Phase Conjugation of Weak Continuous-Wave Optical Signals

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We demonstrate phase conjugation and aberration correction of femtowatt signals using nearly degenerate four-wave mixing in an atomic vapor. Our theoretical and experimental results are in qualitative agreement and show that the conditions under which the minimum signal can be phase conjugated are similar to the conditions under which the phase-conjugate mirror can be operated near its quantum-noise limit. [S0031-9007(96)00871-X]

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Elementary quantum mechanical analysis shows that any optical amplifier is required to add a minimum amount of noise to the input field during the amplification process. The total added noise imposes a fundamental limit on the level of the minimum signal that can be amplified for a specified value of the signal-to-noise ratio (SNR) of the output field. Two types of optical amplifiers are phase-preserving amplifiers and phase-conjugating amplifiers. Examples of phase-preserving amplifiers include laser amplifiers, Brillouin and Raman amplifiers, and nonlinear optical parametric amplifiers, and their quantum-noise properties have been studied extensively [1–3]. Theoretical studies of the quantum-noise properties of phase-conjugating amplifiers (PCA's) [1,2,4–10] show that under conditions in which the amplification of a phase-conjugating amplifier is equal to the amplification of a phase-preserving amplifier, a PCA typically is noisier than a phase-preserving amplifier [2,6]. PCA's have been shown experimentally to compensate for the effects of dispersion and nonlinearities in the propagation of pulses through optical fibers [11] and to remove aberrations in real time from an optical wave front [12]. In the latter case, the PCA is called a phase-conjugate mirror (PCM).

Measurements of the minimum signal that can be phase-conjugated have been performed using PCMs that are based on Brillouin-enhanced four-wave mixing (BEFWM) and on stimulated Brillouin scattering (SBS). High-reflectivity BEFWM-PCMs have been used to conjugate pulses with energy levels as small as 10^{-11} J/pulse (i.e., 286 μW) with a SNR of 1:1 [9]. With a laser preamplifier inserted at the input of the BEFWM-PCM, Andreev *et al.* [10] performed phase conjugation of signals as weak as 4×10^{-17} J/pulse (i.e., 1 nW) and a SNR of 6:1. These PCMs have been used in projection optical systems [13]. Ridley *et al.* [14] have used SBS-PCMs with a high-gain Brillouin preamplifier to perform phase conjugation of signals as weak as 3×10^{-13} J/pulse (i.e., 12 μW) and a SNR of 10:1.

We report that a phase-conjugate mirror that operates via nearly degenerate four-wave mixing (FWM) in an atomic vapor with continuous-wave fields can conjugate weak signals with power levels as small as several femtowatts with near-unity reflectivity. To our knowledge, these power levels are the lowest that have been achieved for any PCM and demonstrate that PCMs based on resonant nonlinearities are attractive candidates for use in optical signal processing of weak signals. We find that the conditions under which the PCM operates nearest its quantum-noise limit (QNL) are similar to the conditions that permit phase conjugation of signals having the lowest power levels. Our observations also agree qualitatively with the results of our recent quantum theory [15] of phase conjugation in an atomic vapor. The quantum-noise properties of other nonlinear optical processes in atomic vapors have been studied previously [16–18].

The origin of quantum noise in phase conjugation can be illustrated with the following phenomenological analysis. For a PCM, the annihilation operator $\hat{a}_c$ of each conjugate field mode is related to the creation operator $\hat{a}_s^\dagger$ of a corresponding signal field mode via $\hat{a}_c = \sqrt{R_{\rm pc}}\,\hat{a}_s^\dagger + \hat{L}$, where $R_{\rm pc}$ is the phase-conjugate reflectivity and $\hat{L}$ is a Langevin noise operator that obeys the commutation relation $[\hat{L}, \hat{L}^\dagger] = R_{\rm pc} + 1$ and that satisfies the condition $\langle \hat{L} \rangle = 0$. For the case in which phase conjugation is achieved via backward FWM in a lossless Kerr medium, $\hat{L}$ is identified with the amplified vacuum field mode incident on the rear port of the PCM [4]. The expectation value n_c of the photon number in the conjugate field is

$$n_c = \langle \hat{a}_c^\dagger \hat{a}_c \rangle = R_{\rm pc} n_s + R_{\rm pc} + N_n \,, \qquad (1)$$

where $n_s = \langle \hat{a}_s^\dagger \hat{a}_s \rangle$ is the expectation value of the photon number in the signal field and $N_n = \langle \hat{L}^\dagger \hat{L} \rangle$ is the number of excess noise photons produced by the PCM. We define the QNL of an ideal PCM to be to the case in which $N_n = 0$. The value of N_n depends on the physical mechanism that gives rise to phase conjugation. For example, in BEFWM, spontaneous scattering due to thermal phonons gives rise to $N_n \gg 1$, whereas in FWM in an atomic vapor, resonance fluorescence of the

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strongly driven atoms produces the excess noise photons. Equation (1) shows that the total number of noise photons per mode, $R_{pc} + N_n$, is minimized for an ideal PCM. Therefore, N_n determines the level of the weakest signal that can be conjugated for a specified value of the SNR of the conjugate field.

