Temperature Dependence of the Coupling of Nucleons to the Nuclear Surface

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The single-particle self-energy is calculated in semi-infinite nuclear matter with use of an effective interaction of strength determined self-consistently. The resulting damping width of a particle at a Fermi energy displays a linear dependence with temperature as a result of the coupling to surface excitations, while the effective mass shows an exponential decay. The results are in overall agreement with detailed calculations carried in finite nuclei and with experimental findings. [S0031-9007(96)00527-3]

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Mean field theory is one of the most useful approximations in nuclear physics. The many-body effects come in via the single-particle potential which is generated by the nucleons themselves. This potential constitutes the basis of the shell model theory of the nucleus, where nucleons feel the presence of the other nucleons through the confining nuclear surface. This quantity is highly dynamic. In fact, it responds collectively to external fields in terms of regular vibrational patterns of different polarity.

The nuclear surface can also be excited by a bound nucleon bouncing inelastically off the surface. For a nucleon at the Fermi surface of a nucleus in its ground state, such a process does not conserve energy. Consequently, it is possible only as a virtual excitation, where the surface vibration is reabsorbed after a finite time by, for example, the same nucleon. Such a dressing process, depicted in Fig. 1(a), leads to a quasiparticle which still has an infinite lifetime but which displays a mass different from the bare nucleon mass (cf., e.g., Ref. [1]). When the nucleus is internally excited, that is, at finite temperature, the dressing process described above provides also a finite lifetime to the quasiparticle [2–5]. Temperature can change both the rate of the collision as well as the elasticity of the nuclear surface and thus the properties of the quasiparticles. A central question in the study of nuclear structure at finite temperature is how the properties of quasiparticles, arising from the coupling of nucleons to small amplitude vibrations of the constrained mean field, change with temperature T.

In the present paper we aim at giving a simple yet realistic answer to this question. For this purpose, the nucleon self-energy is studied as a function of temperature in the slab model of Esbensen and Bertsch [6], where the nucleus is essentially all surface and confines nuclear matter into a semi-infinite region. It will be concluded that the damping width of single-particle motion at the Fermi energy depends linearly on temperature, while the effective mass decreases exponentially with T.

The nucleons of the semi-infinite nuclear matter are confined in the half-space $z < 0$ by the potential barrier

$$V(z) = V_0(1 + e^{-z/a})^{-1}, \tag{1}$$

with $a = 0.75$ fm and $V_0 = 45$ MeV. Single-particle wave functions are thus plane waves in the direction parallel to the surface [(x, y) plane], and are to be calculated numerically in the z direction as eigenstates of the single-particle Hamiltonian

$$H_0 = -\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V(z), \tag{2}$$

where the sum of the nuclear kinetic energy term and of the potential energy are defined in Eq. (1).

The nucleons interact through a two-body separable interaction, which is the product of single-particle fields peaked at the surface

$$u(z, z', K) = k(K)V'(z)V'(z'), \tag{3}$$

where $V'(z)$ is the derivative of the potential in Eq. (1). The coupling constant,

$$k(K) = k_0[1 + (a_r K)^2]^{-1/2}, \tag{4}$$

is given by the Fourier transform of a Yukawa interaction of range $a_r = 1$ fm. The quantity K is relative momentum of the interacting nucleons along the slab surface. Under the assumption of small amplitude quantal fluctuations of the constrained mean field, the quantity k_0 is determined by the equation

$$k_0^{-1} = -\int dz \rho_0'(z)V'(z), \tag{5}$$

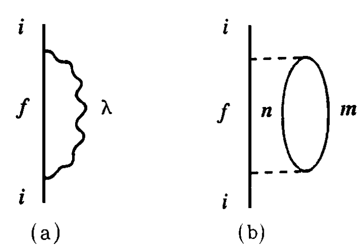

FIG. 1. Lowest order self-energy process due to (a) the coupling of the single-particle state i to a surface vibration λ and (b) the coupling to an uncorrelated particle-hole excitation.

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obtained from the self-consistent relation existing between density and potential variations associated with the normal mode of the system. These modes have been calculated in the thermal random phase approximation (RPA).

