since the Bohm sheath boundary condition forces the plasma to flow at the sound speed to the limiter or divertor plates and this loss must be *balanced* by radial diffusion. Experimentally, this relation is actually used to estimate D from measurements of W [4]. The ordering (5) is different from that usually assumed for the core plasma, where parallel transport is faster than radial diffusion, and results in qualitative changes in the parallel transport, as recently pointed out in the context of diffusion by classical Coulomb collisions [5] and weak turbulence [6]. The drift kinetic equation (1) with (3) for the anomalous diffusion has been solved in simplified geometries at low collisionality in Refs. [7]. Here, we consider the opposite limit of short mean-free path. The collision operator then dominates, and to lowest order the distribution function for each ion species becomes Maxwellian,

$$\overline{f}_{i0} = n_i \left(\frac{m_i}{2\pi T_i} \right)^{3/2} \exp\left(-\frac{m_i v_\perp^2}{2T_i} - \frac{m_i (v_\parallel - V_\parallel)^2}{2T_i} \right),$$

with a common parallel velocity $V_\parallel$, expected to be of the order of the ion thermal speed v_{Ti} at the edge. The temperatures T_i may be different for different species with very disparate masses m_i.

In the next order, we may take f_{i0} to be Maxwellian on the left-hand side of (1), and decompose the correction $\overline{f}_{i1}$ into an odd and an even piece in $u_\parallel \equiv v_\parallel = V_\parallel$, $\overline{f}_{i1} = F_{i1} + G_{i1}$. To obtain the particle and heat fluxes, we need only the odd piece F_{i1}, which obeys

$$\begin{aligned} C(F_{i1}) &= \sum_j [C_{ij}(F_{i1}, f_{j0}) + C_{ij}(f_{i0}, F_{j1})] \\ &= u_\parallel \left[A_{i1} + \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) A_{i2} \right] f_{i0} \,, \end{aligned} \tag{6}$$

where $\mathbf{u} \equiv \mathbf{v} - \mathbf{V}_\parallel$, the sum is taken over all species (including electrons), and

$$\begin{aligned} A_{i1} \equiv \nabla_\parallel \ln (n_i T_i) - \frac{e_i E_\parallel^i}{T} - \frac{m_i}{T_i} \frac{\partial}{\partial r} \left(D \frac{\partial V_\parallel}{\partial r} \right) \\ - \frac{2 m_i D}{T_i} \frac{\partial V_\parallel}{\partial r} \frac{\partial \ln n_i}{\partial r} \,, \end{aligned} \tag{7}$$

$$A_{i2} \equiv \nabla_\parallel \ln T_i - \frac{2 m_i D}{T_i} \frac{\partial V_\parallel}{\partial r} \frac{\partial \ln T_i}{\partial r} \tag{8}$$

are thermodynamic forces. Here we have introduced $E_\parallel^i \equiv E_\parallel - (m_i/e_i)(\mathbf{V}_\parallel + \overline{\mathbf{V}}_E) \cdot \nabla V_\parallel$. Equation (6) has the form of a Spitzer problem for classical transport along the magnetic field. In fact, if the diffusion coefficient D is independent of velocity, (6) is mathematically entirely equivalent to the multi-ion Spitzer problem governing classical parallel ion transport, which has been solved previously [8]. The only difference is that the usual thermodynamic forces are modified by anomalous diffusion through the terms proportional to D. These new terms are of the same order as the classical ones because of the edge ordering (5), and may therefore be expected to be important. We may use the mathematical equivalence to the classical problem to write down the inverse transport laws

$$R_i = \sum_j \left(l_{11}^{ij} u_{\parallel j} + \frac{2}{5} l_{12}^{ij} q_{\parallel j} / p_j \right), \tag{9}$$

$$H_i = \sum_j \left(l_{21}^{ij} u_{\parallel j} + \frac{2}{5} l_{22}^{ij} q_{\parallel j} / p_j \right), \tag{10}$$

relating the total parallel friction force R_i acting on the species i

$$R_i \equiv \int \overline{f}_i m u_\parallel \, d^3u = n_i T_i A_{i1} \,,$$

and the heat friction

$$H_i \equiv \int \overline{f}_i m u_\parallel \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) d^3u = (5/2) n_i T_i A_{i2}$$

to the particle and heat fluxes $u_{\|i}$ and $q_{\|i}$ respectively. The transport coefficients $l_{kl}^{ij} = l_{lk}^{ji}$ are identical to the classical ones [8]. By inverting the system of Eqs. (9) and (10), one obtains the particle and heat fluxes as a linear combinations of the thermodynamic forces. Alternatively, by solving for R_i and $q_{\|i}$ in terms of H_i and $u_{\|i}$, transport equations of the type derived by Braginskii [9] and often used in numerical edge computations are obtained. Thus, when written in this latter form, the usual classical parallel transport laws are modified by the replacement

$$n_j \nabla_\| T_j \longrightarrow n_j \nabla_\| T_j - \frac{2 m_j n_j D}{T_j} \frac{\partial V_\|}{\partial r} \frac{\partial T_j}{\partial r},$$

for all j in the expressions for R_i and $q_{\|i}$.

