

Self-Organized Collective Displacements of Self-Driven Individuals

Ezequiel V. Albano*

*Instituto de Investigaciones Físicoquímicas Teóricas y Aplicadas (INIFTA), Facultad de Ciencias Exactas,
Universidad Nacional de La Plata, Sucursal 4, Casilla de Correo 16 (1900) La Plata, Argentina*

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An archetype model for the collective displacements of self-driven individuals, aimed to describe the dynamic of flocking behavior among living things, is presented and studied. Processes such as growth, death, survival, self-propagation, and competition are considered. It is shown that systems ruled by the model self-organize into a critical state exhibiting power-law behavior in both the distribution of population avalanches and the spatial correlation between individuals. [S0031-9007(96)01121-0]

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The study of self-organizing systems is a fascinating field of multidisciplinary interest. Early ideas about the emergence of large scale complexity from microscopic local rules have been inspired by research in biology (ecology, ethology, evolution, etc.). However, recently these ideas are spreading in many different fields contributing to the understanding of diverse processes and phenomena in physics, economy, geology, chemistry, sociology, etc. [1].

It is known that a variety of biological objects frequently exhibit a tendency to clustering and migration (herds of quadrupeds, flocks of birds, bacterial growth, etc.). Motivated by this observation, Vicsek *et al.* [2], Cshók *et al.* [3], and Toner *et al.* [4] have very recently presented models aimed to describe the collective motion of living individuals. These models are quite simple but they retain basic facts characteristic of biological objects exhibiting collective behavior: (i) The individuals are self-driven, i.e., transforming energy gained from food into mechanical energy they are able to perform displacements [2,3]. (ii) The motion of an individual is, on the one hand, perturbed by environmental conditions and on the other hand, conditioned by communications among their neighbors [2,3]. These models exhibit standard critical behavior [2–4] because criticality is achieved by tuning external parameters such as the noise and the density of individuals. The aim of this work is to further extend these models in order to account for two relevant facts characteristic of the actual cooperative behavior of the individuals: the first one is to allow the onset of self-organization via communications between neighboring individuals, while the second one is to explicitly consider the dynamic evolution of the population.

Following these statements a model based on two rules is formulated.

Rule 1: The displacements.—All individuals have the same absolute velocity $|\mathbf{v}|$. At each time step all individuals assume the average direction of motion of the neighboring individuals within a range R with some random perturbation added. So, the location of the j th individual is updated according to

$$\mathbf{x}_j(t+1) = \mathbf{x}_j(t) + \mathbf{v}_j \Delta t. \quad (1)$$

The direction of the individual is given by the angle $\theta_j(t+1)$ according to

$$\theta_j(t+1) = \langle \theta_j(t) \rangle_R + \pi Q N_R^{-\alpha}, \quad (2)$$

where the first term of Eq. (2) is the average direction of the velocities of the N_R individuals (including the j th one) within a circle of radii R surrounding the j th individual. The second term introduces a random noise, where Q is a random number in the interval $(-1, 1)$ and α is an exponent. So, according to Eq. (2) clusters of individuals tend to self-organize in the same average direction of movement (first term), but this behavior is randomly perturbed (second term). Furthermore, neighboring individuals may self-organize in order to minimize the noise being α a measure of the strength of such ability. Of course, this rule implies communications between the individuals (e.g., via sensing of chemicals, visual, verbal, etc.). The rule of motion is similar to that adopted by Vicsek *et al.* [2]; however, the second term of Eq. (2) involves major formal and conceptual differences: the noise is no longer an external tunable parameter [2], rather the level of noise, inherent to any biological system, depends on the capacity of the individuals to dampen it (e.g., via communications).

Rule 2: The population dynamics.—A live individual such as $N_R > N3$ will die in the next step (decease by overcrowding). Also, a live individual will die in the next step if $N_R \leq N1$ (decease by isolation). Individuals survive if the neighborhood is not too crowded ($N2 \leq N_R \leq N3$) and birth also occurs if N_R satisfies some stringent constraints ($N1 < N_R < N2$). This rule, inspired in the Game of Life [5], allows the population to self-regulate its density.

