Low-Temperature Upper-Critical-Field Anomalies in Clean Superconductors

G. Kotliar*
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855-0849

C. M. Varma†
AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 18 July 1995)

We interpret the upper-critical-field anomalies observed in some high-temperature superconductors as resulting from the proximity to a zero-temperature quantum critical point. We estimate the shape of the phase boundary between the normal and the superconducting phase by modeling the zero-temperature critical point as the second-order end point of the first-order melting line of the vortex lattice. [S0031-9007(96)01126-X]

PACS numbers: 74.60.Ec

In two recent Letters, Mackenzie *et al.* [1] and Osofsky *et al.* [3] observed that in overdoped high-temperature superconductors the upper critical field can be measured at very low temperature and displays a very steep rise as the temperature is decreased [1]. Mackenzie *et al.* studied the single layer system $Tl_2CuO_{6+\delta}$ which is overdoped by incorporating excess oxygen, while Osofsky *et al.* studied Bi_2SrCuO_2.

The positive curvature of the H_{c2} vs T curve and its rapid anomalous low temperature cannot be explained by the classical WHH (Werthamer-Helfand-Hohenberg) theory. In fact, these anomalies have triggered a large number of theoretical interpretations. Schofield and Wheatley suggested that these anomalies are a manifestation result from the Luttinger liquid behavior in the normal state [7]. Alexandrov, Bratkovsky, and Mott [8] account for the anomalous curvature of the H_{c2} vs T curves in terms of bipolaron superconductivity, and Brandow suggested that the negative curvature is the result of pair breaking [6]. Mackenzie *et al.* [2] have suggested that the upper-critical-field anomalies are related to a strong temperature dependence of the effective mass.

In this paper we suggest a very different origin of the upper critical field: the proximity to a zero-temperature critical point. We first argue, on very general grounds, that the observed low-temperature behavior implies the existence of a thermodynamic singularity at zero temperature, which we characterize in terms of two critical exponents. Then we propose a simple model for the relevant combination of exponents controlling the shape of the low-temperature upper-critical-field line which agrees well with the experimental observations.

The microscopic considerations involve the melting of the vortex lattice, a problem which has been a subject of intense study [4]. Most of the work in this area has concentrated on the classical statistical mechanical aspects of this problem with the exception of the recent work by Blatter *et al.* [5], which carried out a microscopic calculation of the quantum and thermal fluctuation of the vortex lattice to one loop order.

Using standard thermodynamic identities, one can relate the slope of the upper-critical-field curve separating the normal and the superconducting phase to the change in entropy and magnetization across the transition line:

$$\frac{S_n - S_s}{M_n - M_s} = \frac{dH_{c2}}{dT}. \qquad (1)$$

If the superconductor-to-metal transition is of the first order, the latent heat and the magnetization jump are finite. If it is second order, the left-hand side of Eq. (1) should be understood as a derivative. At zero temperature $S(T = 0, M) = 0$ in the normal and the superconducting phases. If S is a regular function as M approaches the upper-critical-field line and the temperature tends to zero, then Eq. (1) implies that the slope of the upper-critical-field curve vanishes at zero temperature. In the experiments of Refs. [1,3] the slope of the upper-critical-field curves diverges rather than vanishes as the temperature approaches zero, implying the existence of a singularity in the free energy at zero temperature, namely, a quantum critical point.

Below the upper critical dimension, a second-order transition from an Abrikosov type II superconductor to a normal metal as a function of field is parametrized by two independent exponents ν and z which control the divergence of a length scale and of a time scale as the critical point is approached. The free energy per unit volume has a singular part above and below the superconducting transition which behaves as

$$f = A(H - H_{c2})^{\nu(d+z)}. \qquad (2)$$

At finite temperature scaling implies

$$f(T,H) = A(H - H_{c2}(T = 0))^{\nu(d+z)} \times g(T[H - H_{c2}(T = 0)]^{\nu z}). \qquad (3)$$

 0031-9007/96/77(11)/2296(4)$10.00

The location of the upper critical field vs the temperature line is given by the position in the temperature-field plane where the free energy is singular. If we denote the singular point of the scaling function g by x_c, we obtain a connection between the shape of the upper critical field at low temperatures and the critical exponents:

$$H_{c2}(T) = H_{c2}(T = 0) - x_c T^{1/\nu z}. \tag{4}$$

The steep decrease in H_{c2} observed experimentally is then a measure of the product of the dynamical critical exponent and the correlation length exponent of the zero-temperature critical point. The scaling assumption connects the shape of the critical line to singularities in other physical quantities. We expect the linear term in the normal-state specific heat to have a singular part behaving as $\gamma \approx (H - H_{c2})^{\nu(d-z)}$, while the ac susceptibility acquires a singularity of the form $\chi \approx (H - H_{c2})^{\nu(d+z)-2}$ as we approach the upper critical field at low temperatures.