For instance, in a hydrogen plasma with heavy impurities, the classical expression for the force acting on the hydrogen ions is [10]

$$R_i = -C_1 \frac{m_i n_i (u_{\|i} - u_{\|Z})}{\tau_{iZ}} - C_2 n_i \nabla_\| T_i,$$

where $\tau_{iZ} = 3 m_i^{1/2} T_i^{3/2} / 4 (2\pi)^{1/2} n_Z Z^2 e^4 \ln \Lambda$ is the ion-impurity collision time, and C_1 and C_2 are coefficients depending on the impurity strength $\alpha = n_Z Z^2 / n_i$, tabulated in Ref. [10]. The subscript i refers here to the main hydrogenic species, and Z to the impurities. In a turbulent edge plasma, the force thus becomes [6]

$$R_i = -C_1 \frac{m_i n_i (u_{\|i} - u_{\|Z})}{\tau_{iZ}} - C_2 \left(n_i \nabla_\| T_i - \frac{2 m_i n_i D}{T_i} \frac{\partial V_\|}{\partial r} \frac{\partial T_i}{\partial r} \right). \quad (11)$$

More generally, when there are many ion species present, inverting (10) to obtain $u_{\|i}$ and substituting in (9) gives the force acting on each species as

$$R_i = -\sum_j \left[\alpha_{ij} \frac{m_i n_i (u_{\|i} - u_{\|j})}{\tau_{ij}} + \beta_{ij} \left(n_j \nabla_\| T_j - \frac{2 m_j n_j D}{T_j} \frac{\partial V_\|}{\partial r} \frac{\partial T_j}{\partial r} \right) \right], \quad (12)$$

where the coefficients α_{ij} and β_{ij} are complicated functions of the masses, densities, and charges of all species. Explicit formulas exist in the literature, as well as prescriptions for how to evaluate them numerically [11]. They do not change as a consequence of the edge ordering (5), although the force itself is modified.

A new and unconventional type of thermal force [6] has appeared as the third term in (11) and (12). It arises in a way similar to the usual thermal force (the second term, proportional to $\nabla_\| T$), i.e., as a consequence of an asymmetry in $u_\|$ in the distribution function of the lighter species [in (11) the hydrogenic ions (i)]. For the usual thermal force, the asymmetry arises since the particles traveling in the direction of $\nabla_\| T_i$ are colder than the ones moving in the opposite direction. The former are therefore more collisional and push the heavier particles in the direction of $\nabla_\| T_i$. In an edge plasma with $\partial V_\| / \partial r < 0$ and $\partial T_i / \partial r < 0$, anomalous diffusion transports hot ions with large $V_\|$ outwards in the SOL, where ions with $u_\| < 0$ therefore tend to be colder, and hence more collisional, than the ones with $u_\| > 0$. Thus, again a thermal force arises.

It may be of practical interest for impurity retention in tokamak divertors that if $\partial V_\| / \partial r$ and $\partial T_i / \partial r$ have the same sign, the new thermal force opposes the classical one, which otherwise tends to drive impurity ions from the divertor towards the core plasma. In the divertor plasma, the parallel velocity and the temperature usually both have a maximum near the separatrix, falling off towards the outer SOL and towards the private flux region. In this case the product $(\partial V_\| / \partial r)(\partial T_i / \partial r)$ is positive, and the anomalous thermal force pushes impurities towards the divertor plates. To assess its effect more accurately requires numerical simulation of the fluid equations in realistic geometry.

The usual derivation of these equations by the Chapman-Enskog or Grad expansions does not permit perpendicular gradients large enough to satisfy the edge ordering (5), at least not if the cross-field diffusion is classical [5]. This is why the anomalous thermal force was derived kinetically here. We have not been able to recover it by taking the appropriate turbulent average of Braginskii's equations [9]. Because of the apparently different closure scheme and subtleties in the orderings, it is not clear whether it can be obtained in this manner at all. The anomalous thermal force appears to be a kinetic effect.

In conclusion, we have demonstrated that in a turbulent edge plasma, where the ordering (5) is expected to hold for ions, the anomalous radial diffusion affects the parallel transport by modifying the thermal force between different ion species. The parallel transport is thus not entirely classical as usually assumed, but is affected by a new type of thermal force driven by radial gradients. It is actually a basic property of edge plasmas that transport does not occur within each flux tube separately since radial diffusion feeds in plasma from the core. There can be parallel flow driven entirely by radial diffusion, without parallel gradients. This circumstance is apparent from the fact that the thermodynamic forces (7) and (8) contain terms with only radial gradients.

To make these phenomena explicit, we have adopted the simplest possible turbulence model, with an ion particle diffusion coefficient independent of velocity (as may be expected in the idealized limit of small Larmor radius and long parallel wavelength). While this is clearly a simplification, it is important to realize that the appearance of the new thermal force is independent of the mechanism of radial transport. What matters is that the term $\langle \tilde{\mathbf{V}}_E \cdot \nabla \tilde{f}_i \rangle$ describing anomalous diffusion is allowed to compete with parallel streaming $\mathbf{v}_\| \cdot \nabla \overline{f}_i$ in Eq. (1), which is to be expected in an edge plasma by the ordering (5). In the simple turbulence model adopted, the coefficients of the new thermal force are identical to the classical ones, making its inclusion in existing numerical modeling of the plasma edge relatively straightforward. In a more elaborate description of the anomalous transport, the force between different ion species becomes more complicated. Its precise form depends on the much debated details of the anomalous transport.

Finally, we point out that the appearance of the new type of thermal force is not limited to tokamaks, but could be of importance in any edge plasma existing as a consequence of the balance (5) between diffusion and parallel flow.

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