Textured Edges in Quantum Hall Systems

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We have investigated the formation of spin textures at the edges of quantum Hall systems for several ferromagnetic filling factors. The textures are driven by the same physics that leads to "Skyrmions" in the bulk. For hard confinement and large Zeeman energies, the edges are narrow and spin polarized. Away from this limit and for $\nu = 1/3$, $1/5$, and 1, we find that they widen by developing spin textures. In contrast, for $\nu = 3$, the edge remains spin polarized, even as it reconstructs. We comment on the mode structure of the reconstructed edges and on possible experimental signatures.
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Progress in device fabrication and theoretical interest in moving beyond the basic physics of the quantum Hall effect (QHE) have recently converged in a flurry of activity on multicomponent systems [1]. One of the new themes in this work is the existence of excitations that involve topologically nontrivial configurations of the components. The first examples to be uncovered were "Skyrmions" of the spin degrees of freedom [2] and there is now a considerable amount of experimental evidence for their existence [3–5]. Meanwhile, the Indiana group has studied pseudospin textures in double layer systems [6] and used them to account for a novel phase transition observed as a function of a parallel magnetic field [7].

The physics here is a combination of magnetism, Berry phases, and the density-flux commensuration that is at the heart of the QHE. As a consequence, at ferromagnetic QH fillings, long wavelength variations in the density are most cheaply accomplished by texturing the spins [2,6]. In this Letter we discuss another instance where this tradeoff might be expected to exist, i.e., the edges of QH systems. For a confining potential that rises steeply at the edge, the density falls steeply from its bulk value and the width of the edge region is of order the magnetic length $\ell = \sqrt{\hbar c/eB}$; most work on edge states [8] assumes this simple structure. As the confinement is softened, the edge will reconstruct by the outward motion of the charge and will become wider in the process [9–11]. The precise question we address is the following: For the edges of ferromagnetic QH states [12], under what circumstances does edge reconstruction take place by texture formation?

A subtlety here [13] is that the physics of textures is most evident for long wavelength density fluctuations. Whether it survives at wavelengths of the order of ℓ is an issue of detail and must be settled by explicit calculations. Consequently, we have searched for edge texture formation using both Hartree-Fock and effective action based calculations at $\nu = 1$, 3, $1/3$, and $1/5$. In addition to the width of the edge region, the other important parameter in the problem is the ratio, $\tilde{g} \equiv g\mu_B B/(e^2/\epsilon\ell)$, of the Zeeman energy, $g\mu_B B$, to the typical Coulomb energy, $e^2/\epsilon\ell$; clearly, for large enough $\tilde{g}$ all states are spin polarized.

Our chief results are the following: (i) At $\nu = 1$ and for $\tilde{g}$ less than a critical value $\tilde{g}_c$, the formation of an edge texture preempts the polarized instability [9,10]. (ii) In contrast, at $\nu = 3$, the edge remains polarized even as it reconstructs. This distinction between $\nu = 1$ and $\nu = 3$ parallels that for Skyrmions in the bulk [13] and strongly suggests that spin textures govern the low energy physics at small Zeeman energies only for states in the lowest Landau level, i.e., for $\nu = 1$, $\nu = 1/3$, and $1/5$. (iii) The polarized edge is invariant under spin rotations about the magnetic field as well as under translations along the edge. The textured edge is invariant only under a specific linear combination of these symmetries whereas the orthogonal combination is spontaneously broken. As a consequence, we expect that the reconstructed edge has an additional gapless mode. We now turn to the details of our results.