We use a high-reflectivity, wide-bandwidth PCM that operates via nearly degenerate backward FWM in a 2-mm potassium vapor cell using the setup shown in Fig. 1. A frequency-stabilized continuous-wave titanium-sapphire laser is tuned near the potassium D_2 line, and additional details are in Ref. [19]. Acousto-optic modulators produce a signal field with frequency ν_s that is shifted relative to the frequency ν_0 of the pump field by an amount called the signal-pump detuning $\delta\nu = \nu_s - \nu_0$. Figure 2(a) shows R_{pc} as a function of the pump detuning below resonance for three values of the signal-pump detuning. A peak in R_{pc} is observed for all three values of the signal-pump detuning at a pump detuning of -1.6 GHz, which yields the best compromise between the resonant enhancement of the FWM nonlinearity and absorption of the pump, signal, and conjugate waves.

We use optical heterodyne detection to measure the noise properties of the PCM and the minimum signal level that can be conjugated. The conjugate and local-oscillator (LO) fields are combined at an uncoated glass beam splitter and are detected by a high-quantum-efficiency ($\eta \sim 0.76$), fast (350 MHz) photodiode. The power of the LO field after the beam splitter is $P_0 \sim 4$ mW, and its frequency is the same as the frequency of the pump fields. The photocurrent is amplified, and its frequency content is measured with an electronic spectrum analyzer. The predicted power spectral density of the photocurrent $S(f)$ is derived using Eq. (1) and is

$$S(f) = \left[1 + \frac{\eta P_s}{h\nu_s} R_{pc}(\nu_s)\delta(f - \nu_s + \nu_0) + R_{pc}(\nu_0 + f) + R_{pc}(\nu_0 - f) + N_n(\nu_0 + f) + N_n(\nu_0 - f)\right] S_0, \quad (2)$$

where P_s is the signal power, $R_{pc}(\nu_0 \pm f)$ and $N_n(\nu_0 \pm f)$ are the reflectivity and the number of excess noise photons, respectively, at the frequencies $\nu_0 \pm f$, and S_0 is the power spectral density of the shot noise produced by the LO field. The second term in square brackets on the right-hand side (RHS) of Eq. (2) is the contribution of the conjugate field and is equal to the number of photons detected per unit time per unit frequency. The last four terms on the RHS result from the noise photons.

An expression for the minimum signal power $P_s^{\min}$ (for SNR = 1:1) is obtained by integrating Eq. (2) over a bandwidth Δf and is

$$P_s^{\min} = \frac{h\nu_s \Delta f}{\eta R_{pc}(\nu_s)}[1 + R_{pc}(\nu_s) + R_{pc}(2\nu_0 - \nu_s) + N_n(\nu_s) + N_n(2\nu_0 - \nu_s)], \quad (3)$$

where Δf is the resolution bandwidth of the detection system. The ratio of the SNR of a shot-noise-limited signal field to the SNR of the conjugate field is the noise figure $F = P_s^{\min}\eta/(h\nu_s\Delta f)$. The photon noise factor

$$N_{pc}(f) = 1 + \frac{N_n(\nu_0 + f) + N_n(\nu_0 - f)}{R_{pc}(\nu_0 + f) + R_{pc}(\nu_0 - f)} \quad (4)$$

is the ratio of the total number of noise photons produced by the PCM to the number of noise photons produced by an ideal PCM.

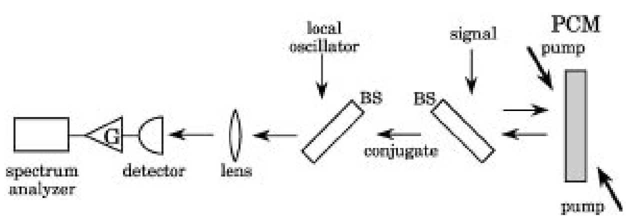

FIG. 1. The experimental setup. BS: beam splitter; PCM: phase-conjugate mirror; PD: photodetector; G: amplifier gain.