The single-particle self-energy associated with the coupling of nucleons to surface vibrations is described, to lowest order perturbation theory, by the process depicted in Fig. 1(a). It was calculated making use of Matsubara's formalism of thermal Green's functions (cf., e.g., [7]). The imaginary part of the retarded self-energy operator is

$$\begin{aligned}\mathrm{Im}\Sigma_i^{\mathrm{ret}}(\epsilon,T) = & -\frac{1}{2}\int_0^{\infty} dK\,K \int_{-\infty}^{+\infty} d\omega \\ & \times R_i(\omega,K)\,(1 - e^{-\beta(\epsilon-\epsilon_i-\omega)}) \\ & \times [1 + n_B(\epsilon - \epsilon_i - \omega) - n_F(\epsilon_i + \omega)] \\ & \times S_T^{\mathrm{RPA}}(\epsilon - \epsilon_i - \omega, K)\,, \end{aligned} \tag{6}$$

where

$$S_T^{\mathrm{RPA}}(\epsilon,K) = -\frac{1}{\pi}\,\mathrm{Im}\,\frac{\Pi_T^0(\epsilon,K)}{1 - k(K)\Pi_T^0(\epsilon,K)} \tag{7}$$

is the thermal RPA strength function, $\Pi_T^0(\epsilon,K)$ being the thermal particle-hole propagator while

$$R_i(\omega,K) = \int d\epsilon_f |\langle f|\hat{F}|i\rangle|^2 \delta(\omega - \epsilon_{fi})\,. \tag{8}$$

The real part of $\Sigma_i^{\mathrm{ret}}(\epsilon)$ can be calculated from the knowledge of the $\mathrm{Im}\Sigma$ by means of the Kramers-Krönig dispersion relation, as the Hillbert transform of the imaginary part of the self-energy,

$$\mathrm{Re}\Sigma_i^{\mathrm{ret}}(\epsilon) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} d\epsilon' \,\frac{\mathrm{Im}\Sigma_i^{\mathrm{ret}}(\epsilon')}{\epsilon - \epsilon'}\,, \tag{9}$$

where P stands for the principal part of the integral. Making use of Eqs. (6) and (9), one can calculate the strength function associated with the dressed state *i*,

$$S_i(\epsilon) = \frac{1}{\pi}\,\frac{\mathrm{Im}\Sigma_i^{\mathrm{ref}}(\epsilon)}{[\epsilon_i - \epsilon - \mathrm{Re}\Sigma_i^{\mathrm{ref}}(\epsilon)]^2 + [\mathrm{Im}\Sigma_i^{\mathrm{ref}}(\epsilon)]^2}\,. \tag{10}$$

The functions $\mathrm{Im}\Sigma_i^{\mathrm{ref}}(\epsilon,T)$ and $\mathrm{Re}\Sigma_i^{\mathrm{ref}}(\epsilon,T)$ for a particle at the Fermi energy ($\epsilon_i = \epsilon_F,\ i = F$) are shown in Figs. 2(a) and 2(b), respectively, as a function of $\epsilon - \epsilon_F$ and for five different values of T within the range $0 \le T \le 2.5$ MeV. It is worth noting that above $T = 3$ MeV the number of thermally excited surface modes is so high that it is likely one has to go beyond lowest order perturbation theory in the calculation of the single particle self-energy. Following the discussion of Ref. [8], a numerical comparison of the results of the slab model and those of finite nuclei requires a scaling of the squared matrix elements appearing in Eq. (8). In the case of ^{208}Pb this scaling factor is 0.1. This is the reason why the results displayed in Figs. 2(a) and 2(b) have been multiplied by this scaling factor.

The quantity $\mathrm{Im}\Sigma_i^{\mathrm{ret}}$ displays a rather smooth behavior with *T*, which can be approximately parametrized by a linear function [inset to Fig. 2(a)]. This result can be understood analytically making use of the fact that, as