To estimate the exponent which determines the shape of the upper critical line at low temperatures we need a more detailed picture of the critical point. Presently, there is no microscopic theory of the superconductor-to-metal phase transition in the presence of a magnetic field in three dimensions. To make progress we assume that at any finite temperature the superconductor-to-normal-metal transition is (weakly) first order, an assumption which is supported by some renormalization group calculations [11], and that this first order line ends at zero temperature in a quantum critical point, whose existence is strongly suggested by the experimental data. Then at any finite temperature, we are dealing with the first-order melting transition of the vortex lattice, and the proximity to the second-order zero-temperature critical point is taken into account by using a renormalized parameters in the determination of the melting line.

More precisely, we regard the phase transition at finite temperatures as a (weakly first order) melting of the Abrikosov lattice of an *anisotropic* three-dimensional superconductor. Then we estimate the locus of the transition line by a modified version of the Lindemann criterion [15] that takes into account the zero-temperature critical behavior near $H_{c2}(0)$. Our basic idea is that at any finite temperature we can integrate out the quantum fluctuations to obtain an effective action that describes the finite-temperature transition. After this integration of the quantum fluctuations is carried out the proximity to the quantum critical point is contained solely in two (renormalized by quantum fluctuations) parameters: a renormalized stiffness and a renormalized Lindemann number.

We assume that melting occurs when the root mean square displacement of a vortex $\sqrt{\langle u^2 \rangle}$ due to thermal fluctuations becomes of the order of d_v a quantity related to the distance between the normal regions surrounding nearby vortices. The square of the distance between the centers of the vortices $l(H)$ defined by $(\sqrt{3} l^2/2 = \Phi_0/H)$ is set by the external magnetic field. The area covered by normal region surrounding the core is given by $\sqrt{(3)} l_c^2/2$ with $l_c \equiv l(H_{c2}(T))$. The quantity $d_v^2 \equiv l^2 - l_c^2$ vanishes as H approaches H_{c2}:

$$d_v^2 \approx l^2 \frac{H_{c2}(T) - H}{H_{c2}(T)}. \tag{5}$$

Our modification of the Lindemann criteria is based on the following picture. After time averaging over many oscillations of the vortex cores, each vortex is associated with an area $\sqrt{3} l_c^2/2 + \text{const} \times \langle u^2 \rangle$ of normal phase. Melting occurs when these normal areas cover the whole sample, that is,

$$\sqrt{\langle u^2 \rangle} = \alpha d_v. \tag{6}$$

Here α is a dimensionless constant similar in spirit to the Lindemann number. Based on this analogy we expect it to be smaller than 1.

We stress that this is our main assumption and is of a phenomenological nature. In Eq. (5) $H_{c2}(T)$ is the upper critical field computed without taking into account the thermal fluctuations which are responsible for the melting of the vortex lattice.

The thermal contribution to the root-mean-square displacement $\sqrt{\langle u^2 \rangle}$ is proportional to the temperature and inversely proportional to a typical elastic constant. For the Abrikosov lattice this estimate is complicated by the fact that some elastic moduli are highly nonlocal [9,12] and softer at short wavelengths than at long wavelengths. At zero wave vector the bulk modulus $c_{11}(0)$ and the tilt modulus $c_{44}(0)$ are noncritical, while the elastic shear modulus is given by $c_{66}(0) \approx (H_{c2} - H)^2$. The expected softening of the bulk and tilt moduli does occur at large wave vectors. For $q \gg k_h$ with $k_h \approx (H_{c2} - H)^{1/2}$, $c_{44}(q) \approx (k_h/q)^2 c_{44}(0)$, and $c_{11}(q) \approx (k_h/q)^4 c_{11}(0)$ giving rise to strong infrared divergences in the evaluation of $\sqrt{\langle u^2 \rangle}$. Fortunately, the relevant estimate of the vortex lattice mean displacement taking into account the nonlocality of the bulk moduli has been carried out by Brandt [12] and by Houghton *et al.* [13] in the anisotropic case. They showed that as H approaches H_{c2} the most divergent contribution to the thermal displacement has the form $\langle u^2 \rangle = B_1 l^2 \kappa^2 \{H_{c2}(T)/[H_{c2}(T) - H]\}^{3/2} kT$ with $B_1 = [\Lambda^2/c(0)_{44} c(0)_{66}]^{1/2} (M_z/M)^{1/2} \kappa \frac{1}{4\pi}$. Here M and M_z are the masses in the Landau-Ginzburg Hamiltonian, $c(0)$ denote the elastic constants at $q = 0$, Λ is an ultraviolet cutoff of the order of the vortex lattice zone boundary wave vector, and κ is the ratio of the penetration depth to the coherence length. Inserting the field dependence of the $q = 0$ elastic moduli they obtained [13]