The model is simulated in a two dimensional off-lattice cell of linear size L with periodic boundary conditions. Individuals, represented by moving points, are updated simultaneously at each time step (cellular automata updating).

Simulations are made taking $|\mathbf{v}| = 0.03$, $R = 1$, $N1 = 2$, $N2 = 6$, and $N3 = 9$. The convenience of the use

of the first two parameters is due to the same reasons as in [2]. The set of numbers N_i is one of the simplest choices which generate highly complex behavior. The global density (ρ_g) is defined as the number of individuals over the total area of the sample. Also, the local density (ρ_l) is measured within the neighborhood (circle of radii R) of each individual.

Starting with a random distribution of live individuals moving in random directions, in most simulations the initial global density is taken $\rho_g(t = 0) = 2.0$; the system is allowed to evolve until it reaches a stationary state [6]. In this state the system self-organizes in order to keep both the global and the local density constant independent of α ; $\rho_g \cong 1.825(8)$ and $\rho_l \cong 2.511(8)$, respectively, as shown in Fig 1. The observed enhancement of the local density reflects a tendency to clustering (“flocking behavior”). This behavior is mostly due to the dynamics of the population, but it is not a consequence of the operation of an attractive potential as observed in most physical systems. However, the behavior of the clusters (“flocks”) as a whole, depends on α . In fact, using the absolute value of the normalized average velocity $|\langle v \rangle|$ [2] as a measure of the flocking behavior, the crossover between two distinct regimes is observed (Fig. 1): for larger α values (e.g., $\alpha \geq 4$) one has $|\langle v \rangle| \rightarrow 1$; that is, individuals self-organize in a single flock with a well defined direction of migration. However, for $\alpha \rightarrow 0$ also $|\langle v \rangle| \rightarrow 0$; that is, many flocks move in random directions resembling a cloud of mosquitoes (see the snapshots in Fig. 2). This behavior becomes also evident in Fig. 2 which shows the angular correlation in the direction of migration of all individuals [$D(\theta)$], i.e., the probability that the difference in the direction of motion between all pairs of individuals lies between θ and $\theta +$

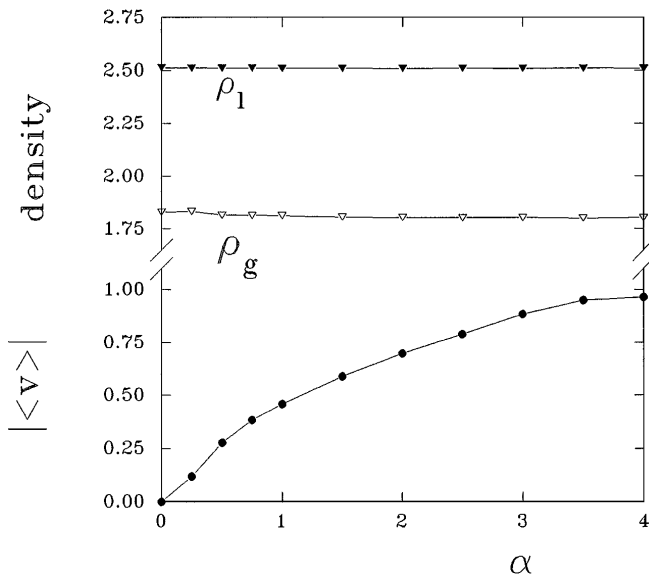


FIG. 1. Plots of ρ_l , ρ_g , and $|\langle v \rangle|$ versus α obtained during the stationary regime in systems of size $L = 40$.