Edge Hamiltonian and polarized instability.—We consider a two-dimensional electron gas in a magnetic field $\mathbf{B}$ and restrict the orbital Hilbert space to the highest occupied Landau level, n. The Hamiltonian is constructed by taking matrix elements of the Coulomb interaction $V(\mathbf{r}) = e^2/(\epsilon|\mathbf{r}|)$ between states in the nth Landau level, in the presence of a specified background charge $\rho_b(\mathbf{r})$ which confines the electron gas. (For $n > 0$, ρ_b is taken to include the charge in all lower, fully occupied Landau levels.) We focus, in the present Letter, on a semi-infinite system with a straight edge; similar effects in quantum dots will be discussed elsewhere [14]. For this geometry, it is convenient to work in Landau gauge ($\mathbf{A} = Bx\hat{\mathbf{y}}$) with periodic boundary conditions in the $\hat{\mathbf{y}}$ direction; the field operators can thus be expanded as $\psi_\sigma = \sum_p \varphi_p c_{p\sigma}$, where $c_{p\sigma}$ annihilates an electron with wave vector p and spin σ. For the $(n+1)$st Landau level and a system of finite length L along the edge, $\varphi_p = (2^n n! \sqrt{\pi} L\ell)^{-1/2} e^{ipy} H_n(\frac{x}{\ell} - p\ell) e^{-(\frac{x}{\ell} - p\ell)^2/2}$, where H_n

is the nth Hermite polynomial and $p = 2\pi n_p/L$, $n_p = 0, \pm1, \pm2$, etc. We take ρ_b to fall linearly from its bulk value $\overline{\rho}$ to zero over a region of width w (Fig. 1), so the "softness" of the edge is set by the dimensionless ratio $\tilde{w} \equiv w/l$.

Chamon and Wen studied the stability of the straight $\nu = 1$ edge as a function of $\tilde{w}$ for polarized electrons [10]. For small $\tilde{w}$ the edge is sharp; the occupation of the up spin one-particle states is unity out to a maximum wave vector and zero thereafter; hence the density falls to zero over a distance of $O(\ell)$ in real space. At $\tilde{w} = \tilde{w}_p = 9.0$ (in Hartree-Fock) an "edge reconstruction" occurs where a lump of QH liquid is split off and deposited a distance $\sim\ell$ from the bulk.

Edge textures.—We study the stability of polarized edges against formation of spin textures by two different methods. The first utilizes an effective action for the spin degrees of freedom at ferromagnetic QH states obtained after the charge dynamics have been integrated out. The Lagrangian density has the form [2,6],

$$\begin{aligned} \mathcal{L}_{\rm eff} = {}& \frac{1}{2}\,\overline{\rho} A(\mathbf{n})\partial_t \mathbf{n} - \frac{1}{2}\,\rho^s(\nabla\mathbf{n})^2 + g\overline{\rho}\mu_B \mathbf{n}\cdot\mathbf{B} \\ & - \frac{\nu_{\rm FM}^2}{2}\int d^2r'[\overline{\rho} - q(\mathbf{r}) - \rho_b(\mathbf{r})] \\ & \times V(\mathbf{r}-\mathbf{r}')[\overline{\rho} - q(\mathbf{r}') - \rho_b(\mathbf{r}')], \end{aligned} \tag{1}$$

where the electron density is the difference $\overline{\rho} - q$, ρ^s is the spin stiffness [15], and $\nu_{\rm FM}$ is the filling factor of the ferromagnetic component (e.g., $\nu_{\rm FM} = 1$ at $\nu = 3$). The spin is described by the unit vector $\mathbf{n}(\mathbf{r})$, $\mathbf{A}(\mathbf{n})$ is the vector potential of a unit monopole, i.e., $\epsilon^{abc}\partial_b A_c = n^a$ and $q(\mathbf{r}) = \mathbf{n}\cdot(\partial_x\mathbf{n}\times\partial_y\mathbf{n})/4\pi$ is the topological (Pontryagin) density of the spin field.

The applicability of this effective action to the edges of a quantum Hall system is not evident. The derivation presented in [2] assumed gapped density fluctuations and the absence of currents, both of which are problematic at the edge. It is our belief, supported by the results of microscopic calculations presented later, that for the *statics* of the textured edge this is not a serious limitation. The energetics of the textures are quite robust and are captured well by the static terms in the effective action.