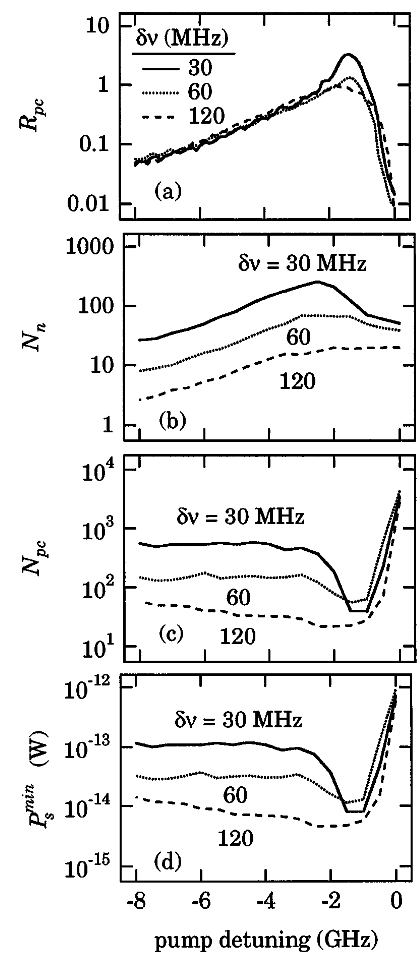

FIG. 2. Measurements of the (a) reflectivity R_{pc}, (b) number N_n of excess noise photons, (c) photon noise factor N_{pc}, and (d) minimum signal power $P_s^{\min}$ as functions of the pump detuning, for three values of the signal-pump detuning $\delta\nu$.

We have measured the dependence of R_{pc} and the noise on the potassium vapor density, signal-pump detuning, and pump detuning below atomic resonance and use these measurements to calculate N_n, N_{pc}, and P_s^{min}. To obtain N_n and N_{pc}, we set $R_{pc}(\nu_0 + f) = R_{pc}(\nu_0 - f)$ and $N_n(\nu_0 + f) = N_n(\nu_0 - f)$ in Eqs. (2) and (4). These assumptions are valid since our theoretical analysis shows that R_{pc} and N_n at each sideband are nearly equal when the frequency $f \leq 120$ MHz is much smaller than the pump detuning (~ -1 to -8 GHz), as in our experiments. In addition, previous measurements in atomic vapors have shown that R_{pc} is symmetric about the frequency of the pump waves under these conditions [20]. Figures 2(b) and 2(c) show N_n and N_{pc}, respectively, as functions of the pump detuning. Inspection of Figs. 2(a) and 2(b) show that N_n is at least several times larger than the number of noise photons expected for an ideal PCM since $N_n \gg R_{pc}$. We find that N_{pc} approaches the QNL of an ideal PCM to within a factor of 20 when the pump frequency is tuned $\sim$1.5 to 2 GHz below resonance.

Figure 2(d) shows the minimum signal power P_s^{min} as a function of the pump detuning for $\Delta f = 300$ Hz. We find that the smallest value of P_s^{min} occurs under conditions in which R_{pc} is maximum. Our results for a signal-pump detuning $\delta\nu = 120$ MHz demonstrate that in principle an atomic-vapor-based FWM-PCM can be used to conjugate a continuous-wave optical signal with a photon-flux spectral density as small as 64 photons/(sec Hz). For detection systems that measure signals in a single sideband, the minimum density can be further reduced by a factor of two. We see from these results that the optimal conditions for performing phase conjugation of the weakest signals are similar to those for which the PCM operates near the QNL.

We use our recently developed theory [15] of phase conjugation by nearly degenerate FWM in a two-level system to model our results. In this model, the pump waves are treated classically, and the signal and conjugate waves are quantized. We use our measurements of the pump transmission through the cell to approximate in the theory the effects of pump absorption. The effects of collisions between the atoms are included, while grating-washout effects due to atomic motion are not included. We find that the theoretical predictions of R_{pc}, N_{pc}, and N_n are in qualitative agreement with the experimental results. Figure 3 is a plot of the minimum signal power P_s^{min} as a function of the pump detuning for three values of the signal-pump detuning. The parameters for the theory correspond to the experimental values for potassium vapor at 212 °C and $\Delta f = 300$ Hz. For the curve in Fig. 3, the ratios of the spontaneous emission rate and the Rabi frequency (associated with each pump field amplitude) to the dipole-dephasing rate $[1/(2\pi T_2) = 10.6$ MHz$]$ are taken to be 0.6 and 153, respectively. The value of the absorption coefficient $\alpha_0 = 1.15 \times 10^4$ cm^{-1} is then chosen to give a good fit to $R_{pc}(\delta\nu = 30$ MHz$)$ as a function of pump detuning. As observed in the experiments, the theoretical

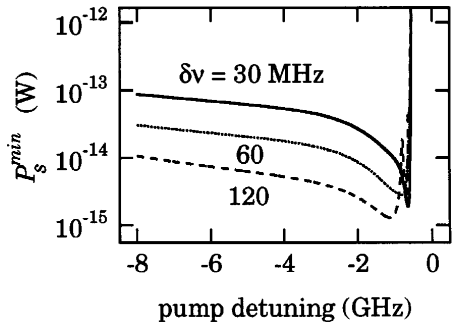

FIG. 3. Theoretical predictions of the minimum signal power P_s^{min} as a function of the pump detuning, for three values of the signal-pump detuning $\delta\nu$.

results show that P_s^{min} reaches a minimum at the pump detuning where R_{pc} is near its maximum, which persists over a wide range of parameters.