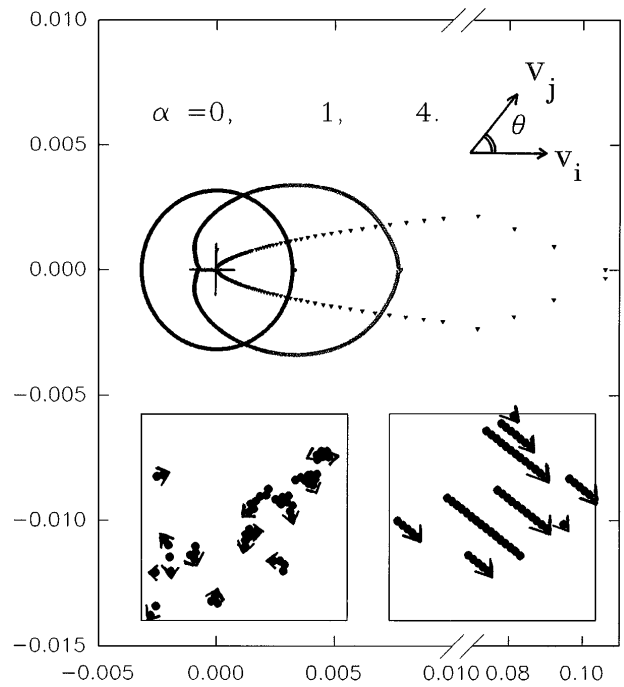


FIG. 2. Polar plot of the angular correlation function of the velocities of the migrating individuals for three values of α . The cross shows the center of the diagram. The snapshot configurations, of small patches (side $l = 1$) of the whole system, show two regimes: $\alpha = 0$, random migration (left) and $\alpha = 4$, single flock behavior (right).

$\Delta\theta$. For $\alpha = 0$ the distribution is isotropic (random migration), while for $\alpha = 4$ a sharp lobule in the direction $\theta = 0$ indicates a preferential direction of displacement (single flock behavior). The onset of a slightly preferential direction of motion can be observed early for $\alpha = 1$. Notice that introducing a periodic variation of α (e.g., due to either daily or seasonal changes), it is possible to describe periodic modifications in the behavior of the flocks. In a more general scope, the evolutionary change of α may be an essential requirement for the successful adjustment to environmental modifications.

A relevant feature of the stationary state is that it exhibits *self-organized-criticality* (SOC) behavior. SOC is a concept proposed [7] to describe the dynamics of a class of nonlinear spatio-temporal systems, which evolve spontaneously toward a critical state (i.e., without having to tune a control parameter). Systems exhibiting SOC have attracted much attention since they might explain part of the abundance of $1/f$ noise, fractal structures, and Lévy distributions in Nature [7] (for examples of systems exhibiting SOC, see also [8]). In order to test for SOC behavior, the stationary state is perturbed by randomly adding a single individual. The evolutionary change triggered by this small perturbation is called an avalanche. The fate of the added individual depends on the environment: some individuals may die while others may succeed to survive and reproduce

generating avalanches of all sizes. So, for a more quantitative evaluation the lifetime of an avalanche t is defined as the time elapsed between the introduction of the perturbation and the extinction of the perturbative individual itself and all its descendants [9]. The size of the avalanche (s) is then computed by counting the number of descendants originated by the perturbative individual during the lifetime of the avalanche [9].

For the stationary state to be SOC the distributions of lifetime [$P(t)$] and size [$P(s)$] must exhibit power-law behavior, i.e., $P(t) \propto t^{-a}$ and $P(s) \propto s^{-b}$ as shown in Figs. 3(a) and 3(b), respectively. The estimates for the exponents are $a \cong 1.7 \pm 0.1$ and $b \cong 1.6 \pm 0.1$, respectively. The propagation of avalanches in all scales is the signature of a critical branching process; see, e.g., [10,11], and references therein. In fact, the fate of the perturbative individual depends on the environment that it encounters upon its introduction in the system. If such an environment is too crowded or too empty the individual may die originating small and short-lived avalanches. If the environment is not adequate for reproduction, the individual may simply survive originating small avalanches covering a broad spectrum of time scales. Another scenario is an environment favorable for reproduction. In this case the individual may have offsprings (“first order” branching) and it and its descendants may become the majority of a flock. Because of the stochastic noise as well as the eventual interaction with neighboring flocks, the perturbation may spread into several flocks (“second order” branching) and the process may continue either triggering subsequent activity or dying out. Because of the error bars the difference between the exponents a and b has to be taken with caution. If $a = b$, one has $t \propto s$ as in simple branching processes [10]. However, due to the highly complex non-linear branching process involved in the present model, one expects $a \neq b$ and the scaling relationship $t \propto s^x$, with $x = (b - 1)/(a - 1)$, holds. Measurements [see inset of Fig. 3(a)] give $x = 0.82 \pm 0.1$ in agreement with the scaling value obtained from the exponents a and b , i.e., $x \cong 0.86$.