$$\langle u^2 \rangle \approx B l^2 \kappa^2 \left(\frac{M}{M_z}\right)^{1/2} \frac{T}{(\frac{H_{c2} - H}{H_{c2}})^{3/2}}, \tag{7}$$

where B is a constant equal to $2.26 \times 10^{-8} K^{-1}$.

To obtain the melting curve, which we denote $H_m(T)$, we insert Eqs. (5) and (7) into the generalized Lindemann equation (6).

The main difference between our equation and the usual Lindemann criterion which takes d_v to be a noncritical constant of the order of the magnetic length is the critical field dependence of d_v and $\langle u^2 \rangle$.

Our assumption for (5), which in a more general case could involve an arbitrary exponent, is akin to a mean-field approximation incorporating the physics of the proximity to the zero-temperature critical point. Temperature appears explicitly in Eqs. (7) and (5) and also implicitly in $H_{c2}(T)$. This temperature dependence only introduces analytic [proportional to $(T/T_c)^2$] corrections which are negligible compared with the nonanalytic terms that we derive below. We therefore evaluate $H_{c2}(T)$ at zero temperature and obtain an equation for $H_m(T)$ given the value of $H_{c2}(T = 0)$.

$$\frac{H_m(T)}{H_{c2}(0)} = 1 - \left(\frac{T}{T^*}\right)^{2/5}. \tag{8}$$

A plot of (5) together with the data of Mackenzie *et al.* [1] in a range including two decades of reduced temperature is shown in Fig. 1. The theoretical fit used the values $Hc2(0) = 17.4$ T and $T^* = 3.99$ K. We find that this estimate fits the low-temperature portion of the data of Ref. [1] remarkably well. At higher temperatures and lower fields the curves depart from this simple power law behavior, but we do not expect our considerations to apply far from the zero-temperature critical point. At the lowest temperatures measured, there is a hint of a crossover to a power law with a power much closer to 1. More experimental data with smaller error bars in the low-temperature region is needed to determine whether our theory holds in the very low-temperature region or whether there is a relevant perturbation such as disorder which is responsible for another type of critical behavior. The theory has a free parameter which is the value of α in Eq. (5). This is very similar to the parameter c in the Lindemann theory of melting [15]. It determines the characteristic temperature T^* via $T^* \approx 0.44 \times 10^8 \alpha/[\kappa^2 (M_z/M)^{1/2}]$. Using the experimental values for the anisotropy ratio in $Tl_2Ba_2CuO_{6+\delta}$, $M/M_z \approx 31.6$ [14], and a Ginzburg parameter $\kappa \approx 200$ [16], we find that our value of T^* corresponds to $\alpha \approx 0.1$.

The zero point motion gives, of course, a T-independent contribution to the mean-square displacement, which can be added to the right-hand side of the Lindemann criterion. If it is only weakly field dependent, it can be considered as merely a renormalization of the core radius, or effectively a renormalization of H_{c2} at $T = 0$. The one-loop calculation of Blatter *et al.* shows that this is the case [5]. We stress that once quantum corrections are explicitly included in a renormalized value of $H_{c2}(0)$, the elastic moduli have to be recalculated in a consistent fashion so that they vanish when the field equals the *renormalized* value of H_{c2}.

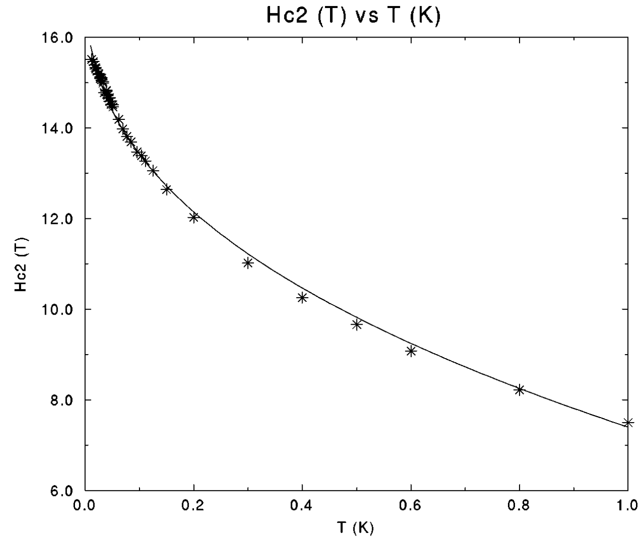

FIG. 1. Upper critical field in tesla vs temperature. The continuous line is given by Eq. (7) with $Hc2(0) = 17.4$ T and $T^* = 3.99$, while the stars are the experimental points from Ref. [1]