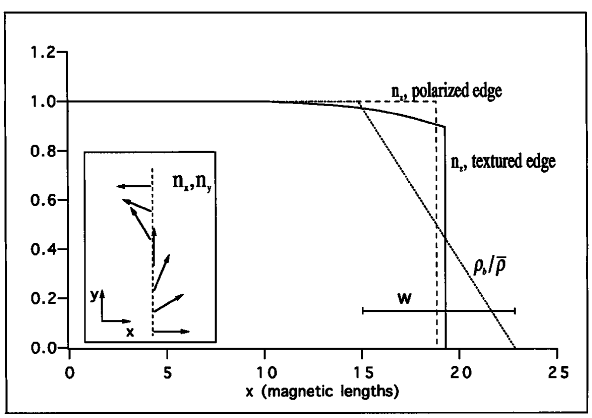

FIG. 1. Sketch of ρ_b and n_z perpendicular to the edge. The inset shows how the planar components of the spin rotate along the textured edge.

Edge textures are configurations of the spin field that possess a topological density at the edge of the system. Denoting the directions perpendicular and parallel to the edge by x and y and the magnetic field axis by z, the spin field takes the form

$$\begin{aligned} n_x &= \sqrt{1 - f^2(x)}\,\cos(ky + \theta_0), \\ n_y &= \sqrt{1 - f^2(x)}\,\sin(ky + \theta_0), \qquad n_z = f(x), \end{aligned} \tag{2}$$

where k and θ_0 are constants and $f(x)$ falls from its value $f(-\infty) = 1$ in the bulk polarized state as the edge is approached. Evidently, the spins tilt away from the direction of the field on going across the edge while they precess about it with wave vector k along the edge; the trivial case, $f \equiv 1$, is the polarized edge (Fig. 1). The topological density of the texture, $q = -(k/4\pi)df/dx$, is proportional both to the gradient of n_z and to the edge wave vector. Note that although q, and hence the electron density $\rho = \overline{\rho} - q$, is constant along the edge, the form (2) breaks translational invariance along the edge, generated by $t_y = -i\partial_y$, as well as spin rotational symmetry about the magnetic field generated by $L_z = \sigma_y$ acting on (n_x, n_y). However, the state (2) is invariant under the combined symmetry generated by $t_y + kL_z$. Thus there is one broken continuous symmetry and the symmetry related states are labeled by θ_0.

To find the optimal texture, we determine $f(x)$ by numerically integrating the equation of motions using a relaxational procedure for a fixed k, which is then fixed by minimizing the energy.

Hartree-Fock technique.—The second method used to search for textured edges is the Hartree-Fock technique introduced by Fertig *et al.* for Skyrmions [16,17]. The form of the wave function is fixed, by the constraints of lowest Landau level occupation and the conservation of $t_y + kL_z$ noted earlier, to be

$$|k\rangle = \prod_p (u_p c^\dagger_{p\uparrow} + v_p c^\dagger_{p+k\downarrow})|0\rangle. \tag{3}$$

The coefficients (u_p, v_p) are determined by numerically iterating the Hartree-Fock equations until a self-consistent solution is found. The wave vector k is again determined by minimizing the energy. Recovering the polarized bulk state requires that $u_p \to 1$ deep in the bulk but its values at the edge are unconstrained. The numerical results in this Letter are for a system of length $L = 100\ell$; we have checked that they are not sensitive to this choice.

We begin with our results on $\nu = 1$ which are illustrated in Figs. 2 and 3. Within one Landau level the dimensionless parameters $\tilde{g}$ and $\tilde{w}$ control the physics.