Finite size effects have also been investigated. Figure 3(b) also shows plots of $P(s)$ vs s for lattices of different sizes ($10 \leq L \leq 30$ [12]). The biggest avalanches that can be monitored are of the order of $s_{\max} \propto \rho_g L^2$ ($t_{\max} \propto s_{\max}^x$), as it follows from the cutoff shown in Fig. 3(b), because in this limit all descendants of the perturbative individual have had the chance to spread over the whole system replacing *completely* the original population which becomes extinct. This phenomenon of *population replacement* naturally introduces the maximum time useful for monitoring avalanches. Typically we used $t = 2000$ time steps for the sake of computer resources. Because of this constraint few large avalanches have to be discarded even if population displacement is not achieved; e.g., for $L = 15$, population replacement has a probability of $\sim 1\%$ while only $\sim 0.6\%$ of the avalanches are discarded.

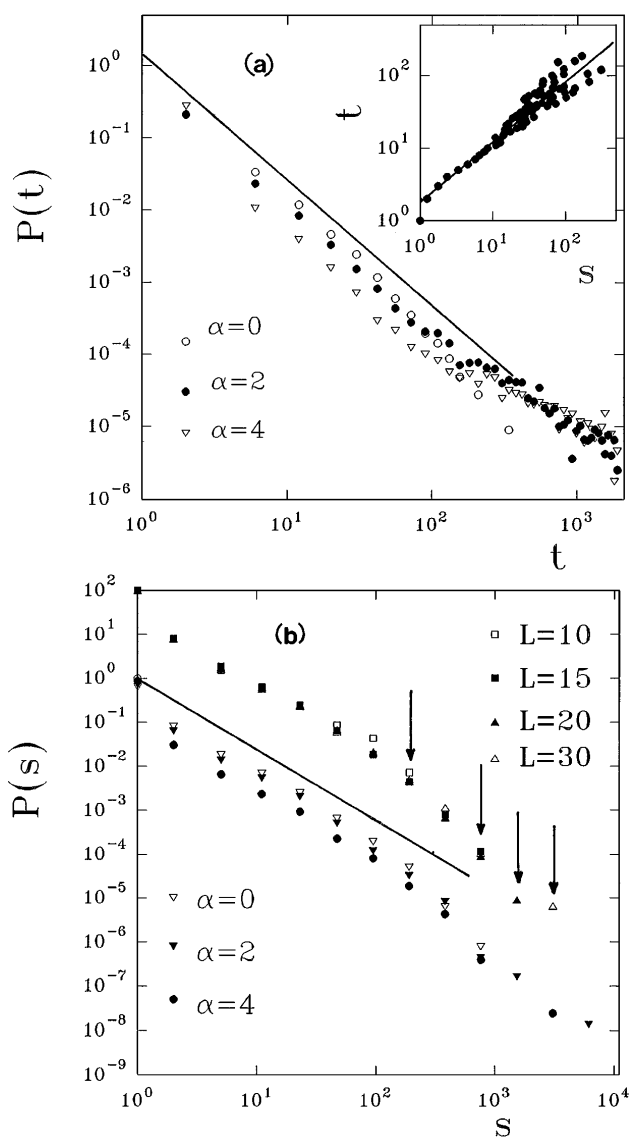


FIG. 3. Log-log plots of the distribution of lifetime [$P(t)$] (a) and size [$P(s)$] (b) of the avalanches within the SOC regime. $L = 30$, results averaged over 2×10^4 avalanches. The lines with slope $a = 1.7$ and $b = 1.6$, respectively, are plotted for reference, and they describe the behavior of the data for intermediate values of t and s . The inset of (a) shows a plot of t vs s with slope $x = 0.82 \pm 0.1$. The upper part in (b) (shifted up 2 orders of magnitude) shows plots of $P(s)$ taken for $\alpha = 4$ and different lattice sizes; for $L \leq 20$ the results are averaged over 10^5 avalanches. The arrows indicate the cutoff due to finite size effects.