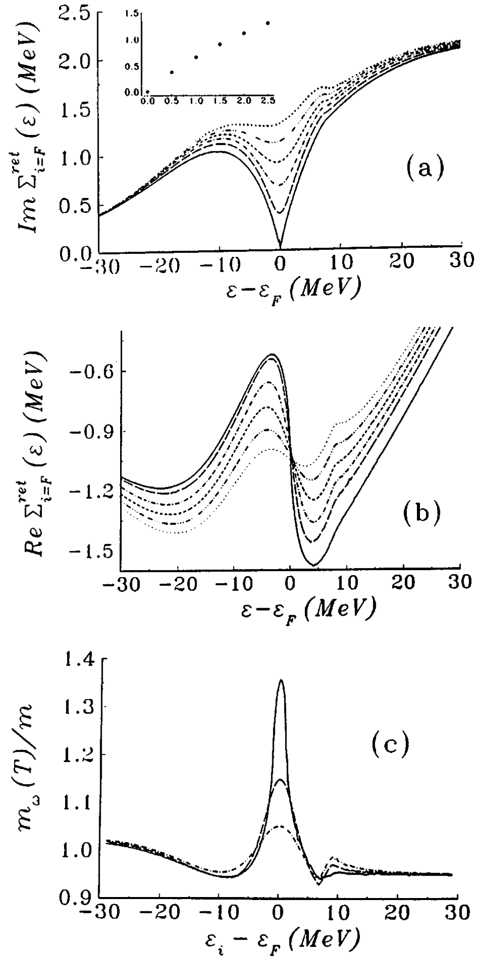

FIG. 2. Imaginary (a) and real (b) parts of $\Sigma_i^{\mathrm{ret}}(\omega,T)$ for a single-particle state at the Fermi energy. The results shown in the figure have been obtained for temperatures in the range $0 \le T \le 2.5$ MeV in steps of 0.5 MeV. The continuum curve displays the $T = 0$ MeV results while the dotted curve displays those associated with $T = 2.5$ MeV. The curves comprised between these two extremes represent the results at 0.5, 1, 1.5, and 2 MeV, respectively. These quantities have been multiplied by 0.1 to scale the results of the slab model to values typical for finite heavy nuclei (^{208}Pb). In the inset of graph (a), the behavior of $\mathrm{Im}\Sigma_i^{\mathrm{ret}}(\omega = 0,T)$ as a function of T is shown (cf. Table I). The quantity $m_\omega(T)/m$ is shown in (c) as a function of the single-particle energy for the temperatures of 0 MeV (continuum), 1 MeV (dashed), and 2 MeV (dash-dotted).

TABLE I. Numerical values of $\mathrm{Im}\Sigma_{i=F}^{\mathrm{ret}}(\epsilon = \epsilon_F, T)$, $m_\omega(T)/m$, and $\Delta m_\omega(T)/\Delta m_\omega(0)$ are displayed in the range of temperatures between 0 and 2.5 MeV.

T (MeV)	$\mathrm{Im}\Sigma_{i=F}^{\mathrm{ret}}(\varepsilon = \varepsilon_F, T)$ (MeV)	$m_\omega(T)/m$	$\Delta m_\omega(T)/\Delta m_\omega(0)$
0.0	0.00	1.36	1.00
0.5	0.38	1.24	0.67
1.0	0.67	1.15	0.42
1.5	0.90	1.09	0.25
2.0	1.11	1.05	0.14
2.5	1.30	1.03	0.06

shown in Refs. [8,9],

$$S^{\mathrm{RPA}}(K,\omega) \approx \mathrm{Im}\,\frac{1}{ia\omega - \sigma K^2}\,, \tag{11}$$

where σ is the nuclear surface tension. Inserting this expression in Eq. (6) one obtains

$$\mathrm{Im}\Sigma_i^{\mathrm{ret}}(\epsilon_i = \epsilon_F, T) = c(\sigma)\int_0^\infty d\omega(1 - e^{-\beta\omega})[n_B(\omega) + n_F(\epsilon_F + \omega) - n_B(-\omega) - n_F(\epsilon_F - \omega)] \times \int \frac{d^2K}{(2\pi)^2}\,\mathrm{Im}\left(\frac{1}{i\omega - K^2}\right) = c(\sigma)\int_0^\infty d\omega(1 - e^{-\beta\omega})[f(\beta\omega) - f(-\beta\omega)] = c(\sigma)4\ln 2T\,, \tag{12}$$

where the constant $c(\sigma)$ is related to the nuclear surface tension, while $f(x) = (1 - e^x)/(e^x - e^{-x})$. This result is consistent with that found by Esbensen and Bertsch at $T = 0$ (cf. Ref. [8])

$$\mathrm{Im}\Sigma_i^{\mathrm{ret}}(\epsilon_i, T = 0) = c(\sigma)|\epsilon_i - \epsilon_F|\,, \tag{13}$$

in keeping the fact that one can view temperature as the energy available to a particle at the Fermi energy to make transitions. The ratio between the slope of the functions (12) and (13) is $4\ln 2 \approx 2.77$, similar to the value of 2.25 found in the case of ^{208}Pb [3].

The linear T and $\omega(= |\epsilon_i - \epsilon_F|)$ dependence of $\mathrm{Im}\Sigma_i^{\mathrm{ret}}$ are a direct consequence of the presence of a surface displaying collective modes in the spectrum under discussion. In fact, in the dressing process depicted in Fig. 1(a), the intermediate state is described by the product of two Green's functions. Each of them implies summations over intermediate frequencies leading to occupation numbers. Because closed loops implying vertices impose frequency conservation conditions, the resulting expression is linear in the occupation numbers [cf. Eq. (12)]. In the case of an infinite system, the lowest order dressing process of single-particle states corresponds to the process shown in Fig. 1(b). In this case three Green's functions are needed to describe the intermediate state, leading to an expression quadratic in the occupation numbers,

$$\mathrm{Im}\Sigma_i^{\mathrm{ret}}(\omega, T) \approx \int_0^\infty d\epsilon_m n_F(\epsilon_m)\int_0^\infty d\epsilon_n \times [g(\beta\epsilon_{nm}) - g(-\beta\epsilon_{nm})] \approx bT^2. \tag{14}$$

This result was first obtained by Landau in the study of oscillations of Fermi liquids [10] and later derived by Morel and Nozières making use of field theoretical methods [11].