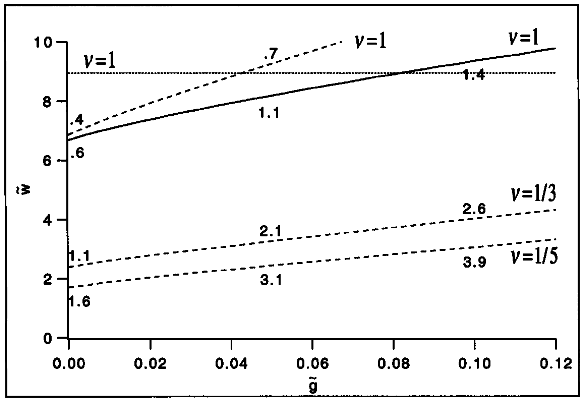

FIG. 2. Stability diagram for the $\nu = 1$ edge showing the region of stability of the sharp edge and the regions where it is unstable to texture formation and to a polarized reconstruction. The solid line marks the onset of the textured edge within Hartree-Fock theory and the dashed line within the effective action calculations. The sharp edge becomes unstable to a polarized reconstruction for $\tilde{w} > \tilde{w}_p = 9.0$ (dotted line) independent of $\tilde{g}$. The wave vectors for the initial textures are indicated along the boundary. Also included are effective action predictions for the onset of the textured edge at $\nu = 1/3$ and $1/5$ (dashed lines).

Figure 2 shows the stability diagram of the edge in the $(\tilde{g}, \tilde{w})$ plane, with phase boundaries obtained both from Hartree-Fock and effective action calculations. The close agreement between these methods at small $\tilde{g}$ where the textures are expected to be slowly varying is encouraging and in line with similar calculations for bulk Skyrmions [14]. The Hartree-Fock treatment represents the sharp edge exactly and hence gives an upper bound for the critical $\tilde{w}$ at a fixed $\tilde{g}$.

Notice that for small Zeeman energies, $\tilde{g} < \tilde{g}_c = 0.082$, and softening confinement, the sharp edge becomes unstable to a textured reconstruction *before* it becomes unstable to polarized reconstruction. In particular, at $\tilde{g} = 0$, the transition to a textured edge with wave vector $k = 0.63$ occurs at $\tilde{w} = 6.7$ while the purely polarized reconstruction happens only at $\tilde{w}_p = 9.0$. As $\tilde{g}$ increases, the critical $\tilde{w}$ and k increase as well till finally at $\tilde{g} = \tilde{g}_c$, the polarized instability becomes the primary instability. The quoted numbers are from Hartree-Fock theory; the effective theory gives that the $\tilde{g} = 0$ transition occurs at $\tilde{w} = 6.9$ with $k = 0.40$. We remind the reader that the effective theory does not allow a computation of the polarized instability and hence of $\tilde{g}_c$. In contrast the Hartree-Fock wave functions allow for both possibilities.

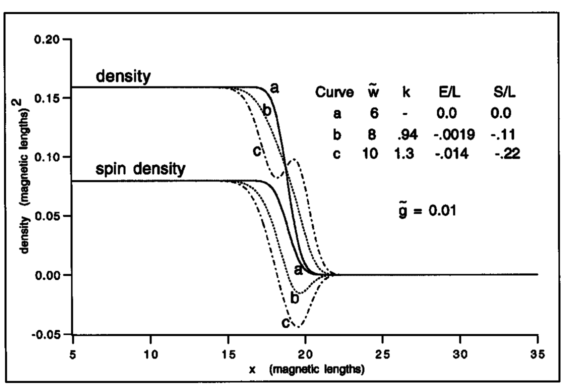

FIG. 3. Density and spin density as a function of x (distance perpendicular to the edge) for $\nu = 1$, $\tilde{g} = 0.01$, and a set of $\tilde{w}$. The curves labeled a are for the polarized edge while c are for a textured edge beyond the polarized instability where the former is still energetically preferred. The table shows the wave vector k, as a well as the spin, S/L, and energy, $E/(Le^2/\epsilon\ell)$, per unit length relative to the polarized edge.