Other scenarios, such as the naive application of the ordinary Lindemann criterion, or the theory of Kosterlitz-Thouless two-dimensional melting [10], give an exponent of $2/3$ and do not agree well with the experimental data.

We now turn to the reasons why the upper-critical-field singularity was observed in only Refs. [1,3] and not in other high-temperature superconductors where fluctuations are clearly important. For our considerations to be applicable a sharp phase transition should take place between the superconducting and normal phases without an intermediate wide crossover region, the so-called "vortex liquid regime." In the materials we discuss in this paper sharp resistive transitions take place. The sharpness is due to a combination of the purity of the samples and their strong anisotropy which eliminates the kinetic barriers responsible for the vortex liquid phase. Disorder and the smaller anisotropy of YBCO cause pinning and vortex entanglement which result in a new intermediate asymptotic regime, the vortex liquid, broadening the transition between the superconducting and the normal phases.

Our ideas relating the H_{c2} anomalies to a zero-temperature quantum critical point can be tested experimentally by looking for critical behavior in specific heat and susceptibility measurements.

If the quantum critical point is connected to melting, we expect that controlled addition of impurities to the sample will result in a broadening of the transition.

The considerations in this paper are largely phenomenological and motivated by experiments. To justify these ideas from microscopic considerations, one is led to the problem of the quantum melting of the vortex lattice. If the considerations presented here are correct, quantum effects can turn the weakly first order finite-temperature melting transition into a continuous transition at zero temperature.

We thank G. Blatter, B. I. Halperin, A. Sudbo, and especially A. Schofield for useful discussions. Interactions with D. Vollhardt resulted in an improved understanding of the material and in a more clear presentation. G. K. was supported by NSF Grant No. 95-291382.

*Electronic address: kotliar@physics.rutgers.edu
†Electronic address: cmv@physics.att.com

[1] A. P. Mackenzie *et al.*, Phys. Rev. Lett. **71**, 1238 (1993).
[2] A. P. Mackenzie *et al.*, Phys. Rev. Lett. **76**, 3786 (1996).
[3] M. Osofsky *et al.*, Phys. Rev. Lett. **71**, 2315 (1993).
[4] For recent reviews, see G. Blatter, M. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994); H. Brandt (to be published).
[5] G. Blatter *et al.*, Phys. Rev. B **40**, 13 013 (1994); G. Blatter and B. Ivlev, Phys. Rev. B **50**, 10 272 (1994).
[6] B. H. Brandow, Int. J. Mod. Phys. **8**, 3859 (1994).
[7] A. Schofield, Phys. Rev. B **51**, 11 733 (1995); R. Dias and J. Wheatley, Phys. Rev. B **50**, 13 887 (1994).
[8] S. Alexandrov, Phys. Rev. B **48**, 10 571 (1993).
[9] E. H. Brandt, J. Low Temp. Phys. **24**, 409 (1976); **26**, 735 (1977).
[10] B. Huberman and S. Doniach, Phys. Rev. Lett. **43**, 950 (1979); D. Fisher, Phys. Rev. B **22**, 1190 (1980).
[11] E. Brezin, D. R. Nelson, and A. Thiaville, Phys. Rev. B **31**, 7124 (1985).
[12] E. H. Brandt, Phys. Rev. Lett. **63**, 1106 (1989).
[13] A. Houghton, R. Pelcovits, and A. Sudbo, Phys. Rev. B **40**, 6763 (1989).
[14] T. Manako, Y. Kubo, and Y. Shimakawa, Phys. Rev. B **46**, 11 019 (1992).
[15] F. Lindemann, Z. Phys. **11**, 609 (1910).
[16] A. Carrington, A. P. Mackenzie, D. C. Sinclair, and J. R. Cooper, Phys. Rev. B **49**, 13 243 (1994); J. Wade *et al.*, J. Superconductivity **7**, 261 (1994).