From the knowledge of $\mathrm{Re}\Sigma^{\mathrm{ret}}$ [cf. Fig. 2(b)], the single-particle effective mass, the so called ω mass, can calculated through the relation (cf., e.g., Ref. [1])

$$m_\omega = m\left[1 - \left(\frac{d\mathrm{Re}\Sigma}{d\omega}\right)_{\epsilon F}\right] = m + \Delta m_\omega\,. \tag{15}$$

As seen from Fig. 2(b), the derivative $d\mathrm{Re}\Sigma/d\omega|_{\epsilon_F}$ is negative for all values of T. For $T = 0$, $m_\omega > m$ within an interval of the order of ± 5 MeV around the Fermi energy, in keeping with the fact that to effectively dress the nucleon, the surface modes have to display frequencies which are similar to that of the particle, and that the isoscalar response of the slab is mainly concentrated in the lowest few MeV of excitation energy. At $\epsilon = \epsilon_F$, the quantity m_ω reaches a value of the order of $\approx 1.4m$

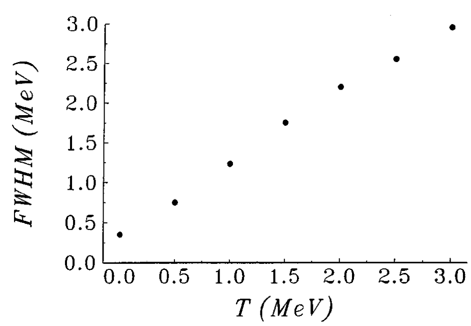

FIG. 3. Full width at half maximum of the strength functions of a single-particle state at the Fermi energy as a function of temperature.

[cf. Fig. 2(c) and Table I]. The quantity $d\mathrm{Re}\Sigma/d\omega|_{\epsilon_F}$ decreases markedly, in absolute value, with temperature, and $\Delta m_\omega(T)$ can be accurately parametrized according to

$$\Delta m_\omega(T) = \Delta m_\omega(0)e^{-T/T_0}, \tag{16}$$

with $T_0 \approx 1.1$ MeV.

Making use of the results displayed in Figs. 2(a) and 2(b), the strength function defined in Eq. (10) was calculated. The full width at half maximum (FWHM) of this function can be viewed as the damping width of single-particle states at the Fermi energy. The results shown in Fig. 3 display a linear behavior with temperature. The finite value at $T = 0$ is connected with the fact that $\mathrm{Re}\Sigma(\epsilon = \epsilon_F, T = 0)$ is different from zero and shifts the single-particle level originally at the Fermi energy away from it, allowing for real transitions. The results displayed in Figs. 2(c) and 3 and in Table I provide an overall account of the experimental findings and are consistent with detailed calculations carried out in finite nuclei (cf., e.g., Refs. [1,12,13]).

We conclude that the damping width of single-particle levels at the Fermi energy arising from the coupling of nucleons to the nuclear surface increases linearly with the temperature of the system. Furthermore, the nucleon effective mass due to the same coupling displays an almost exponential decrease with temperature.

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[1] C. Mahaux, P. F. Bortignon, R. A. Broglia, and C. H. Dasso, Phys. Rep. **120**, 1 (1985).
[2] A. H. Blin *et al.*, Phys. Lett. **161B**, 211 (1985).
[3] P. F. Bortignon *et al.*, Nucl. Phys. **A460**, 149 (1986).
[4] P. F. Bortignon and C. H. Dasso, Phys. Lett. B **189**, 381 (1987).
[5] P. F. Bortignon and R. A. Broglia, *The Response of Nuclei under Extreme Conditions,* edited by R. A. Broglia and G. F. Bertsch (Plenum, New York, 1988), p. 115.
[6] H. Esbensen and G. F. Bertsch, Ann. Phys. (N.Y.) **157**, 255 (1984).
[7] G. D. Mahan, *Many-Particle Physics* (Plenum Press, New York, 1988).
[8] H. Esbensen and G. F. Bertsch, Phys. Rev. Lett. **52**, 2257 (1984).
[9] H. Esbensen and G. F. Bertsch, Phys. Lett. **161B**, 248 (1985).
[10] L. D. Landau, Zh. Eksp. Teor. Fiz. **32**, 59 (1957).
[11] P. Morel and P. Nozières, Phys. Rev. **126**, 1909 (1962).
[12] P. Donati *et al.*,Phys. Rev. Lett. **72**, 2835 (1994).
[13] M. Gonin *et al.*, Nucl. Phys. **A495**, 139c–154c (1989).