The reconstruction in going across the phase boundary is continuous, as illustrated in Fig. 3 where we show (two-dimensional) density and spin-density profiles. The density profile evolves smoothly from that of the sharp edge towards the fragmented edge produced by polarized reconstruction. As the initial instability involves moving a small amount of charge locally, textured reconstruction is really an edge instability; in contrast, for $\tilde{g} > g_c$, the polarized instability sets in when the edge is still locally stable [9,10]. The inset table in Fig. 3 details the increase in the edge wave vector and the depolarization of the edge away from the phase boundary.

For $\nu \neq 1$, it is straightforward to extend the effective action calculations and one again finds instabilities of the sharp edge to texturing (Fig. 2). At $\tilde{g} = 0$ these set in at critical values $\tilde{w}_\nu$ where $\tilde{w}_{1/3} = 2.4$, $\tilde{w}_{1/5} = 1.7$, and $\tilde{w}_3 = 12$. The corresponding edge wave vectors are 1.11, 1.57, and 0.23. One notes that $\tilde{w}_\nu$ are related to the value at $\nu = 1$ by $\tilde{w}_\nu \approx (1/\nu_{\mathrm{FM}}^2)(\rho_\nu^s/\rho_1^s)\tilde{w}_1$, where ρ_ν^s is the spin stiffness at filling factor ν.

For $\nu = 3$, we have carried out an analogous calculation to that of Chamon and Wen to find the critical value of $\tilde{w}_p$ at which the polarized instability occurs. We find that $\tilde{w}_p = 8.3$ which is obviously less than $\tilde{w}_3$. (This is corroborated by the fact that no texturing is observed in the Hartree-Fock solutions for $\tilde{w} < \tilde{w}_p$, even at $\tilde{g} = 0$.) Consequently, the $\nu = 3$ edge (really, the innermost of its three edges) will not display texturing and should reconstruct to a polarized, fragmented edge with softening confinement.

For $\nu = 1/3$ and $1/5$ no estimate for a polarized instability is available and we do not know if it preempts texture formation. Nevertheless, the moral of the $\nu = 3$ calculations is that the relative energetics of the polarized and textured instabilities roughly follow those of polarized quasiparticles and Skyrmions [13]. This strongly suggests that they exhibit textured edges at small $\tilde{g}$ as well.

Edge dynamics.—The limitations of the effective action, detailed earlier, preclude a proper treatment of the dynamics at this time; hence, we limit ourselves to a qualitative discussion.

Close to the phase boundary, the most striking attribute of the mean field solutions is the presence of a broken symmetry. As this is a broken continuous symmetry in a one-dimensional quantum system, it is likely that the true long-range order in these solutions will be reduced to quasi- (algebraic) long-range order when quantum fluctuations are included. Consequently, the true ground state will conserve both t_y and L_z although the transverse spin-spin correlation function will show evidence of the textures discussed in this Letter. A true broken symmetry would imply an extra gapless, Goldstone mode and we expect that a modified version of this will survive fluctuations as well.

Experimental consequences.—At $\nu = 1$, the range of Zeeman energies for which there is an instability to textured reconstruction is quite large; indeed it covers all values of interest for GaAs systems ($0.005 < \tilde{g} < 0.02$ for fields between 1 and 10 T). The major experimental barrier to realizing the textured edge is tuning the confining electric field to the right range. Recent theory [18] suggests that standard samples may contain highly reconstructed edges with several channels even for integer states. If this is correct then it might be necessary to investigate systems with ultrasharp confinement produced by cleaving which have been fabricated recently [19].

The principal signatures of textured edges are likely to be their sensitivity to the value of the Zeeman energy and the associated depolarization of the edge. The former could be investigated by tunneling into the edge of the electron gas at various values of a tilted magnetic field. The latter might be amenable to a local NMR probe. As with Skyrmions, the contrast between $\nu = 1$ and 3 should prove to be a useful diagnostic.