Another signature of the critical behavior spontaneously achieved by the system is the development of long-range spatial correlations between individuals. These correlations are independent of α as it is shown in Fig. 4. Within the interaction radii ($r < R$) the correlation function $G(r)$ decays algebraically [$G(r) \propto r^{-c}$] with exponent $c \cong 0.72 \pm 0.02$. For $r \sim R$, $G(r)$ becomes enhanced most likely due to the operation of *rule 2* (population dynamics) which prevents overcrowding for $r \leq R$ while

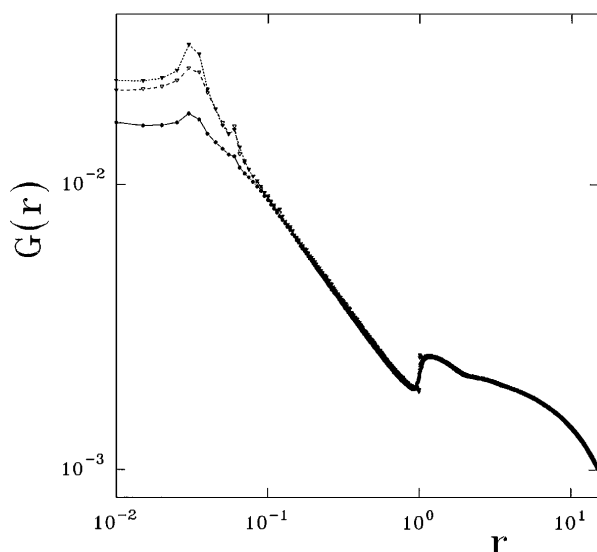


FIG. 4. Log-log plots of the spatial correlation function $G(r)$ vs r , within the SOC regime, measured for different values of α in systems of side $L = 40$. \bullet : $\alpha = 0$, ∇ : $\alpha = 2$ and \blacktriangledown : $\alpha = 4$.

this stringent condition is abruptly relaxed just outside the interaction radii. For larger r values $G(r)$ recovers an algebraic decay but the slope cannot accurately be determined due to finite size effects.

It has to be stressed that, while the flocking behavior of the individuals strongly depends on α (i.e., single flock behavior for large α and random displacements of individuals for $\alpha \rightarrow 0$), all power laws describing the distribution of avalanches and spatial correlations are independent of α , pointing out that the model exhibits robust critical behavior which is achieved spontaneously without the necessity of tuning any external parameter, i.e., SOC. It should also be noted that the conjecture on the occurrence of SOC behavior in the Game of Life [13], i.e., a cellular automata which simulates the evolutionary dynamics of a population [5] according to similar rules than *rule 2* of the present work, has originated an interesting debate [9,14,15]. The most reliable conclusion is that Life is subcritical, but with a rather long length scale. That is, power laws can be obtained in spatially small lattices, and the SOC behavior disappears for larger systems due to the low density and limited displacements of the living sites [15]. These constraints do not operate in the present model, which exhibits robust critical behavior, because, in contrast to Life, one has *dense* flocks of self-driven individuals exhibiting *high mobility*. Under this circumstance one can observe large avalanches which eventually may cause the replacement of the population, even in the largest lattice used [12], evidence which further supports the claim that the stationary state is critical.

Summing up, a model which describes the self-organized cooperative displacement of self-replicating

individuals is proposed and studied. The emergence of a very rich and complex critical behavior at global scale is originated in simple local rules. So, this model exhibits relevant property characteristics of actual biological systems and it can easily be generalized to account for more realistic situations.

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*Electronic address: ealbano@isis.unlp.edu.ar

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- [6] The stationary state (SS) can quickly be achieved when the starting density $\rho_g(0)$ lies within the range $N1/(\pi R^2) < \rho_g(0) < N2/(\pi R^2)$. A simple approach may suggest that outside that range the SS will not be achieved because the system may evolve to extinction either by isolation or by overcrowding. However, a detailed study of the spreading of *single* colonies of N_c individuals ($N1 < N_c < N2$) reveals that roughly 70% of such colonies can successfully spread the population over the whole system achieving the SS. So, even if $\rho_g(0)$ is outside of the favorable range, due to fluctuations in the stochastic distribution of individuals, the occurrence of small patches "colonies" having a favorable local density may be expected. Thus, while the remaining system becomes annihilated, the SS could be established by these colonies even under unfavorable initial conditions.
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