We now demonstrate aberration correction of weak optical signals under the conditions in which P_s^{min} reaches its minimum value. For this experiment, the pump frequency is tuned 1.6 GHz below resonance, $\nu_s - \nu_0$ is 109 MHz, and $R_{pc} \sim 90\%$. From our noise measurements, we estimate that $N_n = 18$, $F = 42$, $N_{pc} = 20$, and $P_s^{min} = 6$ fW for $\Delta f = 300$ Hz. We measure F directly using the SNR's of the weak signal and conjugate beams, as discussed below, and this value is within a factor of 2.5 of the predicted value. To measure the fidelity of the phase conjugation process, we insert a spatial filter into the path of the conjugate field after it is combined with the LO field and before it is detected by the photodiode. The spatial filter consists of a 60 μm pinhole placed at the focus between two 50 mm focal-length lenses and is aligned to pass the conjugate beam in the absence of an aberrator. Figures 4(a) and 4(a′) show an image of a strong signal ($P_s = 350$ μW) and the power spectrum of the photocurrent of a highly attenuated ($P_s = 250$ fW) signal, respectively. Figure 4(b) shows the image (taken before the spatial filter) of the conjugate of the strong signal, and Fig. 4(b′) shows the photocurrent power spectrum (taken after the spatial filter; SNR = 18:1) generated by the conjugate of the attenuated signal. To impart spatial aberrations on the signal wave front, we insert an aberrator (an HF-etched microscope glass slide) just in front of the lens that focuses the signal into the cell, and we verify that the conjugate beam retains its optical quality, as shown in Fig. 4(c). Figure 4(c′) shows that the highly attenuated conjugate beam is detected with a SNR of 16:1, which corresponds to $P_s^{min} = 15.6$ fW. To verify that aberrations are removed from the wave front of the attenuated conjugate beam, we translate the pinhole in the spatial filter by one pinhole diameter in a direction transverse to the propagation direction of the conjugate beam and observe that the conjugate beam disappears. As an additional check, we replace the PCM with an ordinary dielectric mirror and retroreflect the signal through the aberrator and lens.

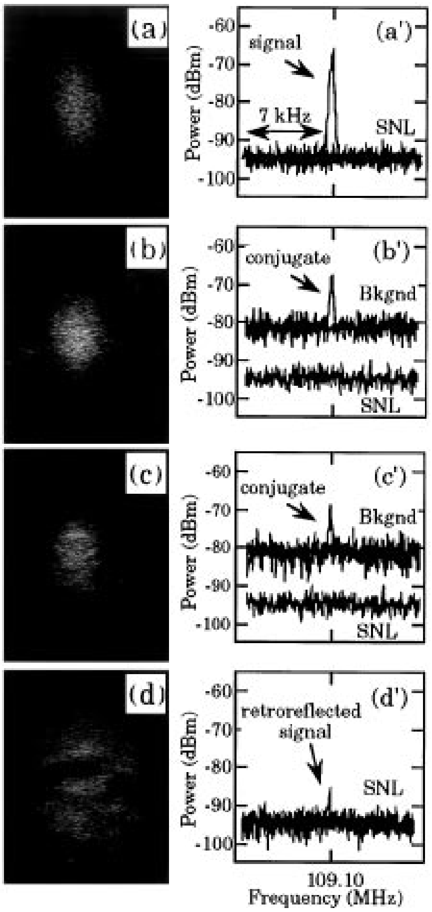

FIG. 4. Demonstration of aberration correction. Images (a)–(d) of strong (350 μW) beams and electronic power spectra (a′)–(d′) of the corresponding highly attenuated (250 fW) beams: (a), (a′) signal; (b), (b′) conjugate; (c), (c′) conjugate (aberrator in); (d), (d′) retroreflected beam (aberrator in). SNL: shot-noise level; Bkgnd: background level. In Figs. 2(b) and 2(c), the light surrounding the conjugate beams is scattered pump light and is at the frequency ν_s of the pump beams.

Figure 4(d) shows that the retroreflected beam is highly distorted after the double-pass through the aberrator and lens, and Fig. 4(d′) shows that only a small fraction of the attenuated retroreflected beam is transmitted through the spatial filter.

In summary, we have demonstrated that a phase-conjugate mirror based on nearly degenerate FWM in an atomic vapor can be used in applications that require aberration correction of weak optical signals with power levels as low as several femtowatts. Furthermore, this system has the advantage that the use of an optical preamplifier is not required to achieve these results.

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