Finally, we note that spin effects in the reconstruction of small droplets were found in exact diagonalization studies of small systems by Yang, MacDonald, and Johnson [20]. As we were writing up this work we received a preprint by Oaknin *et al.* [21] which discusses the existence of a branch of textured excitations for QH droplets with $\nu \simeq 1$.

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[1] For a recent, extensive review containing references to the literature, see S. M. Girvin and A. H. MacDonald, in *Novel Quantum Liquids in Low-Dimensional Semiconductor Structures,* edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1995).

[2] S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16 419 (1993); see also E. H. Rezayi, Phys. Rev. B **36**, 5454 (1987); **43**, 5944 (1991); D.-H. Lee and C. L. Kane, Phys. Rev. Lett. **64**, 1313 (1990).

[3] S E. Barrett, G. Dabbagh, L. N. Pfeiffer, K. W. West, and R. Tycko, Phys. Rev. Lett. **74**, 5112 (1995); see also R. Tycko, S. E. Barrett, G. Dabbagh, L. N. Pfeiffer, and K. W. West, Science **268**, 1460 (1995).

[4] A. Schmeller, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **75**, 4290 (1995).

[5] E. H. Aifer, B. B. Goldberg, and D. A. Broido, Phys. Rev. Lett. **76**, 680 (1996).

[6] K. Yang, K. Moon, L. Zheng, A. H. McDonald, S. M. Girvin, D. Yoshioka, and S.-C. Zhang, Phys. Rev. Lett. **72**, 732 (1994); K. Moon, H. Mori, K. Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka, and S.-C. Zhang, Phys. Rev. B **51**, 5138 (1995).

[7] S. Q. Murphy, J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **72**, 728 (1994).

[8] X. G. Wen, Int. J. Mod. Phys. B **6**, 1711 (1992).

[9] A. H. MacDonald, S. R. Eric Yang, and M. D. Johnson, Aust. J. Phys. **46**, 345 (1993).

[10] C. de C. Chamon and X. G. Wen, Phys. Rev. B **49**, 8227 (1994).

[11] Other work on edge reconstruction includes D. B. Chklovskii, L. I. Glazman, and B. I. Shklovskii, Phys. Rev. B **46**, 4026 (1992); J. Dempsey, B. Y. Gelfand, and B. I. Halperin, Phys. Rev. Lett. **70**, 3639 (1993); Y. Meir, Phys. Rev. Lett. **72**, 2624 (1994); L. Brey, Phys. Rev. B **50**, 11 861 (1994); D. B. Chklovskii, Phys. Rev. B **51**, 9895 (1995), and references therein.

[12] The same question arises for nonferromagnetic states, e.g., $\nu = 2$, which are hierarchical descendents of ferromagnetic states and hence contain them in their edge structure [14].

[13] X. G. Wu and S. L. Sondhi, Phys. Rev. B **51**, 14 725 (1995); see also J. K. Jain and X. G. Wu, Phys. Rev. B **49**, 5085 (1994).

[14] A. Karlhede, K. Lejnell, S. A. Kivelson, and S. L. Sondhi (to be published).

[15] For $\nu = 3$, 1, 1/3, and 1/5 $\rho^s/(e^2/\epsilon\ell)$ equals 4.36×10^{-2}, 2.49×10^{-2}, 9.23×10^{-4}, and 2.34×10^{-4}, respectively.

[16] H. A. Fertig, L. Brey, R. Cote, and A. H. MacDonald, Phys. Rev. B **50**, 11 018 (1994).

[17] S. L. Sondhi and A. Karlhede (unpublished).

[18] See Chklovskii *et al.* and Chklovskii in [11].

[19] L. N. Pfeiffer *et al.*, J. Cryst. Growth **127**, 849 (1993).

[20] S. R. Eric Yang, A. H. MacDonald, and M. D. Johnson, Phys. Rev. Lett. **71**, 3194 (1993).

[21] J. H. Oaknin, L. Martin-Moreno, and C. Tejedor, Report No. cond-mat 9510086.